IN PURSUIT OF A MEDIEVAL MODEL THEORY

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Logicians have always worked with some notion of logical consequence. Usually their notion of logical consequence belongs to one of the following two types. We say that θ is a logical consequence of the premises Φ if:

> (Proof-theoretic) *There is* a pattern of inference steps (a 'derivation') that leads from Φ to θ.
>
> (Model-theoretic) *Every* interpretation making Φ true (i.e. every model of Φ) also makes θ true.

The difference between 'There is' and 'Every' implies that these two notions of consequence will be used in very different ways.

The proof-theoretic notion goes back to Aristotle. The model-theoretic notion was introduced as a basic notion of model theory in papers of Abraham Robinson and Tarski in 1949–1954, after Tarski had called the attention of philosophers to this notion in a paper of 1936. In fact truth-tables (Wittgenstein and Post around 1920, Peirce a little earlier) had already introduced a propositional version of model-theoretic consequence. For countable first-order logic, the agreement between proof-theoretic and model-theoretic notions of consequence was stated and proved in Gödel's doctoral dissertation in 1929. In these ways the notion began to be used in Western logic in the 20th century, over two thousand years after Aristotle had first introduced logic. Why so late?

During the last six months it came to light that in the mid 12th century Abū al-Barakāt al-Baghdādī, a Jew based in Baghdad, had a system of syllogistic logic up and running, in which he used only the model-theoretic notion of consequence. Barakāt was already known as a perceptive philosopher and physicist—he was the first to state that bodies fall with constant acceleration. But it was not realised that he broke the mould in logic too. The present talk will be to some extent a preliminary report on this discovery.

In brief, Barakāt showed how we can deduce the conclusion of a productive syllogism (i.e. one that has a conclusion) by listing representatives of all possible models of the premises, and looking to see what propositions are true in all these models. He also invented a notation to accompany these calculations. His notation is interesting as the earliest known system of logical diagrams for proving consequences, anticipating

Leibniz by 500 years. Unlike the diagrams of Leibniz, Euler and Venn, Barakāt's diagrams represent models, not propositions.

About a hundred years before Barakāt, Ibn Sīnā (= Avicenna) started to develop model-theoretical consequence for proving non-entailments—similar to Hilbert's *Grundlagen der Geometrie*, though done entirely within logic. Unlike Barakāt, Ibn Sīnā made significant mistakes. But his models are more concrete than those of Barakāt, and there are clear signs that he took them to consist of a set on which relations are defined, just as in today's model theory.

In keeping with the interest in 'perspectives' at this meeting, we discuss how this sudden appearance of model theory in the 11th and 12th centuries, and its equally sudden disappearance after the death of Barakāt, make sense within the history of logic as a whole. For example, what logical tools needed to be developed to sustain model theory, and why was there an incentive to build these tools in the 20th century but not in the 13th? We note that Barakāt's ideas were in a sense already implicit in Aristotle's work in the 4th century BC. This raises further questions: can we trace a development from Aristotle to Barakāt? (The 6th century logician Paul the Persian is a likely intermediary.) Why was there no development along these lines for maybe a thousand years after Aristotle himself?

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