

# UNIVERSALLY BAIRE SETS AND BOREL CANONIZATION

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Using the axiom of choice, one can construct sets of reals which are quite pathological: e.g. non-measurable, not having the Baire property etc. But the guiding principle of descriptive set theory is that if the set is “nicely” definable then it is not pathological. An possible definition of the maximal family of “nice” sets of reals (or of any Polish space) is the family of Universally Baire sets, introduced in [4].

In this talk we shall survey the definition and some of the basic “niceness” properties of the family of universally Baire subsets of a Polish space. Some of these properties depends on the assumptions of strong axioms of infinity. (“The existence of large cardinals”).

As an example of the regularity properties of universally Baire sets, we shall discuss the problem of Borel canonization. This problem was introduced by Kanovei, Sabok and Zapletal ([5]). In the original setting we are given an analytic equivalence relation  $E$  and an ideal  $I$  on the reals. The problem is to find a Borel set  $B$  which is not in the ideal such that  $E$  restricted to  $B$  is Borel. In this generality the answer is “NO”, but if we put some “nicety” conditions on  $I$  and the equivalence relation  $E$  one can get a positive answer, assuming some large cardinals. (These results are due to W. Chan and O. Drucker, independently: [3] and [1].)

In the talk we shall survey some possible generalizations of these results. For instance when we assume that the relation  $E$  is universally Baire. (Some of the results are joint results with W. Chan [2].)

## REFERENCES

- [1] W. Chan. Equivalence relations which are Borel somewhere. *Journal of Symbolic Logic* 82(3):893–930, 2017. DOI: [10.1017/jsl.2017.22](https://doi.org/10.1017/jsl.2017.22)
- [2] W. Chan and M. Magidor When an equivalence relation with all Borel classes will be Borel somewhere? Preprint arXiv:1608.04913v1, 2016. Available at <https://arxiv.org/>
- [3] O. Drucker. Borel canonization of analytic sets with Borel sections. Preprint arXiv:1512.06368, 2016. Available at <https://arxiv.org/>
- [4] Q. Feng, M. Magidor and H. Woodin. Universally Baire sets of reals. In: H. Judah, W. Just and H. Woodin (eds.), *Set Theory of the Continuum*, Springer, 1992, pp. 203–242. DOI: [10.1007/978-1-4613-9754-0\\_15](https://doi.org/10.1007/978-1-4613-9754-0_15)
- [5] V. Kanovei, M. Sabok and J. Zapletal. *Canonical Ramsey Theory on Polish Spaces*, Cambridge Tracts in Mathematics 202. Cambridge University Press, 2013. DOI: [10.1017/CBO9781139208666](https://doi.org/10.1017/CBO9781139208666)

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