ON THE COMPUTATIONAL CONTENT OF THEOREMS

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To analyze the computational content of theorems is a research topic at least since Turing's seminal work on computable numbers in which he started the investigation of computable versions of theorems in analysis. In the sequel this topic was taken up by many other researchers such as Specker, Lacombe, Shore and Nerode, Pour-El and Richards [2], and Weihrauch [4]. A related but formally different approach has been started by Friedman and Simpson [3] who have characterized axioms that are sufficient and often necessary to prove certain theorems in second-order arithmetic. In recent years the interaction between these two research trends has been intensified and overlaps in what is called Weihrauch complexity. Weihrauch complexity is a computability theoretic approach to the classification of the computational content of theorems that yields results that can be seen as a uniform and resource sensitive version of reverse mathematics. The benefit of this theory is that it yields fine grained computational results that answer typical questions from the computable analysis perspective, while being compatible with reverse mathematics. Sometimes results can be imported from reverse mathematics and computable analysis, but often completely new methods and techniques are required. We will present a survey on this approach that is based on a recent survey article [1] on this topic.

References

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