Generic torus orbit closures in Schubert varieties

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The standard action of a torus $(\mathbb{C}^*)^n$ on $\mathbb{C}^n$ induces an action of $(\mathbb{C}^*)^n$ on the flag variety $\mathcal{F}l(\mathbb{C}^n)$. As is well-known, if a torus orbit is *generic* in $\mathcal{F}l(\mathbb{C}^n)$, then its closure is a smooth toric variety called a permutohedral variety. If a torus orbit is not generic, its closure is not necessarily smooth but normal ([1]); so any torus orbit closure in $\mathcal{F}l(\mathbb{C}^n)$ is a toric variety. Therefore we are naturally led to study toric varieties which appear as torus orbit closures in $\mathcal{F}l(\mathbb{C}^n)$.

The Schubert variety X_w associated to a permutation w on n letters is a torus invariant subvariety of $\mathcal{F}l(\mathbb{C}^n)$. In this talk, I will define a *generic* torus orbit in X_w and discuss the fan associated to its closure. It turns out that the generic torus orbit closure in X_w is not necessarily smooth and the smoothness at the fixed point w is equivalent to acyclicity of a graph associated to w. As a result, we will see that the smoothness of the generic torus orbit closure in X_w is closely related to (but not necessarily same as) the smoothness of X_w. This is joint work with Eunjeong Lee.

References

[1] J. B. Carrell, Normality of torus orbit closures in G/P, *J. of Algebra* **233** (2000), 122–134.