Cyclohedron and Kantorovich-Rubinstein polytopes

Rade T. Živaljević *(Mathematical Institute of the Serbian Academy of Sciences and Arts),*
rade@mi.sanu.ac.rs

Theorem 1 ([2, Theorem 31]) *There exists a quasi-metric (asymmetric distance function) ρ on the set $[n]$ such that the associated Kantorovich-Rubinstein polytope (introduced in [3]),*

$$KR(\rho) = \mathrm{Conv}\left\{\frac{e_i - e_j}{\rho(i,j)} \mid 1 \leqslant i \neq j \leqslant n\right\}$$

is affinely isomorphic *to the dual W_n° of the cyclohedron W_n.*

A close relative of Theorem 1 is the following theorem. .

Theorem 2 *There exists a triangulation of the boundary of the $(n-1)$-dimensional type A root polytope $Root_n$ parameterized by proper faces of the $(n-1)$-dimensional cyclohedron. More explictly there exists a map $\phi_n : \partial(W_n^\circ) \to \partial(Root_n)$, inducing a piecewise linear homeomorphism of boundary spheres of polytopes W_n° and $Root_n$. The map ϕ_n sends bijectively vertices of $\partial(W_n^\circ)$ to vertices of the polytope $Root_n$, while higher dimensional faces of $Root_n$ are triangulated by images of simplices from $\partial(W_n^\circ)$.*

In the lecture we will explore topological and combinatorial consequences of these results, for example the map described in Theorem 2 defines a 'canonical' quasi-toric manifold over a cyclohedron W_n.

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References

[1] J. Gordon, F. Petrov. Combinatorics of the Lipschitz polytope. *Arnold Math. J.* **3**(2), 205–218 (2017).

[2] F.D. Jevtić, M. Jelić, R.T. Živaljević. Cyclohedron and Kantorovich-Rubinstein polytopes, *Arnold Math. J.* (accepted for publication).

[3] A.M. Vershik. Classification of finite metric spaces and combinatorics of convex polytopes. *Arnold Math. J.* **1**(1),75–81, 2015.