Billiards within quadrics and Chebyshev type polynomials

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A comprehensive study of periodic trajectories of billiards within ellipsoids in d-dimensional Euclidean space is presented. The novelty of the approach is based on a relationship established between periodic billiard trajectories and extremal polynomials on the systems of d intervals on the real line. By leveraging deep, but yet not widely known results of the theory of generalized Chebyshev polynomials, fundamental properties of billiard dynamics are proven for any d, viz., the monotonicity of sequences of winding numbers and the injectivity of frequency maps. As a byproduct, for $d = 2$ a new proof of the monotonicity of the rotation number is obtained and the result is generalized for any d. The case study of trajectories of small periods T, $d \leq T \leq 2d$ is given. It is proven that all d-periodic trajectories are contained in a coordinate-hyperplane and that for a given ellipsoid, there is a unique set of caustics which generates $d + 1$-periodic trajectories. A complete catalog of trajectories with small periods is provided for $d = 3$. This is a joint work with M. Radnović [1].

References

[1] V. Dragović, M. Radnović, Periodic ellipsoidal billiard trajectories and extremal polynomials, 2018, arXiv:1804.02515.