Cohomology formulae of real toric spaces

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For a simplicial complex K on $[m]$ and a mod 2 simplicial complex $\lambda\colon \mathbb{Z}_2^m \to \mathbb{Z}_2^n$, we have the associated real toric space $M^{\mathbb{R}}(K,\lambda) := \mathbb{R}\mathcal{Z}_K/\ker\lambda$.

In this talk, we provide an explicit R-cohomology ring formula of a real toric space in terms of K and Λ, where R is a commutative ring with unity in which 2 is a unit. Interestingly, it has a natural $(\mathbb{Z}\oplus \operatorname{row}\Lambda)$-grading.

Theorem 1 *There are $(\mathbb{Z}\oplus\operatorname{row}\Lambda)$-graded R-algebra isomorphisms*

$$H^*(M) \cong \bigoplus_{\omega\in\operatorname{row}\Lambda} \widetilde{H}^{*-1}(K_\omega),$$

where the product structure on $\bigoplus_{\omega\in\operatorname{row}\Lambda}\widetilde{H}^{*-1}(K_\omega)$ is given by the canonical maps

$$\widetilde{H}^{k-1}(K_{\omega_1})\otimes\widetilde{H}^{\ell-1}(K_{\omega_2}) \to \widetilde{H}^{k+\ell-1}(K_{\omega_1+\omega_2})$$

which are induced by simplicial maps $K_{\omega_1+\omega_2} \to K_{\omega_1}\star K_{\omega_2}$ when $\star$ denotes the simplicial join.

If time allows, we also discuss about the integral cohomology of real toric space. This work is partially jointly with Hanchul Park [1] and Li Cai [2].

References

[1] Suyoung Choi and Hanchul Park, *Multiplicative structure of the cohomology ring of real toric spaces*, preprint. arXiv:1711.04983

[2] Li Cai and Suyoung Choi, *Integral cohomology groups of real toric manifolds and small covers*, preprint. arXiv:1604.06988