Surgery and cell-like maps

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The main goal of surgery theory is the classification of manifolds and manifold structures. The structure set $\mathcal{S}^{CAT}(X)$ of the Poincare complex X measures the number of distinct CAT-manifolds in the simple homotopy class of X where CAT is the category. The surgery theory was initiated for $CAT = DIFF$ but it works better for $CAT = TOP$. In this talk we consider the later but then we apply our results to differentiable manifolds.

Contrary to the DIFF, in the case of topological manifolds $\mathcal{S}^{TOP}(M)$ is a group. We define a subset $\mathcal{S}^{CE}(M) \subset \mathcal{S}^{TOP}(M)$ generated by homotopy equivalences $h : N \to M$ that come as homotopy lifts of g in the diagram

$$\begin{array}{ccc} N & \xrightarrow{h} & M \\ {\scriptstyle g}\downarrow & & {\scriptstyle f}\downarrow \\ X & \xrightarrow{=} & X \end{array} \tag{1}$$

where f and g are cell-like maps. Quinn's theorem implies that if X is finite dimensional then h is homotopic to a homeomorphism and hence h defines a trivial element $[h] = 0 \in \mathcal{S}^{TOP}(M)$. Thus, to have $\mathcal{S}^{CE}(M) \neq \emptyset$ one needs use cell-like maps that raise dimension to infinity. Such maps were constructed in the 80s [2]. The construction is based on Edwards' theorem and results of Anderson-Hodgkin and Buchstaber-Mishchenko [1]. The important feature of the construction is that a cell-like map of a manifold can kill a K-theory class [3].

We give a complete description of $\mathcal{S}^{CE}(M)$ which is a bit technical. A special case of that is

Theorem 1 *For any manifold M the set $\mathcal{S}^{CE}(M)$ is a group.*

For a simply connected manifold M with finite $\pi_2(M)$ the group $\mathcal{S}^{CE}(M)$ equals the odd torsion subgroup of $\mathcal{S}^{TOP}(M)$.

As a corollary we construct two smooth nonhomeomorphic manifolds that admit cell-like maps with the same image. We use this result to construct exotic convergence of Reimannian manifolds in the Gromov-Hausdorff moduli space.

This is a joint work with Steve Ferry and Shmuel Weinberger [4].

References

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