Combinatorial Hopf algebras and generalized permutohedra

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Hopf algebra structures naturally arise if we know how to compose and decompose combinatorial objects. In addition, a multiplicative functional gives rice to a combinatorial Hopf algebra. If we employ the formalism of combinatorial Hopf algebras we can reconstruct various old and obtain some new algebraic, enumerative combinatorial invariants.

The generalized Dehn-Sommerville relations are defined in an arbitrary combinatorial Hopf algebra and we solve these relations in the case of hypergraphs [?]. This is the first non-standard solution different from the classical that is given by eulerian posets.

The universal morphism of combinatorial Hopf algebras produces a quasisymmetric function invariant. To a variety of combinatorial objects we can associate convex polytopes which are generalized permutohedra. In general, this quasisymmetric function invariant has a geometric meaning as the enumerator function of lattice points associated to generalized permutohedra. The prominent example is the Stanley chromatic symmetric function of simple graphs which is interpreted as the enumerator function of lattice points associated to graphic-zonotopes. We studied the cases of nestohedra and matroid base polytopes [?], [?].

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References

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