Relative phantom maps

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De Bruijn-Erdős theorem [1] states that the chromatic number of an infinite graph equals the maximum of the chromatic numbers of its finite subgraphs. To a graph one associates a simplicial complex called the box complex, and the chromatic number of a graph is related with a homotopy invariant of its box complex called the index. Then one may ask whether the index of a box complex has the same property as the chromatic number stated in de Bruijn-Erdős theorem. This leads one to the relative version of a phantom map, where a phantom map [3] is a map from a CW-complex such that its restriction to any finite subcomplex is trivial.

In this talk, the triviality of a relative phantom map will be discussed, and criteria for triviality in terms of rational cohomology will be given. Then a problem on a relative phantom map coming from combinatorics will be partially solved.

This is joint work with K. Iriye and T. Matsushita [2].

References

[1] N. G. de Bruijn and P. Erdős, A colour problem for infinite graphs and a problem in the theory of relations, _Nederl. Akad. Wetensch. Proc. Ser. A_, **54** (1951), 369-373.

[2] K. Iriye, D. Kishimoto, and T. Matsushita, Relative phantom maps, _arXiv:1710.00475v2._

[3] C. A. McGibbon, Phantom maps, *Handbook of algebraic topology* (I.M. James, ed.), North-Holland (1995), pp. 1209-1257.