Moment-angle manifolds and linear programming

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Moment-angle manifolds have originated in [1] and studied in a greater detail in [2]. Our motivation starts with the following statement:

Lemma 1 *Any linear function on the convex polytope gives rise to a Morse function on the corresponding real moment-angle manifold (Morse-Bott in the complex case).*

So maximizing a linear function over a simple convex polytope is equivalent to maximizing some Morse function on the corresponding real moment-angle manifold.

Optimizing a linear function on a convex polytope is a very well-known problem, known as linear programming. One can use gradient descent on the moment-angle manifold to optimize a linear function on the original polytope, thus solving a linear programming problem. Convex optimization methods operating in the polytope interior are known as interior-point (or path-following) methods. They started to gain popularity with [3]; see [4].

Moment-angle manifolds are well-defined for simple polytopes which are dense in the space of all convex polytopes. Since the method is interior-point, it can be used to tackle generic convex linear problems as well.

An alternative formulation of the method is running a gradient flow on a polytope itself but using a pushforward of Riemannian metric from the moment-angle manifold to form a gradient. Riemannian metrics in the context of convex optimization have been discussed in [5].

The code for the method is available online:

- https://github.com/kustarev/malp-python (Python version);
- https://github.com/kustarev/malp-cpp (C++ version).

The code also contains examples of solving actual optimization problems: optimizing linear function on a high-dimensional simplex and solving a portfolio optimization task.

References

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