Poincaré's rotation number in dynamics and knot theory

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Let $\varphi\colon S^1 \to S^1$ be an orientation preserving homeomorphism of the circle $S^1 = \mathbb{R}/\mathbb{Z}$, and let $\tilde{\varphi}\colon \mathbb{R} \to \mathbb{R}$ be a lift of φ. Then, for each $x \in \mathbb{R}$, the limit

$$\tau(\tilde{\varphi}) := \lim_{n\to\infty} \frac{\tilde{\varphi}^n(x)}{n}$$

exists and does not depend on the choice of x. This limit is called the *translation number* of $\tilde{\varphi}$. Considered modulo integers, it is called the *rotation number* of φ.

These invariants were first defined by Poincaré and play a significant rôle in modern dynamics [1]–[3].

It turns out that Poincaré's rotation and translation numbers have useful applications in knot theory, braid theory, the theory of mapping class groups of surfaces [4]–[9]. We will overview main concepts and results in this research area.

References

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