On one–point commuting difference operators of rank one

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One-point commuting difference operators of rank 1 are considered. The coefficients in such operators depend on one functional parameter, and the degrees of shift operators in difference operators are positive. These operators are studied in the case of hyperelliptic spectral curves, where the base point coincides with a point of branching. Examples of operators with polynomial and trigonometric coefficients are constructed. Operators with polynomial coefficients are embedded in differential operators with polynomial coefficients. This construction provides a new method for constructing commutative subalgebras in the first Weyl algebra. A relationship between one-point commuting difference operators of rank 1 and one-dimensional finite-gap Schrödinger operators is investigated. In particular, a discretization of the finite-gap Lamé operators is obtained.