Diagonal complexes

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(The talk is based on a joint work with J. Gordon)

Given an n-gon, the poset of all collections of pairwise non-crossing diagonals is isomorphic to the face poset of some convex polytope called *associahedron*. We replace in this setting the n-gon (viewed as a disc with n marked points on the boundary) with an arbitrary oriented surface equipped with a number of labeled marked points ("vertices"). The surface is not necessarily closed, and may contain a number of punctures. With appropriate definitions (in a sense, we mimic the construction of associahedron) we arrive at cell complexes $\mathcal{D}$ and its barycentric subdivision $\mathcal{BD}$. If the surface is closed, the complex $\mathcal{D}$ (as well as $\mathcal{BD}$) is homotopy equivalent to the space of metric ribbon graphs $RG_{g,n}^{met}$, or, equivalently, to the decorated moduli space $\widetilde{\mathcal{M}}_{g,n}$ [2], [1]. For bordered surfaces, we prove the following:

(1) Contraction of a boundary edge does not change the homotopy type of the complex.

(2) Contraction of a boundary component to a new marked point yields a forgetful map between two diagonal complexes which is homotopy equivalent to the Kontsevich's tautological circle bundle [3]. Thus, contraction of a boundary component gives a natural simplicial model for the tautological bundle. As an application, we compute the first Chern class (also its

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powers) in combinatorial terms. The latter result is an application of the Mnev-Sharygin local combinatorial formula [4].

(3) In the same way, contraction of several boundary components corresponds to Whitney sum of the tautological bundles.

(4) Eliminating of a puncture gives rise to a bundle which equals to a surgery on the universal curve. In particular, the bundle incorporates at a time all the M. Kontsevich's tautological S^1-bundles.

References

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