Smooth actions of compact Lie groups on complex projective spaces

Krzysztof M. Pawałowski *(Adam Mickiewicz University in Poznań),* `kpa@amu.edu.pl`

The goal of the talk is to present constructions of smooth actions of compact Lie groups G on complex projective spaces such that the manifold of points fixed under the action of G on the complex projective space $\mathbb{C}P^n$ in question –

$$M = \{x \in \mathbb{C}P^n \mid g \cdot x = x \ \text{ for all } \ g \in G\}$$

– is not stably almost complex,

– is stably almost complex but not almost complex,

– is almost complex but not homotopically symplectic,

– is homotopically symplectic but not symplectic,

– is symplectic but not Kähler.

We give examples of manifolds M with specific properties listed above, and we prove that the manifolds M can occur as the fixed point sets of smooth actions of compact Lie groups on complex projective spaces.

In particular, following arguments in [4], we prove that for every compact Lie group G, there exists a smooth action of G on a complex projective space $\mathbb{C}P^n$ such that the fixed point set is not a symplectic manifold and therefore, the action of G on $\mathbb{C}P^n$ is not symplectic with respect to any symplectic structure on $\mathbb{C}P^n$.

Examples of non-symplectic smooth actions on symplectic manifolds were obtained for the first time in [2], for actions of the circle S^1 on products $\mathbb{C}P^1 \times \cdots \times \mathbb{C}P^1$ and $N \times \mathbb{C}P^1$ for some 4-dimensional closed symplectic manifold N.

The article [3] presents a description of manifolds M which can occur as the fixed point sets of smooth actions of finite Oliver groups G on complex projective spaces.

The recent work [1] (now in progress) focuses on answering the question which symplectic manifolds M can occur as the fixed point sets of symplectic actions of a given compact Lie group G on specific symplectic manifolds.

References

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