Isoperimetric inequalities for Laplace eigenvalues on the sphere and the real projective plane

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This talk contains two recent results concerning isoperimetric inequalities on the sphere and the real projective plain.

The first result (joint work with Nadirashvili) is an isoperimetric inequality for the second non-zero eigenvalue of the Laplace-Beltrami operator on the real projective plane. For a metric of unit area this eigenvalue is not greater than 20π. This value is attained in the limit by a sequence of metrics of area one on the projective plane. The limiting metric is singular and could be realized as a union of the projective plane and the sphere touching at a point, with standard metrics and the ratio of the areas $3:2$.

The second result (joint work with Karpukhin, Nadirashvili and I. Polterovich) is an isoperimetric inequality for all eigenvalues of the Laplace-Beltrami operator on the sphere. It is shown that for any positive integer k, the k-th nonzero eigenvalue of the Laplace-Beltrami operator on the two-dimensional sphere endowed with a Riemannian metric of unit area, is maximized in the limit by a sequence of metrics converging to a union of k touching identical round spheres. This proves a conjecture posed by Nadirashvili in 2002 and yields a sharp isoperimetric inequality for all nonzero eigenvalues of the Laplacian on a sphere. Earlier, the result was known only for $k = 1$ (J. Hersch, 1970), $k = 2$ (N. Nadirashvili, 2002; R. Petrides, 2014) and $k = 3$ (N. Nadirashvili and Y. Sire, 2017). In particular, it is proven that for any $k \geqslant 2$, the supremum of the k-th nonzero eigenvalue on a sphere of unit area is not attained in the class of Riemannian metrics which are smooth outsitde a finite set of conical singularities.