On Zeros of Yamada Polynomial for Spatial Graphs

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Polynomial invariants of knots, links, and spatial graphs are studied from various points of views. One of interesting problems is to describe the distribution of zeros of the polynomial invariant.

The mostly investigated case is the Jones polynomial for a knot. The roots of the Jones polynomial for all prime knots with at most ten crossings were computed numerically in [1], that lead to interesting observations. In [2] was shown that zeros of Jones polynomials of (pretzel) links are dense in the whole complex plane.

In [3] S. Yamada introduced a polynomial invariant for spatial graphs which is known now as Yamada polynomial. The behaviour of Yamada polynomial under replacing of an edge by a sub-diagram of a link was described in [4] for some classes of graph. Using the exact formulae of Yamada polynomial for some classes of spatial graphs we get the following result.

Theorem 1 [4] *Zeros of the Yamada polynomial for spatial graphs are dense in the following region:*

$$\Omega = \big\{ z \in \mathbb{C} \,:\, |z+1+z^{-1}| \geq \\ \min\{1, |z^3+2z^2+z+1|, |1+z^{-1}+2z^{-2}+z^{-3}|\}\big\}.$$

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We will also discuss the extension of the method from [4] which leads to the proof of density of Yamada polynomial for spatial graphs in the whole complex plane.

References

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