Microformal geometry and homotopy algebras

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I will speak about the notion of a "thick morphism", which generalizes ordinary smooth maps. Like ordinary maps, a thick morphism induces an action on functions (pullback), but unlike the familiar case, such pullbacks are, in general, non-linear transformations. They have the form of formal non-linear differential operators and are constructed by some perturbative procedure. (Thick morphisms themselves are defined as formal canonical relations between the cotangent bundles specified by generating functions of particular type.) Being non-linear, these pullbacks cannot be algebra homomorphisms; however, their derivatives at each point turn out to be homomorphisms.

The non-linearity is a feature essential for application to homotopy bracket structures on manifolds. Roughly, "non-linearity" = "homotopy". A thick morphism intertwining odd master Hamiltonians of two S_∞-structures (which is practically described by a Hamilton-Jacobi type equation for the generating function) induces an L_∞-morphism of the corresponding homotopy Poisson algebras. Application to homotopy Poisson structures was our primary motivation; but there are also applications to vector bundles and Lie algebroids.

There are two parallel versions: "bosonic" (for even functions) and "fermionic" (for odd functions). The bosonic version has a quantum counterpart. "Quantum pullbacks" have the form of particular Fourier integral operators. There is also an application to "quantum brackets" induced by BV-type operators.

References

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