Converse of Smith Theory

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In 1942, P. A. Smith [3] showed that the fixed point of a p-group action on a finite $\mathbb{Z}_p$-acyclic complex is still $\mathbb{Z}_p$-acyclic. In 1971, Lowell Jones studied the converse problem and showed that any $\mathbb{Z}_p$-acyclic finite CW-complex is the fixed point of a $\mathbb{Z}_p$-action on a finite contractible CW-complex. In 1974, Robert Oliver [2] extended Jones' work to the problem that, for a given finite group G and a finite CW-complex F, whether F is the fixed point of a (semi-free or general) action of G on a finite contractible CW-complex.

We study the following problem. Suppose G is a finite group, and $f: F \to Y$ is a map between finite CW-complexes. Is it possible to extend F to a finite G-CW complex X satisfying $X^G = F$, and extend f to a G-map $g: X \to Y$ (G acts trivially on Y), such that g is a homotopy equivalence after forgetting the G-action? The work of Jones and Oliver can be regarded as the special case that Y is a point.

In case of general G-action, we find that Oliver's theory largely remains true. In case of semi-free G-action, the problem has an obstruction in K_0, and we calculate some examples.

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References

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