Iterated higher Whitehead products in topology of moment-angle complexes

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The moment-angle complex $\mathcal{Z}_\mathcal{K}$ is a cell complex built of produts of polydiscs and tori parametrised by simplices in a finite simplicial complex $\mathcal{K}$. The moment-angle complex is a special case of polyhedral products that are interesting in themselves. Polyhedral products provide a wonderful basis for applying the unstable homotopy theory methods.

In this talk we will study the topological structure of moment-angle complexes $\mathcal{Z}_\mathcal{K}$ from the point of view of iterated higher Whitehead products. Higher Whitehead products in the homotopy groups of moment-angle complexes and polyhedral products were first studied by T. Panov and N. Ray in [8]. They obtained structural results and proposed several problems, some of which will be discussed in the talk. Further important results on the structure of higher Whitehead products for special classes of simplicial complexes were obtained in the works of Grbic and Therialut [6], Iriye and Kishimoto [7].

Consider two classes of simplicial complexes. The first class B_Δ consists of simplicial complexes $\mathcal{K}$ for which $\mathcal{Z}_\mathcal{K}$ is homotopy equivalent to a wedge spheres. The second class W_Δ consists of $\mathcal{K} \in B_\Delta$ such that all spheres in the wedge are

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realized by iterated higher Whitehead products. Buchstaber and Panov asked in [4, Problem 8.4.5] if it is true that $B_\Delta = W_\Delta$.

In this talk we will show that this is not the case.

Theorem 1 *Let $\mathcal{K}$ be the simplicial complex $(\partial\Delta^2 * \partial\Delta^2) \cup \Delta^2 \cup \Delta^2$. The moment-angle complex $\mathcal{Z}_\mathcal{K}$ is homotopy equivalent to a wedge of spheres $(S^7)^{\vee 6} \vee (S^8)^{\vee 6} \vee (S^9)^{\vee 2} \vee S^{10}$, but the sphere $S^{10} \subset \mathcal{Z}_\mathcal{K}$ cannot be realized by a linear combination of iterated higher Whitehead products.*

On the other hand, we show that the class W_Δ is large enough.

Theorem 2 *Let $\mathcal{K} \in W_\Delta$. Then the simplicial complex $\mathcal{J}_n(\mathcal{K}) = (\partial\Delta^n * \mathcal{K}) \cup \Delta^n$ also belongs to W_Δ.*

Theorem 3 *If $\mathcal{K}_1, \mathcal{K}_2 \in W_\Delta$ then $\mathcal{K} = \mathcal{K}_1 \cup_I \mathcal{K}_2 \in W_\Delta$ for any common face I.*

Then using these operations we prove that there exists a simplicial complex that realizes any given iterated higher Whitehead product. Also, we describe the smallest simplicial complex that realizes the given product.

References

[1] S. Abramyan, Iterated Higher Whitehead products in topology of moment-angle complexes, *arXiv:1708.01694*.

[2] V. Buchstaber,T. Panov, *Toric Topology*, Math. Surv. and Monogr., 204, Amer. Math. Soc., Providence, RI, 2015.

[3] J. Grbić, T. Panov, S. Theriault, J. Wu, Homotopy types of moment-angle complexes for flag complexes, *Trans. Amer. Math. Soc.* **368**:9 (2016), 6663–6682.

[4] J. Grbić, S. Theriault, Homotopy theory in toric topology, *Uspekhi Mat. Nauk*, **71**:2(428) (2016), 3–80; *Russian Math. Surveys*, **71**:2 (2016), 185–251.

[5] K. Iriye, D. Kishimoto, Polyhedral products for shifted complexes and higher Whitehead products, *arXiv:1505.04892*.

[6] T. Panov, N. Ray, Categorical aspects of toric topology, Toric topology, 293–322, *Contemp. Math.*, 460, Amer. Math. Soc., Providence, RI, 2008.