Totally normally split quasitoric manifolds

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A closed stably complex manifold M is called a totally normally split, or TNS-manifold for short, if its stably normal vector bundle $NM \to M$ is stably isomorphic to a Whitney sum of some complex linear vector bundles. (Only topological locally trivial vector bundles will be discussed here.)

Theorem 1 ([1]) *Let M^4 be a stably complex simply connected closed 4-manifold. Then M^4 is a TNS-manifold iff the intersection form of 2-cycles of M^4 is non-definite.*

Quasitoric TNS-manifolds are simply connected. There are many diverse examples of such a family of manifolds. Among the smooth projective toric TNS-manifolds one has: any toric surface not isomorphic to $\mathbb{C}P^2$; Bott towers (towers of $\mathbb{C}P^1$-bundles); equivariant blow-up of an invariant submanifold of (complex) codimension 2 of any toric TNS-manifold. A remarkable property of quasitoric manifolds is given by the following

Theorem 2 ([2]) *Let M^{2n} be a quasitoric TNS-manifold. Then any complex vector bundle $\xi \to M$ is is stably isomorphic to a Whitney sum of complex linear vector bundles.*

The work was partially supported by RFFI grant 16-51-55017

There is a criterion for a quasitoric manifold M^{2n} to be TNS. For any element $\alpha \in H^{2(n-k)}(M;\mathbb{R})$ of the cohomology ring of M consider the homogeneous real k-form

$$Q_\alpha : \ H^2(M;\mathbb{R}) \to \mathbb{R}, \ x \mapsto \langle \alpha x^k, [M] \rangle,$$

where $\langle \alpha x^k, [M] \rangle$ is the evaluation at the fundamental class of M, and $k = 1, \dots, n$. We say that Q_α is admissible if it takes values of opposite signs as a real-valued function.

Theorem 3 ([2]) *Let M^{2n} be a quasitoric manifold. Then it is TNS iff the form Q_α is admissible for any $\alpha \in H^{2(n-k)}(M;\mathbb{R})$, $k = 1, \dots, n$.*

Theorem 3 generalises Theorem 1 in the family of quasitoric manifolds. Using Theorem 3 one deduces

Theorem 4 ([2]) *Let M^6 be a smooth projective toric TNS-manifold. Then the respective moment polytope $P^3 \subset \mathbb{R}^3$ is a flag polytope.*

In the talk we will discuss different versions of the above TNS-criterion for a quasitoric manifold M: in terms of K-theory of M and the volume polynomial of the respective multifan of M.

References

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