Список литературы

1. *Дряженков А.А., Потапов М.М.* Численное решение задачи позиционного граничного управления для волнового уравнения с неизвестными начальными данными // Труды ИММ УрО РАН. 2016. Т. 22, № 2. С. 138–146.
2. *Potapov M.M., Dryazhenkov A.A.* Numerical solution to one guaranteeing control problem for the wave equation in classes of weak generalized solutions // Proc. 2016 Int. Conf. "Stability and Oscillations of Nonlinear Control Systems" (Pyatnitskiy's Conference), June 1–3, 2016, Moscow, P. 1–4.
3. *Дряженков А.А.* Численное решение некоторых задач граничного управления волновым уравнением в присутствии неопределенностей // Динамические системы: обратные задачи, устойчивость и процессы управления: Тез. докл. Междунар. конф., посв. восьмидесятилетию Ю.С. Осипова, Москва, 22–23 сентября 2016 г. М.: МИАН, 2016. С. 48–50.

On linking optimization models under asymmetric information

Yuri Ermoliev[a], Tatiana Ermolieva[a], Petr Havlik[a], Michael Obersteiner[a], Elena Rovenskaya[a,b]

[a]*International Institute for Applied Systems Analysis, Laxenburg, Austria*
[b]*Faculty of Computational Mathematics and Cybernetics, Lomonosov Moscow State University, Moscow, Russia*

Detailed sectorial and regional models have been traditionally used to anticipate and plan desirable developments of the respective sectors and regions. These models operate with a set of feasible decisions and aim to select a solution optimizing a sector- or region-specific objective function, depending on various input scenarios. Nowadays, sectors and regions become more and more interconnected utilizing common resources. The competition for natural resources becomes more and more pronounced; for example, energy and agricultural sectors often compete for land and water both needed for growing crops and also the production of biofuel, hydroelectric power generation, and coal mining in the same location. Also, both sectors contribute to the deterioration of the common environment, polluting soil, water and air, and emitting greenhouse gases. In such situations, an independent analysis of sectors and regions ignoring their interconnectedness can become highly misleading. Hence, the sectorial and regional models must be linked together to produce truly integrated solutions optimal for the overall economy, in which they are a part.

The mathematical formulation of a model linkage problem considered in this paper is as follows. Let us consider sectors/regions utilizing some common resources. Let $x^{(k)}$ be the vector of decision variables in sector/region k and assume that each sector/region $k = 1, \ldots, K$ aims to choose such $x^{(k)}$ to maximize its objective function of the form

$$\langle c^{(k)} | x^{(k)} \rangle \to \max \tag{1}$$

such that

$$x^{(k)} \geq 0, \tag{2}$$

$$A^{(k)} x^{(k)} \leq b^{(k)}, \tag{3}$$

$$B^{(k)} x^{(k)} \leq y^{(k)}, \tag{4}$$

where $c^{(k)}$ and $b^{(k)}$ are given vectors, $A^{(k)}$ and $B^{(k)}$ are given matrices. Vectors $y^{(k)}$ define common inter-sectorial/inter-regional constraints as follows:

$$\sum_{k=1}^{K} D^{(k)} y^{(k)} \leq d, \qquad y^{(k)} \geq 0, \tag{5}$$

where matrix $D^{(k)}$ and vector d are given.

Thus, each sector/region k maximizes its objective function (1) by choosing $x^{(k)}$ and $y^{(k)}$ from the feasible set defined by (2), (3), so that (4) and (5) are also fulfilled.

Clearly, there may be different ways how sectors/regions can distribute the quotas $y = (y^{(1)}, \ldots, y^{(K)})$ needed to achieve joint constraints (5). Truly integrative solutions, by definition, imply cooperation between sectors; from this perspective, the problem of model linkage can be considered essentially as a multi-criteria optimization problem, in which a resource-efficient Pareto solution is to be found. That is, assuming some weights w_k, $w_k > 0$, $\sum_{k=1}^{K} w_k = 1$, a single welfare function should be maximized producing a Pareto optimal solution:

$$\sum_{k=1}^{K} w_k \langle c^{(k)} | x^{(k)} \rangle \to \max, \tag{6}$$

subject to (2)–(5). If implementing literally, it would require collecting all models in a single place (code).

In case the original sectorial/regional models are high dimensional and are possessed by different modeling teams, such a straightforward linkage may be problematic. It would require all parties to agree to reveal and share the codes; also notations and data should be harmonized and (presumably substantial) efforts should be invested to re-programming.

In this paper, we show that re-coding is actually unnecessary, as, instead, one can arrange an iterative procedure, which converges to a Pareto optimal solution. In this procedure, there exists a central hub who must know $D^{(k)}$ and d, while the information on sectorial/regional $c^{(k)}$, $A^{(k)}$, $B^{(k)}$, $x^{(k)}$ does not need to be shared with other sectors/regions. The central hub recalculates the resource quotas y by shifting their current approximation in the direction defined by the corresponding vectors of dual variables (shadow prices of resources) from the primal optimization problems. These quotas are received by sectorial/regional models enabling parallel computations of solutions and fast adjustments of vector y. The foundations of this algorithm were initially introduced by Ermoliev [1]. In this work we prove the convergence rigorously. Current computer capacities enable the implementation of this algorithm to large-scale models used to support decisions.

We demonstrate an application of the developed iterative linkage procedure to the case study linking coal and agricultural sectorial models used to define optimal management strategies in the water-scarce Shanxi province in China [2].

References

1. *Ermoliev Y.* Some problems of linkage systems: IIASA Working Paper WP-80-102, 1980.
2. *Xiangyang X., Gao J., Cao G.-Y., Ermoliev Y.M., Ermolieva T., Kryazhimskii A.V., Rovenskaya E.* Systems analysis of coal production and energy-water-food security in China // Cybern. Syst. Anal. 2015. V. 51, N 3. P. 370–377.