The method for global extremum search of objective functional based on Pontryagin maximum principle*

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The problem of constructing algorithms for global extremum search in optimal control problems is one of the topical problems of control theory. The report discusses solutions of nonconvex optimization problems for dynamical systems for one class of optimization models.

We consider a nonlinear controlled system with linear controls. The initial state and the interval of determining the independent variable (time) are fixed:

$$\dot{x} = f(x, u, t), \qquad x(t) \in R^n, \quad u(t) \in R^r, \tag{1}$$

$$x(t_0) = x^0, \qquad t \in T = [t_0, t_1]. \tag{2}$$

The controls satisfy the direct constraints of the traditional ("parallelepiped") type

$$\alpha_l(t) \le u(t) \le \beta_l(t), \qquad l = \overline{1, r}, \quad t \in T. \tag{3}$$

The quality of the process is described by a linear terminal functional

$$I_0(u) = \varphi^0(x(t_1)) \to \min. \tag{4}$$

The idea of the proposed approach goes back to papers of L.S. Pontryagin (see, for example, [1]). To construct a computational scheme, a $2n$-dimensional system of differential equations is formulated, which includes both the direct and the conjugate subsystems. For this system, a Cauchy problem is formed, in which the initial values for the direct subsystem are given by the initial state of the process, and for the conjugate one, they are selected from the unit sphere (circle in the two-dimensional case). To close the system, a "local synthesis" for control is used, chosen from the maximum condition of Pontryagin's function at every point of the time interval. In the absence of degeneracy of the maximum principle (which is a limitation of the proposed

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approach), in the linear case under investigation, this problem can be solved in a closed form, "by formulas".

To select the initial state of the conjugate system in the two-dimensional case (two phase variables), one-parameter representation of the circle is possible; in three-dimensional case (three phase variables), a two-parameter representation of the sphere is possible, and so on. With this formalization, the optimal control problem can be reduced to the problem of minimum search of a one-dimensional function in the two-dimensional case, of a two-dimensional function in three-dimensional case, etc.

The software implementation of the proposed approach is performed in the C language within the OPTCON software complex [2].

We present the result of solving a test optimal control problem by using the proposed computing technology:

$$\dot{x}_1 = x_2 \cdot u_1, \qquad \dot{x}_2 = \frac{x_1 + u_1}{x_1 \cdot x_1 + x_2 \cdot x_2}.$$

The initial phase vector and the time interval are fixed: $x_1(t_0) = 1$, $x_2(t_0) = -1.2$, $T = [t_0, t_1] = [0, 2]$. It is necessary to find a control $u \in U = [-1, 1]$ that minimizes the functional

$$I(u) = e^{0.926 \cdot x_1(t_1)} \left(\sqrt{2.72 - x_1(t_1)} - \frac{2.72 - x_1(t_1)}{7} \right) - 1.481 \cdot x_1(t_1)$$
$$- 0.014 \cdot x_2(t_1) \cdot x_2(t_1) \to \min.$$

The minimum value of the functional $I(u^*) = -2.83027$ is found in 4 s of CPU time, with 2049 Cauchy problems solved using a computer with a processor Intel Core i5-2500K and 16Gb RAM. The optimal control and the corresponding trajectories of the system are shown in Fig. 1. A reachable set with global extremum (dark marker) and local extrema (light markers) are shown in Fig. 2.

The results of a series of computational experiments made it possible to demonstrate the effectiveness of the proposed approach for the class of problems under consideration.

References

1. *Pontryagin L.S.* Mathematical theory of optimal processes and differential games // Tr. Mat. Inst. im. V.A. Steklova. 1985. V. 169. P. 119–158 (in Russian).

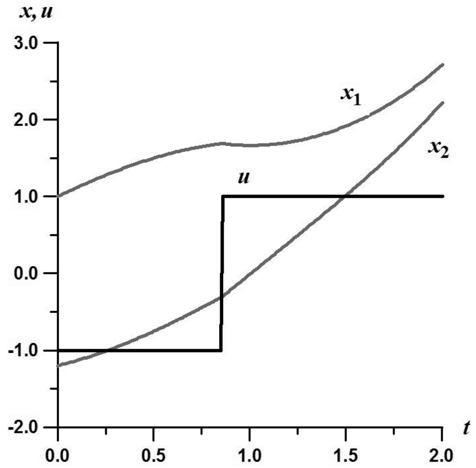

Figure 1. Optimal control and trajectories for the test problem

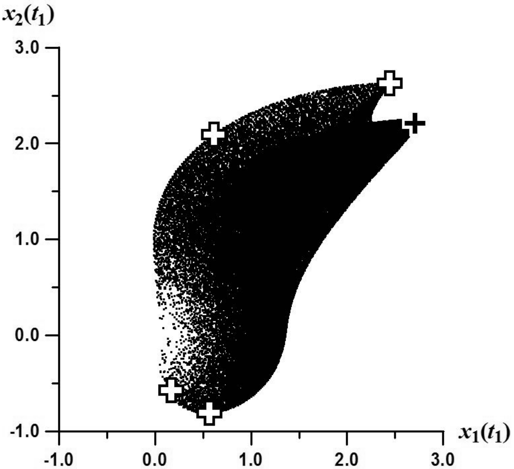

Figure 2. Optimal control, trajectories and reachable set with extreme points for the test problem

2. *Gornov A.Yu., Zarodnyuk T.S., Anikin A.S., Finkelshtein E.A.* Software for numerical analysis of optimization problems // Proc. XX Int. Conf. on Computational Mechanics and Modern Applied Software Systems, May 24–31, 2017, Alushta. P. 680–682 (in Russian).