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On necessary conditions in the Mayer problem with differential inclusion*

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The author developed a direct method for obtaining necessary optimality conditions for the solution of the Mayer problem, in which the differential inclusion is introduced as a constraint under the conditions of unboundedness and pseudo-Lipschitz property of the right-hand side of the differential inclusion. The necessary optimality conditions are obtained in the form of a differential inclusion of the Euler–Lagrange type and generalize the results from the works of F. Clarke and the author (see [1, 2]).

Statement of the problem and conditions. We consider the interval $T := [0,1]$, closed sets $C_0, C_1 \subset \mathbb{R}^n$, a locally Lipschitz function $\varphi\colon \mathbb{R}^n \to \mathbb{R}^1$ and a multivalued mapping $F\colon T \times \mathbb{R}^n \rightrightarrows \mathbb{R}^n$, with the help of which we have the differential inclusion of the form

$$x'(t) \in F(t, x(t)) \qquad \text{for a.e. } t \in T. \tag{1}$$

The symbol $\mathcal{R}_T(F, C_0)$ denotes the (possibly empty) set of all trajectories $x(\cdot) \in \mathcal{R}_T(F, C_0) \subset \mathrm{AC}(T, \mathbb{R}^n)$ of the differential inclusion (1) with the initial condition $x(0) \in C_0$.

The *Mayer problem* is to find the minimum of the values $\varphi(x(1))$ over all end points $x(1) \in C_1$ of the trajectories $x(\cdot) \in \mathcal{R}_T(F, C_0)$.

Let $\widehat{x}(\cdot) \in \mathcal{R}_T(F, C_0)$ be a trajectory that solves the Mayer problem; i.e., its end value $\widehat{x}(1) \in C_1$ is such that $\varphi(\widehat{x}(1))$ takes a minimum value for all

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trajectories (1). To obtain the necessary conditions for optimality, it suffices to formulate local conditions on the mapping F near the trajectory $\widehat{x}(\cdot)$.

We assume that the mapping $F\colon T \times \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ is $(\mathcal{L} \times \mathcal{B})$-measurable and for almost every $t \in T$ the set $\operatorname{Graph} F(t, \cdot) := \{(x, y) \mid y \in F(t, x)\}$ is a closed subset in $\mathbb{R}^n \times \mathbb{R}^n$.

Let there be numbers $\varepsilon > 0$, $\nu > 0$, a function $l(\cdot) \in L^1(T, \mathbb{R}^n)$ and a measurable function $R\colon T \to (0, +\infty]$ such that the following two conditions are satisfied:

(i) pseudo-Lipschitz condition: for almost every $t \in T$ and any $x_1, x_2 \in B_\varepsilon(\widehat{x}(t))$, the following inclusion holds:

$$F(t, x_1) \cap (\widehat{x}'(t) + R(t) B_1(0)) \subset F(t, x_2) + l(t)\|x_1 - x_2\| \, \overline{B_1(0)};$$

(ii) non-degeneracy condition: $l(t) \le \nu \, R(t)$ for a.e. $t \in T$.

As usual, we denote by $T_L(A; a)$ and $T_C(A; a)$, respectively, the lower tangent cone and the Clarke tangent cone to the set A at the point $a \in \overline{A}$ (see [2]).

Let there be given a measurable multivalued mapping $K\colon T \rightrightarrows \mathbb{R}^n \times \mathbb{R}^n$, whose values are closed cones, that satisfies for a.e. $t \in T$ the inclusion

$$T_C(\operatorname{Graph} F(t, \cdot); (\widehat{x}(t), \widehat{x}'(t))) \subset K(t) \subset T_L(\operatorname{Graph} F(t, \cdot); (\widehat{x}(t), \widehat{x}'(t))).$$

Examples of such a map $K(t)$ are the Clarke tangent cone, the Michel–Peno tangent cone, and the asymptotic lower tangent cone to the set $\operatorname{Graph} F(t, \cdot)$ at the point $(\widehat{x}(t), \widehat{x}'(t))$ (see [1, 2]).

Let K_0 and K_1 be Boltyanskii tents to the sets C_0 and C_1 at the points $\widehat{x}(0)$ and $\widehat{x}(1)$, respectively (see [3]). Let $\psi\colon \mathbb{R}^n \to \mathbb{R}^1$ be a convex positively homogeneous function that is the upper convex approximation of the function φ at the point $\widehat{x}(1)$. For every cone K we denote its polar cone by K^0.

Main result. The necessary conditions for the optimality of the solution of the Mayer problem take the following form.

Theorem. *Let $\widehat{x}(\cdot)$ be the solution of the Mayer problem and the above conditions be satisfied in the neighborhood of $\widehat{x}(\cdot)$. Then there exist a number $\lambda \ge 0$ and an arc $p(\cdot) \in \mathrm{AC}(T, \mathbb{R}^n)$ satisfying the nontriviality condition $\lambda + \|p(\cdot)\|_{\mathrm{AC}} \ne 0$ and the transversality condition $p(0) \in K_0^0$, $-p(1) \in K_1^0 + \lambda \, \partial\psi(0)$ and such that the arc p satisfies the Euler inclusion*

$$(p'(t), p(t)) \in K^0(t) \qquad \textit{for a.e. } t \in T. \tag{2}$$

Corollary. *If in addition for all* $t \in T$ *and* $x \in B_\varepsilon(\widehat{x}(t))$ *the set* $F(t,x) \cap (\widehat{x}'(t) + R(t)B_1(0))$ *is convex, then the arc* p *satisfies the Pontryagin maximum principle*

$$\langle p(t), \widehat{x}'(t)\rangle \geq \langle p(t), y\rangle \qquad \forall y \in F(t, \widehat{x}(t)) \cap (\widehat{x}'(t) + R(t)B_1(0))$$

for a.e. $t \in T$.

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Задачи оптимального управления динамическими системами дробного порядка с сосредоточенными и распределенными параметрами (Optimal control problems for fractional-order dynamical systems with lumped and distributed parameters)

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Одно из заметных направлений развития современной теории управления составляют исследования вопросов оптимального управления системами дробного порядка [1]. Наличие интегрального представления для систем дробного порядка позволяет применять для поиска оптимальных управлений метод моментов по аналогии с системами целого порядка. Данный метод позволяет строить в явном виде оптимальные управления и исследовать их свойства, в том числе в случаях, когда