On shift of the Lyapunov spectrum for linear stationary control systems in Banach spaces*

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Let X and U be Banach spaces. Consider a linear control system

$$\dot{x}(t) = A(t)x(t) + B(t)u(t), \qquad 0 \le t < +\infty, \tag{1}$$

where $x(t) \in X$, $u(t) \in U$, $A(t) \in L(X, X)$, $B(t) \in L(U, X)$, $t \ge 0$; $L(Q_1, Q_2)$ is the Banach space of linear bounded operators $P\colon Q_1 \to Q_2$. We suppose that $A(\cdot)$ and $B(\cdot)$ are piecewise continuous and, for some $M_1 > 0$, for all $t \ge 0$, the following inequalities hold: $\|B(t)\| \le M_1$ and

$$\|A(t)\| \le M_1. \tag{2}$$

Consider the corresponding free system

$$\dot{x}(t) = A(t)x(t), \qquad 0 \le t < +\infty. \tag{3}$$

Denote by Σ_A the (*upper*) *Lyapunov spectrum* of system (3) [1, Ch. III, Sect. 4, p. 117]. By (2), we have $\Sigma_A \subset [-M_2, M_2] \subset (-\infty, +\infty)$ for some $M_2 > 0$. In particular, if $X = \mathbb{R}^n$ and $A(t) \equiv A$ then $\Sigma_A = \{\operatorname{Re} \mu_j \colon j = \overline{1, n}\}$, where μ_j ($j = \overline{1, n}$) are eigenvalues of A [1, p. 117].

Let us construct a linear state feedback control

$$u(t) = K(t)x(t), \qquad 0 \le t < +\infty, \tag{4}$$

where $K(t) \in L(X, U)$ for $t \ge 0$, $K(\cdot)$ is piecewise continuous, and $\|K(t)\| \le M_3$, $t \ge 0$, for some $M_3 > 0$ (we will call such an operator $K(t)$ admissible). The closed-loop system has the form

$$\dot{x}(t) = (A(t) + B(t)K(t))x(t). \tag{5}$$

Definition 1. We say that *the Lyapunov spectrum of system* (5) *is arbitrarily shiftable* if for any $\lambda \in \mathbb{R}$ there exists an admissible operator $K(t)$ such that $\Sigma_{A+BK} = \Sigma_A + \lambda$.

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Consider a stationary system (1):

$$\dot{x}(t) = Ax(t) + Bu(t), \qquad 0 \le t < +\infty, \tag{6}$$

where $A \in L(X, X)$ and $B \in L(U, X)$.

Definition 2. The control system (6) is said to be *exactly controllable on* $[0, T]$ [2, Ch. 3, p. 51] if for any points $x^0, x^1 \in X$ there exists a control function $\widehat{u} \in L^2([0, T], U)$ such that the solution $x(t)$ of system (6) with $u(t) = \widehat{u}(t)$ with the initial condition $x(0) = x^0$ satisfies the condition $x(T) = x^1$.

System (6) in closed loop with (4) becomes

$$\dot{x}(t) = (A + BK(t))x(t). \tag{7}$$

Theorem. *Assume that X is a reflexive Banach space and U is a Hilbert space. Let system* (6) *be exactly controllable on some* $[0, T]$*. Then the Lyapunov spectrum of the closed-loop system* (7) *is arbitrarily shiftable.*

For the case $X = \mathbb{R}^n$, $U = \mathbb{R}^m$, the theorem follows from [3, 4]. For system (6), obvious corollaries on stabilization or destabilization (by means of linear state feedback (4)) follow from the above theorem.

References

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