of equations:

$$
\begin{aligned}
-V^i_\tau(t,\tau,x) &= \max_{\phi_i}\{g^i(\tau,x,\widetilde{u}^{\mathrm{NE}}_{-i}) + V^i_x(t,\tau,x)f(\tau,x,\widetilde{u}^{\mathrm{NE}}_{-i})\} \\
&= g^i(\tau,x,\widetilde{u}^{\mathrm{NE}}) + V^i_x(t,\tau,x)f(\tau,x,\widetilde{u}^{\mathrm{NE}}), \\
& V^i(t,t+\overline{T},x) = 0, \qquad i \in \mathbb{N},
\end{aligned}
\tag{3}
$$

where $\widetilde{u}^{\mathrm{NE}}_{-i} = (\widetilde{u}^{\mathrm{NE}}_1, \ldots, \phi_i, \ldots, \widetilde{u}^{\mathrm{NE}}_n)$.

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Cooperative differential games

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Game theory has greatly enhanced our understanding of decision making. As socioeconomic and political problems increase in complexity, further advance in the analytical content of the theories, methodology, techniques and applications as well as case studies and empirical investigations are urgently required. In the social sciences, economics and finance are the fields which most vividly display the characteristics of games.

The origin of differential games traces back to the late 1940s. Rufus Isaacs modeled missile versus enemy aircraft pursuit schemes in terms of descriptive and navigation variables (state and control).

For various reasons Isaacs' work did not appear in print until 1965 [3]. In the meantime control theory reached its maturity in the optimal control theory of Pontryagin et al. [9] (1962) and Bellman's dynamic programming [1] (1957). Research in differential games focused in the first place on extending control theory to incorporate strategic behavior. First papers about differential games appeared in the Soviet Union in 1962–1967 (Pontryagin [8], Krasovskii [4], Petrosyan [5]).

Research in differential game theory continues to extend over a large number of fields and areas. Some non-zero games were first investigated by Petrosyan and Murzov [6] (1967), Case [2] (1969), and Starr and Ho [10] (1969), where as a solution the Nash equilibrium was considered.

It is well known that non-cooperative behavior among participants would, in general, lead to an outcome which is not Pareto-optimal. Worse still, highly undesirable outcomes (like the prisoners dilemma) and even devastating results (like tragedy of commons) could appear when the involved parties only care about their self-interests in a non-cooperative situation. In a dynamic world, non-cooperative behavior guided by short-slighted individual rationality could be a source for series of disastrous consequences in the future. Cooperation suggests the possibility of obtaining socially optimal and group efficient solutions to decision problems involving strategic actions.

However, dynamic cooperation cannot be sustainable if there is no guarantee that the participants will be always better off within the entire duration of the cooperation. More than anything else, it is due to the lack of this kind of guarantees that cooperative schemes fail to last till their end. Dynamic cooperation represents one of the most intriguing forms of optimization analysis. To ensure that the cooperation will take place in the game during the whole time interval, the solution concepts (optimality principles) must be time or subgame consistent. But unfortunately solutions taken from the classical one-shot cooperative game theory are time or subgame inconsistent. And by using them in differential cooperative games we cannot be sure that the cooperation will be sustained on the whole time interval.

To overcome this problem, we introduced in 1979 a new control variable IDP (imputation distribution procedure) which prescribes a payment distribution over time in such a way that cooperation at each time instant is more preferable than non-cooperative behavior of players. For different game-theoretical models, the approach was successfully used by Petrosyan and Zaccour [7] (2003) and Yeung and Petrosyan [11, 12] (2012, 2016).

In this presentation we shall stress our attention to new directions and problems in cooperative differential and dynamic games and show the way of constructing time-consistent (subgame-consistent) and strongly time-consistent cooperative solution concepts.

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К задаче группового преследования с дробными производными (To the problem of group pursuit with fractional derivatives)*

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Одним из направлений современной теории дифференциальных игр преследования является разработка методов решения задач конфликтного взаимодействия группы преследователей с группой убегающих [1–3]. Следует отметить, что, кроме углубления классических методов решения, активно ведется поиск новых задач, к которым применимы уже разработанные методы. В частности, в работе [4] рассматривалась задача преследования двух лиц, описываемая уравнениями с дробными производными, где были получены достаточные условия поимки.

В данной работе рассматривается задача преследования группой преследователей группы убегающих с равными возможностями всех участников в дифференциальной игре, описываемой уравнениями с дробными производными. Целью группы преследователей является поимка заданного числа убегающих, причем поимку каждого убегающего должны осуществить заданное число преследователей.

Определение 1 [5]. Пусть $f\colon [0,\infty) \to \mathbb{R}^k$ — функция такая, что f' абсолютно непрерывна на $[0,+\infty)$, $\mu \in (1,2)$. Производной по Капуто порядка μ функции f называется функция $D^{(\mu)}f$ вида

$$\big(D^{(\mu)}f\big)(t) = \frac{1}{\Gamma(2-\mu)}\int_0^t \frac{f''(s)}{(t-s)^{\mu-1}}\,ds, \qquad \text{где} \quad \Gamma(\beta) = \int_0^\infty e^{-s}s^{\beta-1}\,ds.$$

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