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Weighted universality of the Hurwitz zeta-function

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In mathematics, often weighted statements are discussed. They generalize usual statements when the weight function is identically 1. In probability theory, we have weighted limit theorems, in number theory the weighted sieves are used. Analytic number theory considers weighted universality theorems for zeta- and L-functions.

The first weighted universality theorem was obtained for the Riemann zeta-function $\zeta(s)$, $s = \sigma + it$, in [3]. Let $T_0 > 0$ be fixed. Suppose that $w(t)$ is a positive function of bounded variation on $[T_0, \infty)$ such that, for every subinterval $[a, b] \subset [T_0, \infty)$, the variation $V_a^b w$ satisfies the inequality $V_a^b w \leqslant cw(a)$ with a certain constant $c > 0$, and that

$$\lim_{T\to\infty} U(w,T) = \lim_{T\to\infty} \int_{T_0}^{T} w(\tau)\,\mathrm{d}\tau = +\infty.$$

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Denote by $I(A)$ the indicator function of the set A, and $D = \{s \in \mathbb{C} : 1/2 < \sigma < 1\}$. Let $\mathcal{K}$ be the class of compact subsets of the strip D with connected complements, and $H_0(K)$ with $K \in \mathcal{K}$ the class of continuous non-vanishing functions on K that are analytic in the interior of K. Then the following statement is true [3].

Theorem 1. *Let $K \in \mathcal{K}$ and $f(s) \in H_0(K)$. Then, for every $\varepsilon > 0$,*

$$\liminf_{T\to\infty} \frac{1}{U(T,w)} \int_{T_0}^{T} w(\tau) I\left(\left\{\tau \in [0,T] : \sup_{s\in K} |\zeta(s+i\tau) - f(s)| < \varepsilon\right\}\right) \mathrm{d}\tau > 0.$$

We consider the weighted universality for the Hurwitz zeta-function. Let α, $0 < \alpha \leqslant 1$, be a fixed parameter. The Hurwitz zeta-function $\zeta(s,\alpha)$ is defined, for $\sigma > 1$, by the Dirichlet series

$$\zeta(s,\alpha) = \sum_{m=0}^{\infty} \frac{1}{(m+\alpha)^s},$$

and has analytic continuation to the whole complex plane, except for a simple pole at the point $s = 1$ with residue 1. Moreover, we have that $\zeta(s,1) = \zeta(s)$, and

$$\zeta(s,1/2) = (2^s - 1)\zeta(s).$$

Thus, the function $\zeta(s,\alpha)$ is a generalization of the Riemann zeta-function.

It is well known that the function $\zeta(s,\alpha)$ for some classes of parameter α is universal in the Voronin sense. Denote by $H(K)$ with $K \in \mathcal{K}$ the class of continuous functions on K that are analytic in the interior of K. Then the universality of $\zeta(s,\alpha)$ is contained in the following theorem [2].

Theorem 2. *Suppose that the parameter α is transcendental or rational $\neq 1, 1/2$. Let $K \in \mathcal{K}$ and $f(s) \in H(K)$. Let $K \in \mathcal{K}$ and $f(s) \in H(K)$. Then, for every $\varepsilon > 0$,*

$$\liminf_{T\to\infty} \frac{1}{T} \mathrm{meas} \left\{\tau \in [0,T] : \sup_{s\in K} |\zeta(s+i\tau,\alpha) - f(s)| < \varepsilon\right\} > 0.$$

The universality of $\zeta(s,\alpha)$ with algebraic irrational α is an open problem.

The weighted universality of $\zeta(s,\alpha)$ was began to study in [1]. Define the set

$$L(\alpha) = \{\log(m+\alpha) : m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}\}.$$

Theorem 3. *Suppose that the set $L(\alpha)$ is linearly independent over the field of rational numbers $\mathbb{Q}$. Let $K \in \mathcal{K}$ and $f(s) \in H(K)$. Then, for every $\varepsilon > 0$,*

$$\liminf_{T\to\infty} \frac{1}{U(T,w)} \int_{T_0}^{T} w(\tau) I\left(\left\{\tau \in [0,T] : \sup_{s\in K} |\zeta(s+i\tau,\alpha) - f(s)| < \varepsilon\right\}\right) \mathrm{d}\tau > 0.$$

For example, the set $L(\alpha)$ is linearly independent over $\mathbb{Q}$ for transcendental α.

Hurwitz zeta-functions also are jointly universal, i. e., a collection of analytic functions can be approximated by a collections of shifts $\zeta(s+i\tau,\alpha_1), \ldots, \zeta(s+i\tau,\alpha_r)$. Define the set

$$L(\alpha_1,\ldots,\alpha_r) = \{\log(m+\alpha_j) : m \in \mathbb{N}_0,\ j = 1,\ldots,r\}.$$

Then the following statement is known [4].

Theorem 4. *Suppose that the set $L(\alpha_1,\ldots,\alpha_r)$ is linearly independent over $\mathbb{Q}$. For $j = 1,\ldots,r$, let $K_j \in \mathcal{K}$ and $f_j(s) \in H(K_j)$. Then, for every $\varepsilon > 0$,*

$$\liminf_{T\to\infty} \frac{1}{T} \mathrm{meas} \left\{\tau \in [0,T] : \sup_{1\leqslant j\leqslant r} \sup_{s\in K_j} |\zeta(s+i\tau,\alpha_j) - f_j(s)| < \varepsilon\right\} > 0.$$

For example, the set $L(\alpha_1, \dots, \alpha_r)$ is linearly independent over $\mathbb{Q}$ for algebraically independent $\alpha_1, \dots, \alpha_r$.

Theorem 4 has the following weighted generalization [5].

THEOREM 5. *Suppose that the set $L(\alpha_1, \dots, \alpha_r)$ is linearly independent over $\mathbb{Q}$. For $j = 1, \dots, r$, let $k_j \in \mathcal{K}$ and $f_j(s) \in H(K_j)$. Then, for every $\varepsilon > 0$,*

$$\liminf_{T\to\infty} \frac{1}{U(T,w)} \int_{T_0}^{T} w(\tau) I\left(\left\{\tau \in [0,T] : \sup_{1\leqslant j\leqslant r} \sup_{s\in K_j} |\zeta(s+i\tau, \alpha_j) - f_j(s)| < \varepsilon\right\}\right) \mathrm{d}\tau > 0.$$

Moreover, the limit

$$\lim_{T\to\infty} \frac{1}{U(T,w)} \int_{T_0}^{T} w(\tau) I\left(\left\{\tau \in [0,T] : \sup_{1\leqslant j\leqslant r} \sup_{s\in K_j} |\zeta(s+i\tau, \alpha_j) - f_j(s)| < \varepsilon\right\}\right) \mathrm{d}\tau > 0$$

exists for all but at most countably many $\varepsilon > 0$.

For example, we can take $\alpha_1 = \frac{1}{\pi}, \dots, \alpha_r = \frac{1}{r\pi}$.

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Classical generators for category of coherent sheaves and the regular locus

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We give a necessity and sufficiency condition for a singularity category $\mathbf{D}_{sg}(X)$ of a Noetherian scheme X to have a classical generator. This is due to the openness of the regulor locus $Reg(X)$.

keywords: singularity category; classical generator; regulor locus; Noetherian scheme

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