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Leonhard Euler's Letters to a German Princess

A milestone in the history of physics textbooks and more

Ronald S Calinger
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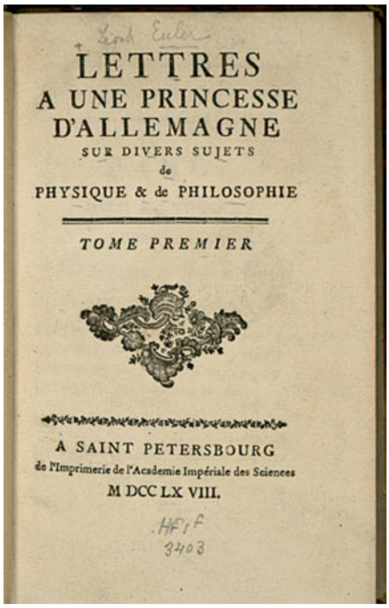

LETTRES
A UNE PRINCESSE
D'ALLEMAGNE
SUR DIVERS SUJETS
de
PHYSIQUE & de PHILOSOPHIE

TOME PREMIER

A SAINT PETERSBOURG
de l'Imprimerie de l'Academie Impériale des Sciences
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Frontispiece of *Lettres à une Princesse d'Allemagne sur divers sujets de Physique & de Philosophie* (1768).
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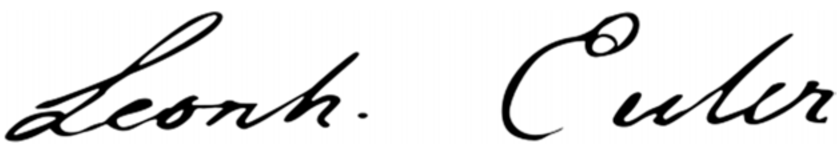
To Elena for the proposal and to Betty for technical assistance and in loving memory of Aleksey Gavril (1974–2019), who never stopped questioning.

Contents

Preface

Leonhard Euler's *Lettres à une Princesse d'Allemagne sur divers sujects de physique & de philosophie* (*Letters to a German Princess on Different Subjects in Physics and Philosophy*) is a milestone in the history of physics textbooks, the popularization of the sciences, and the teaching of women in them. From April 1760 to May 1762, Euler sent 234 letters to two German princesses of Brandenburg-Schwedt instructing them in the natural sciences, philosophy and religion, and physical science questions. This came roughly to one or two letters a week. The portion of the *Letters* addressing physics or natural philosophy encompassed a range of subjects that Euler had pursued outside of pure mathematics. He gives basics in ten disciplines for non-scientists: astronomy, celestial mechanics, cosmography, mechanics, mechanics of elastic systems, music theory, physics, optics, electricity, and magnetism, and dealing less with philosophy and religion. He treats at length the composition of the air and changes caused by heat, cold, and altitude as well as new instruments and technology, such as the latest thermometers, microscopes, telescopes, and camera obscura; he refers also to ballistics and hydromechanics. Among the topics in physics are the theory of sounds and the theory of light. The extent of his discussion of electricity, letters 138–54, and magnetism, 169–87, reflects the great interest in them at the time. The popular letters, written in an easy form, cover with genial clarity all the natural philosophy of the mid-eighteenth century. Euler taught everything that he believed his young correspondents should know in the physical sciences.



Euler's signature.
Source: https://upload.wikimedia.org/wikipedia/commons/d/d3/Euler%27s_signature.svg.

During his final years in Berlin from 1762 to late 1766, Euler gathered and edited the separate letters, which the Imperial Academy of Sciences in St. Petersburg (hereafter, Petersburg Academy) published in a large three-volume book in French. The first two volumes appeared in 1768, the third in 1772. The resulting *Letters to a German Princess* also offer the views of its author on epistemology and the history of science. For its scope and depth, which exceed a mild curiosity and the basic instruction of an adolescent, it may be surmised that Euler actually wrote them for the European and Russian reading publics. The *Letters to a German Princess*, containing materials that taxed even the wits of savants, have been described as the chief treasury for physical knowledge made during the eighteenth century.

The two German princesses taught were the fifteen-year-old Frederike Charlotte Leopoldine Ludovica Louise (1745–1808) and her younger sister, Louise Henriette Wilhelmine (1750–1811), to whom the letters were addressed[1]. This introduction outlines the subjects in the *Letters to a German Princess.* Chapter 2 briefly examines two errors in identifying the princesses, to whom they were addressed, and the resolution in 2002. It continues with biographical notes on each, the course of writing and publishing the *Letters,* and reviews three of the first translations as well as principal natural philosophies during the Enlightenment. The two pupils were educated together, and Euler probably recognized both as '*Votre Altesse*' (Your Highness). From their father's title of prince of Prussia, both were princesses of Prussia.

Euler's 234 letters may be divided into three general subject matter sections: general natural science: 1–79, philosophy and religion: 80–120, and a diverse range of physical science questions: 121–234. An outline of topics in these three sections follows.

Section 1: General natural science Basic topics: magnitude, sound, air, heat, light, colors, optics, eye, glass (Letters 1—44)

Magnitude (*grandeur*) or extension and velocity (1–2)
Sound, the harpsichord, and pleasures from fine music (3–8)
Air, the atmosphere, heat, gunpowder, air guns, and the barometer (9–16)
Light, including the systems of Descartes and Newton, and its propagation (17–20)
A digression on the distances of heavenly bodies and the nature of the Sun (21)
Opaque and luminous bodies, colors relative to optics, catoptrics, dioptrics, vision, burning mirrors, the structure of the eye, and a *camera obscura* (22–44)

Newton's inverse-square law of gravitational attraction, gravity on Earth's surface, the tides, celestial motion, lunar motion, impenetrable bodies, mechanics, acceleration, criticism of Wolff's monads (Letters 45–79)

Gravity as a general property of bodies, debates over its validity and objections to its effects, the universal gravitation of Newton, its influence on planetary motion, including small irregularities and other celestial motion (45–61)
Differing explanations of the tides, flux and reflux, and disputes over the impact of universal gravitation (62–8)
The nature of bodies: extension, impenetrability and inertia (69–70)
Mechanics, universal, accelerated, and retarded motion of bodies, disputes over the principal laws of motion, and inertia (71–5)

[1] See Judith Kopelevich, 'History of the Creation of the 'Letters' and their Addressees' in *Leonard Euler: Letters to a German Princess about Various Physical and Philosophical Matters* (2002) ed Nina Nevskaya *et al* (Saint-Petersburg: Nauka), pp 535–54 (in Russian).

Objections to the monadic doctrine of Wolff in the theory of matter, and then to Leibniz's pre-established harmony (76–9)

Section 2: Philosophy and religion Review of liberty, perfection of languages, abstraction, the permission of evil, happiness, and truth (Letters 80–120)

Natural, supernatural, or moral events; liberty of intelligent beings and spirits; the soul, prayer, and disproving materialists and idealists (80–97)
Memory; perfection of languages; the senses, abstraction, and different modes of syllogisms and logic (98–108)
Origins of Euler–Venn diagrams (102–6)
The origins of evil and permission for it, true happiness, the sources of truth, the senses and types of truth, certainty, and Pyrrhonian skepticism (109–20)

Section 3: Physical science questions electricity; astronomical methods for determining latitude and longitude on the open sea; magnetism and magnetic devices; the optics of lenses and optical devices (telescope, optical tubes, etc); and stellar distances (Letters 121–234)

Magnitude or extension and divisibility *ad infinitum* (121–32)
Detailed revisit to Wolff's monad doctrine with objections and support, the principle of sufficient reason (125, May 5, 1761–132 May 30, 1761)
Colors and the human voice (133–7)
A lengthy examination of electricity, the Leyden jar, lightening, thunder, and averting their effects (138–54)
The magnitude of Earth, methods of determining latitude, meridians, and the 'celebrated problem of finding longitude' on ships at sea (six methods, 155–68)
Ships and propulsion, magnetic forces, precision (169–86)
Dioptrics, lenses, pocket glasses, camera *obscura*, constructing microscopes, quality of telescopes, remedying defects (187–222)
The distance of the stars, the size of the moon, eclipses of the Moon and Jupiter's satellites, and the color of the heavens (223–34)

The content and selection of topics for the *Letters to a German Princess* came from the scientific interests of Euler himself and research of the time, conducted mainly at the Royal Society of London, the Royal Prussian Academy of Sciences and Arts in Berlin (or Berlin Academy), the Petersburg Academy, and especially the Royal Academy of Sciences in Paris (Paris Academy). Most important were the annual competitions at the Paris Academy that Euler won twelve times. He also kept up to date the journals of each, including the *Philosophical Transactions* of the Royal Society, at times ordering them for the science libraries in St. Petersburg and Berlin. The letters reflect Euler's mature scientific views rather than tracing in a few cases his evolving thought. In earlier books and articles he had described natural phenomena in detail.

Euler's *Letters to a German Princess* became the most exhaustive and authoritative encyclopedia for physical knowledge written during the eighteenth-century Enlightenment. He explained this knowledge clearly and simply to literate Europeans, Russians, and the English colonists in North America. Euler's idea of the ether, an extremely tenuous and elastic form of matter filling in otherwise empty space, was the fundamental concept in his physics. His concept of the ether appeared first in the initial volume of the *Letters to a German Princess.* He applied it to explain most of the principles underlying physical phenomena: mechanical, celestial, electrical, magnetic, and optical. From his study and computations for motions of the moon, planets, and comets, he was to confirm the exactness obtained from the law of mutual attraction acting alone according to the inverse squares of the distances between them, thereby showing that Isaac Newton was correct. Euler had changed after first thinking that a small modification of Newton was needed for explaining irregularities in lunar motion. His application of the ether to explain celestial motions was later often interpreted as Cartesian. But his ether was not that of Descartes. For Euler it was a new concept of space set within a new physics.

From 1787 to the present, the *Letters to a German Princess* have been hugely successful. The European reading public quickly recognized their merits. They were the most popular scientific text of the late eighteenth century and at least the first half of the nineteenth. Inspired by preceding best-selling popular works in the sciences, Bernard Fontenelle's *Entretiens sur la Pluralité des Mondes* (Conversations on the Plurality of Worlds, 1686), simply known as *Mondes*, with its more ingenious explanation of the system of vortices in the heavens and Cartesian physics, the *Letters to a German Princess* quickly surpassed them. By 1800 there were 30 editions and translations from French into eight other languages: Russian, German, Danish, Dutch, English, Italian, Spanish, and Swedish, which were most of the major languages in Europe. Most of their early translations were based on Condorcet's French edition (1787–89), which is described in chapter 2. The *Letters to a German Princess* were a principal document of the Enlightenment. By 1840 there were in Europe over 40 editions of the *Letters to a German Princess*. The Petersburg editions from 1768 and 1772, published in French, were the source for the French edition in Paris in 1843 and the enlarged version there in 1859 that was the basis for the *Letters to a German Princess* printed in volumes 11 and 12 of the third series of Euler's *Opera omnia* in 1960. The work is still in use.

Mainly a popular but fundamental presentation of the principles of physics, the *Letters to a German Princess* have generated the interests of generations of scientists for the 250 years since their first publication. Euler wrote a unique and important text that is a monument to education, teaching, the history of science, and the basics and development of physics. In all this, it makes a contribution to world culture.

Prologue: physics pedagogy

The pedagogical reforms of the Common Core and Next Generation Science Standards (NGSS) require K-12 and college physics educators take a different look at the way physics is taught to twenty-first-century students. The standards push for the shift from instructor-delivered facts, concepts, and relationships to students' direct engagement with empirical evidence and to conceptual modeling with the purpose of *explaining natural phenomena*. NGSS place greater emphasis on helping students understand not only science content (*products* of science), but also the processes and practices scientists use to *create* new knowledge, including argument, observation, and experimentation. This change in teaching practice, as a reflection of the constructivist view of knowledge acquisition, makes the role of a science textbook in twenty-first century interesting and unique. Science education community recommends more rigorous hands-on student investigations, and student argumentation based on learners' prior knowledge and experiences and favors a lesser use of textbook, as a 'collection of facts'. Many physics educators believe that textbooks remove many of the exciting discovery and *aha moments* for learners.

Modern science textbooks, including online versions, are seen by teachers as resources that invite students to think, ask new questions, and design new investigations to make meaning of the world around them. In the glossary, we include a list of the most prominent physics textbooks, used both at high school and college level in the US.

By reading Euler's *Letters*, published almost 300 years ago, we come to a conclusion that teaching physics by attempting to explain everyday situations culturally familiar to the learner (Why is the sky blue, Letter 32; vibrations of bells, Letter 3; human vision, Letter 41; air pressure in guns, Letter 13, etc), is clearly not a new approach. The most remarkable in this sense are letters on physics of musical instruments, and nature of sound and music (Letters 4–8). Euler starts the discussion with the description of sound propagation through air and gradually transitions to his views on music and the evaluation of the musical pleasure to the arithmetic measurement of the proportions attached to the sounds. In Letter 9, Euler describes air as a medium of sound propagation and then transitions to the discussion of air compression, atmosphere and gas properties. He uses music as an engaging and familiar concept, which connects a variety of fundamental physics ideas into a rich tapestry, which nowadays are classified as acoustics, material science, and earth science.

In the *Letters*, we also find several cases when Euler uses the modern NGSS-endorsed pedagogy of evidence-based argument: to make sense of a phenomenon, he presents opposing points of view and provides justifications of his ideas by providing evidence from observations and explains his reasoning. For example, in letters 23 and 24, Euler presents two opposing views on light reflection off opaque objects (houses, moon, planets), arguing that 'we do not see opaque bodies by means of the rays reflected from their surface' and provides comparison to the process of light reflecting off a mirror. By including this conceptual debate in his thought process, he naturally taps into one of the areas of most severe and persistent 'student misconceptions', the study of which are also a foundation of modern science teaching strategies. In the same manner, in Letter 17 Euler argues with Newton's theory that the luminous rays are separated from the body of the Sun, and 'the particles of light thence emitted with that inconceivable velocity which brings them down to us in eight minutes.... This opinion is called the system of emanation: it being imagined that rays emanate from the Sun and other luminous bodies, as water emanates or springs from a fountain.' He gives his reasons why he disagrees with Newton's corpuscular theory and why he did not accept it, even if many thinkers of his time did[2].

[2] Musielak D 2014 'Euler and the German Princess', arXiv preprint (arXiv:1406.7417).

Author biographies

Ronald S Calinger

Ronald S Calinger received his doctorate in the history of science from the University of Chicago in 1971. He is professor of history emeritus at the Catholic University of America in Washington, DC. He specializes in the history of mathematics and the mathematical sciences during the Enlightenment and the early nineteenth century. He has taught year-long courses on the history of mathematics, the history of science, and imperial Austria. He received the Austrian Cross, for the Sciences and Arts, First Class, 1996. He was the Founding Chancellor of the Euler Society, 2003, a Dibner Library Resident Scholar, 2007 and 2010, and was invited to lecture on imperial Austria during the Mozart celebration by the Smithsonian Associates, 2006–7. He has written more than 70 research articles and reviews in such journals as *Isis, Archive for History of Exact Sciences, Science*, and *Annals of Science*. Among his eight books, he edited *Vita Mathematica: Historical Research and Integration with Teaching*, 1996, and *Classics of Mathematics,* 1999, and wrote *A Contextual History of Mathematics: to Euler*, 1999, and *Leonhard Euler: Mathematical Genius in the Enlightenment*, 2016.

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Ekaterina (Katya) Denisova is a physics educator and teacher professional developer at Baltimore City Public Schools, Maryland. She earned a Master's degree in Science Education from University of Northern Iowa, Iowa and a PhD in Physics Education from Herzen State Pedagogical University of Russia. Her research focuses on common students' difficulties and misconceptions in conceptual physics courses and strategies of framing science teaching around students' culture. Katya is an active member of the American Association of Physics Teachers (AAPT).

Elena N Polyakhova

Elena N Polyakhova graduated from the Mathematics and Mechanics Department (Astronomy Division) of the Leningrad University in 1957 and has been teaching Celestial Mechanics at the St. Petersburg State University Astronomy Department since then. The scope of Polyakhova's research interests include celestial mechanics, astrodynamics (space flight dynamics and solar sailing theory), history of natural sciences (physics, mechanics, astronomy, astrodynamics), biographies and scientific legacies of scientists (Leonhard Euler, Sofya Kovalevskaya, Michael Ostrogradsky, Alexander Lyapunov), and of classical and celestial mechanics scholars of St. Petersburg. Polyakhova has published more than 200 research articles, several books, and reference materials on celestial mechanics and history of sciences. In 2012 The Princess Ekaterina Romanovna Dashkova's Society (founded by the Dashkova's Moscow Humanitarian Institute) awarded Elena Polyakhova with the gold medal *For Freedom and Enlightenment*. Elena Polyakhova is honored by the name of a minor planet (asteroid): the numbered minor planet (NMP) *4619 Polyakhova* is named after her.

Chapter 1

Physics textbooks: origins before 1650 and principal natural philosophies and physics textbooks of the Enlightenment

1.1 Physics textbooks: origins before 1650

Up to the mid-seventeenth century, the forerunners of modern physics textbooks rested on scholastic or peripatetic physics, until they were supplanted by Cartesian mechanics. The sciences were changing, for example with the role of magic. The name *scholastic* comes from its teachers, the schoolmen, at medieval universities in the thirteenth and fourteenth centuries. Their textbooks began with the critical method and thought of Aristotle (384 BC–322 BC) with their syllogisms and added commentary from medieval Islamic and western authorities and thinkers. The scholastics sought to explain the entire Universe, stars, earth, animals, plants, and inanimate matter. Written in Latin, the language of learning of the time, their textbooks did not employ mathematics, made no references to experiments, and did not analyze any scientific problems. Their chief purpose was to confirm truths, not to make discoveries. The procedure in this early teaching of science was to read the text, which was the authority, and discuss and dispute its main principles by the use of deduction based on various interpretations of the accuracy of observations. To explain puzzling phenomena the scholastics appealed to passive occult qualities.

Having enjoyed a two-millennia dominance, scholastic physics with its Aristotelian origins was challenged and largely surpassed in the seventeenth century, mainly by the French philosopher, mathematician, and physicist René Descartes (1596–1650) with his new rationalism and the English statesman Francis Bacon (1561–1626), who appealed to an inductive, empirical method (figure 1.1). Like Renaissance humanists, both were skeptical of accepted thought from antiquity, in this case the scientific, and attacked blind submission to it. Descartes displaced it with his mechanics, its laws of bodies in motion, including the conservation of momentum. The Cartesians found scholastic explanations with occult qualities

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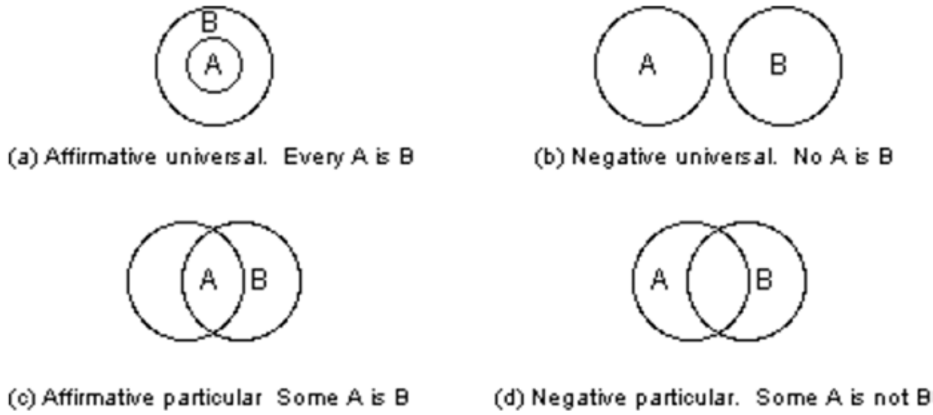

Figure 1.1. Euler diagrams for the four Aristotelian propositions.
Source: Picturing probability: the power of Venn diagrams, the richness of Eikoseograms (http://sas.uwaterloo.ca/~rwoldfor/papers/venn/eikosograms/paperpdf.pdf).

inadequate and rejected them. They ridiculed the metaphysical explanations of phenomena by the doctrine of substantial forms. These forms or ideas were the source of properties of order and unity. Among the four types of Aristotelian causes, formal, material, efficient and final, they are closest to the efficient. But the new Cartesian metaphysics did not resolve the connections in the Cartesian dualism of mind and extension in the nature of basic matter, active or passive and elastic or hard, nor did it eliminate inconsistencies.

Descartes had set forth a purely rationalist system in his greatest work, *Discourse de la Méthode* (*Discourse on Method*, 1636) (figure 1.2). Written in an elegant French, he presented an original, exhaustive deductive method by which to find truth in the sciences. He rested upon an axiomatic foundation, universal and self-evident, the structure of the sciences that provided an absolute grounding for deduction. In methodology, he subordinated observation and experiment to reason. Descartes' dualism divided all created existence into matter (beginning with extension) and mind (un-extended thinking substance). In *Principia Philosophia* (*Principles of Philosophy*, 1644) Descartes offered a mechanistic cosmology. Rejecting the idea of occult qualities and that of the void, Descartes proposed the Universe was a plenum consisting of a system of vortices (or *tourbillions*), whirlpools of ether, in the heavens with movement provided by action at contact (impulsion).

An inventor of analytic geometry, Descartes, especially in his *Principia philosophiae* (*Principles of Philosophy*), published in 1644, rooted physics in mathematics and a thorough deductive method. Descartes employed a textbook form to explain all of physics. By contrast in method, Bacon, the author of *Novum Organum Scientiarum* (*New Scientific Method*, 1620), had set out a new, empirical and inductive procedure (figure 1.3). He urged gathering relevant data, a thorough analysis of them, and the performing of organized experimentation to uncover the secrets of nature. His critical empirical methods led to modern scientific inquiry.

Figure 1.2. René Descartes.
Source: https://www.britannica.com/biography/Rene-Descartes/images-videos.

Figure 1.3. Francis Bacon.
Source: https://www.biography.com/people/francis-bacon-9194632.

Three of the leading physics textbooks used in Europe during the late seventeenth to the mid-eighteenth century were by the Genevan theologian and biblical scholar Jean Leclerc (1657–1736), the French philosopher, mathematician, and physicist Jacques Rohault (1618–72), and the English inventor Charles Morton. Leclerc wrote *Physica sive de rebus corporeis* (*Physics, or, of corpuscular bodies*, 1696), which had several editions. It took a mechanical approach with the action of small bodies underlying physics. Rohault wrote the masterful *Traité de Physique*, 1671. His last book, *Systeme de philosophie et philosophie naturelle (System of Philosophy and Natural Philosophy)* was only published posthumously in 1720. Charles Morton had an unpublished but semi-popular *Compendium Physicae.* Among these scholars Jacques Rohault, one of the most important Cartesians, mediated between Aristotle and Descartes. Accepting the Cartesian mechanistic philosophy and its laws, he attacked the uncritical acceptance of ancient authority, and his own strain of Cartesian physics required experiment. He viewed reason and experiment both as essential for scientific studies and designed instruments that included an air pump. The *Traité de physique* was rapidly translated into Latin and became the standard textbook in physics up to the mid-eighteenth century. It dealt with hydrostatics, optics, solids, animate bodies, and machines. On the recent studies of the circulation of the blood, Rohault preferred William Harvey to Descartes. The English Newtonian Samuel Clarke translated Rohault's *Traité de Physique* and added corrections. This book helped spread Newtonian thought, but Rohault had considered action at a distance absurd.

1.2 Principal Enlightenment natural philosophies

The Enlightenment had four main natural philosophies: Cartesian, Newtonian, Leibnizian, and Wolffian. Of these four, the Cartesian, just summarized, was the oldest. In continental western Europe, mid-century controversies at the Paris Academy centered around vestiges of Cartesian science and Isaac Newton's *Principia Mathematica* over the shape of Earth, the tides, irregularities in lunar motion, and the paths of comets. The quest was to find exact answers and formulate mathematical equations in differential calculus to reach them. Leiden University was central in developing Newtonian science from the start, and from the mid-century so were the Paris Academy and the Royal Prussian Academy of Sciences and Arts in Berlin (hereafter, the Berlin Academy). Leibnizian and Wolffian sciences were investigated in German lands at schools and universities. Observations, experiments, and measurements were done with new improved equipment, including timepieces, surveying equipment, microscopes, and telescopes, and by proposing annual challenge problems and expeditions. Answers to problems were sometimes known beforehand. Differing results were possible until obtaining exact answers. Leonhard Euler excelled in formulating the new differential calculus computations to achieve precise answers, so even when others found them he would occasionally be asked to devise the required mathematics. New royal and imperial science academies and their observatories in London, St. Petersburg, Berlin, and especially Paris now surpassed universities in scientific discoveries, research, and applications.

Except for the case of London, they now had funding or helped raise funds for projects. In one subject, astronomy, there was cooperation, principally in developing a new lunar theory to cover irregularities and to describe accurately the courses of comets and the transit of Venus.

Writers of physics textbooks had the work of two extraordinary scientific geniuses of the late seventeenth and the early eighteenth century to present, explain, and elaborate. The first of these was the English natural philosopher and mathematician Isaac Newton (1642–1727) (figure 1.4). In the *Principia Mathematica*, published in 1687, Newton, the Lucasian Professor at the University of Cambridge in England, developed a general dynamics providing sound definitions, introducing new concepts and recasting separated phenomena and laws. Under the universal law of gravitational attraction by inverse-squares of distances between bodies, he unified celestial and terrestrial physics. Previously, celestial motion had been seen as separate, the heavenly bodies moving in perfect circles. Newton accepted Copernican heliocentric astronomy, but even after Johannes Kepler's three laws of planetary motion projected elliptical planetary orbits in our solar system, some astronomers still attempted to modify the Copernican model. In his method of fluxions, Newton also independently invented the beginnings of calculus. In Book 1 of the *Principia Mathematica,* he wrote about prime and ultimate ratios, roughly

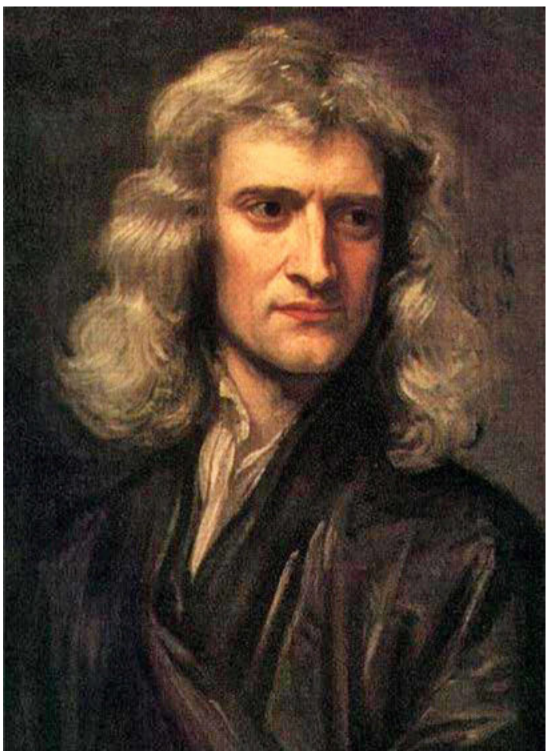

Figure 1.4. Isaac Newton.
Source: https://en.wikipedia.org/wiki/Isaac_Newton#/media/File:GodfreyKneller-IsaacNewton-1689.jpg.

derivatives. Though *most* convergency tests were a century away, he knew the importance of convergence tests for infinite series. For the new fluxional calculus, Newton later developed as central a hazy concept of limit. Although his *Principia Mathematica* is geometric in format, Newton must have had the new calculus, which offered a powerful method for solving an expanded range of problems in physics not possible with Euclidean geometry.

In his *Opticks,* published in 1704, Newton argued that the basic matter in the Universe is passive, impenetrable, indivisible atoms, advanced a corpuscular (emission) theory of light, and gave his critical empirical methodology. He accepted the concept of the void and maintained that the Universe is nearly vacuous, essentially a nutshell. In his universe matter occupies but a small portion of space and is extremely tenuous. During the Enlightenment, the *Opticks* was as influential as Newton's *Principia mathematica.* But religious leaders believed that his idea of atomistic corpuscles led to atheism. Even as science continued discarding magic and the occult, Newton made extensive alchemical studies, part of which included the occult. His experiments with some form of 'sophick mercury' or lead may have harmed his health.

In German lands a similar situation existed toward the development of the physical sciences as in the rest of western Europe. German areas, especially Brandenburg-Prussia and Hanover, had their own Enlightenment natural philosophers, teachers, lecturers, and educational specialists. The most famous were the German philosopher, polymath, mathematician, natural philosopher, logician, and historian Gottfried Wilhelm Leibniz (1646–1716), who also made significant contributions to jurisprudence, epistemology, language, and geology along with the German rationalist philosopher, mathematician, writer on politics and natural law Christian Freiherr (Baron) von Wolff (1697–1754).

Leibniz, known as the last 'universal genius', was the foremost successor to Descartes in continental rationalism (figure 1.5). In *De Rerum Originatione* (*Ultimate Origination of the Universe*, 1697), he described a Platonic universe in which God geometrizes. He believed that the mind is active, not a passive *tabula rasa*. In the debate between which is superior, faith in written revelation or reason, he opposed Pierre Bayle's Pyrrhonistic skepticism. Leibniz anonymously published *Théodicée* (*Theodicy*, 1710), which asserted that truths of faith in written revelation and reason must agree, but when they do not, he espoused the primacy of reason. At the core of his metaphysics and science, he distinguished between the necessary truths of reason and the contingent truths of facts. His deductive methodology had two axioms: the principles of contradiction (POC) and sufficient reason (PSR). He followed ancient thought that nothing is without a reason (in Latin, *nihil est sine ratione*). To cover all physical causality, he coupled sufficient reason with his metaphysical law of optimism, that the real world is 'the best of all possible worlds'. As articulated by Johann I Bernoulli and Isaac Newton, sufficient reason became the guiding principle of classical physics. Leibniz described two 'labyrinths' from the continuum of knowledge that connect them and the concept of human freedom, which could lead reason astray. Above all, his world was intelligible.

Figure 1.5. Gottfried Wilhelm Leibniz.
Source: https://en.wikipedia.org/wiki/Gottfried_Wilhelm_Leibniz#/media/File:Gottfried_Wilhelm_Leibniz,_Bernhard_Christoph_Francke.jpg.

The foundation of Leibniz's physics was his doctrine of monads, the theory of energy underlying matter. It was the cornerstone of his physics. He rejected Descartes' extension and the hard, indivisible, and passive atoms of the ancient Greeks as well as Pierre Gassendi (1592–1655). His final account of animate monads described them as geometric points of energy, well ordered in degrees of clarity. The ordering proceeds directly from God, the absolute clarity, to the basest things. The monads are windowless (autarkic), so the human body and soul had to be connected by a pre-established harmony set perfectly by God. Leibniz used the analogy of two ideally designed, synchronized clocks that run totally independently of each other. In believing that all natural events are mechanistic, he contributed to the foundations of dynamics. His universe was a dynamic organism. He based his dynamics on two scalar quantities, *vis viva* (in modern notation mv^2, where m = quantity of matter and v = velocity, essentially twice kinetic energy) and work function (now known as potential energy). He understood neither in its modern sense. As a unifying principle for his dynamics, Leibniz posited the conservation of *vis viva*.

In mathematics Leibniz independently of Newton invented and named an early stage of differential calculus. Integration and differentiation, he recognized, are directly inverse operations, the fundamental theorem of calculus. He published his first articles on calculus in *Acta Eruditorum* in 1684, '*Nova methodus* …', and in 1686, '*De geometria recondita* …', which addressed the inverse tangent problem or integral calculus. But these were so difficult that almost only Jakob (1655–1705) and Johann I Bernoulli (1667–1748) could understand them. Leibniz was a master notation builder, whose additions included the cap 'S' ($\int$) for the integral sign and the d notation, dx, dy, and dt for differentials. In a series of papers dating to 1695, he obtained the result $\pi/4 = 1 - 1/3 + 1/5 - \ldots$ (Leibniz's series). During the eighteenth century on the Continent, first the Bernoulli brothers, and most of all Leonhard Euler, Jean d'Alembert (1717–83), and Joseph-Louis (Luigi) Lagrange (1736–1813), adopted and expanded his notation and incorporated his results in their rapid and extensive elaboration of calculus.

From his student days in Jena and Leipzig, Christian Wolff was known as a brilliant epigone of the great Leibniz, who recommended him for a position at the University of Halle in Saxony, held by Wolff from 1717 to 1723. He was brought to lecture on mathematics, which was in a poor state in Halle's curriculum, and the new calculus. He wrote his first books in German rather than Latin (figure 1.6). He was the first to attempt to establish German as the main language for instruction and scholarly research. The shift from Latin to the vernacular was spreading. When Frederick William I, the King of Prussia, exiled him from Halle for disputes with the Lutheran Pietists, he proceeded to the University of Marburg, where he was a professor of philosophy and natural sciences from 1724 to 1740. He lectured on mathematics, physics, astronomy, geography, politics, philosophy, psychology, and aesthetics. Wolff was a talented and popular teacher who drew to his classes students from across Europe and Russia. His academic reputation soon began to grow.

Leibniz was the first president of the Berlin Society of Sciences from 1700 to 1716. In 1710/11 he was named an ordinary member, and in 1711 a foreign member of the Royal Society of London. In 1716 after several meetings with Peter the Great,

Figure 1.6. Christian Wolff.
Source: https://www.biography.com/people/christian-wolff-9535799.

Leibniz urged the Czar to accept Wolff as a scientific adviser in planning his new science academy in St. Petersburg. In 1723 Peter sent his secretaries to offer Wolff the vice presidency of the projected institution. Wolff declined all offers but agreed to recommend members from western Europe and became a foreign member of the Petersburg Imperial Academy of Sciences (hereafter Petersburg Academy) from its founding in 1725 as professor of mathematics with an academic financial pension. Three of the six new members whom he recommended were Daniel Bernoulli (1700–82), Jakob Hermann (1668–1733), and Joseph Nicolas Delisle (1688–1768). As was the case in many western European universities and colleges, most of the new members were not Cartesians: they were partisans of Newton, Leibniz, and Wolff, who in 1733 was named a foreign member of the Paris Academy.

Wolff introduced into German philosophy its spirit of exactness, rigor, and clarity, its *Gründlichkeit.* His many writings included subjects in mathematics and the sciences. In 1710 he wrote the four-volume *Der Anfangs-Gründe aller mathematischen Wissenschaften* (Elements of All the Mathematical Sciences)*,* which covers arithmetic, astronomy, hydraulics, and algebra. It is more for an engineer than a mathematician. It explains Wolff's axiomatic method. From the start of his career he recognized the value of good textbooks. In 1716 he edited a practical treatise, *Mathematisches Lexikon …* (Mathematical Dictionary …) that contained useful knowledge about the theoretical foundations of cartography (projections and construction), geodesy, astronomy, geography, and hydrography. It did not include Leibniz's new calculus, which had appeared in *Acta Eruditorum*. It explained new mathematical words and reports on the history of the exact sciences. It was printed in Frankfurt and Leipzig. The *Mathematisches Lexikon …* was re-edited in 1738.

From 1713 to 1725 Wolff published the seven volumes of his *Vernünftige Gedanken* (Rational Thoughts), including *Vernünftige Gedanken von der Wirkung der Natur* (Rational Thoughts on the Operation of Nature, 1723), which could not be used as a textbook because the books had new terms in German that were complicated and too difficult to follow; they were not available in Latin, the official language of learning.

Wolff, who was famous as a lecturer, had articles on physics that included his *Principia Dynamica* (Principles of Dynamics), which appeared in the *Commentarii* of the Petersburg Academy for 1726 and was published two years later in 1728, as was his *Cosmologia Generalis* (Universal Cosmology), in 1730 and again in 1737, and his *Elementa Matheseos Universae* (Universal Mathematical Elements) in 1731. The *Cosmologia Generalis* consisted of two parts: the scientific and the experimental (general principles of visible nature). Wolff's articles described the mechanics of the visible world as a perfect machine and gave the laws of motion by which it operates. No scientific explanations or proofs were given, but descriptions of experiments were useful and interesting.

With some good and some poor reasons, Wolff's philosophy has been treated as Leibnizian or Leibnizo-Wolffian. But many of Leibniz's ideas lay scattered in writings published posthumously, and thus were not yet known in 1716. Another title given is Wolffian Dogmatic Philosophy, corrected or opposed by the views of Leibniz as posthumously discovered. Leibniz's *New Essays Concerning Human Understanding*, completed in 1707, was, for example, not published until 1765. His first collected works, *Leibnitii Opera Omnia,* was prepared by Ludovici Dutens in Geneva in 1768. The word 'dogmatic' here does not have the present meaning. It signifies extremely rational.

Major differences existed between the philosophy of Leibniz and that of Wolff. All knowledge, for example, Wolff divided into mathematical, philosophical, and historical. Mathematical knowledge is the quantity of things, and historical is empirical. The foundation of Wolff's mathematics was geometric with its strict deduction and not the new analysis of the infinite, developed by Leibniz and two generations of mathematicians on the Continent during the Enlightenment, above all by Euler. Analysis of the infinite gained autonomy as a branch of mathematics and formed a triumvirate with algebra and geometry which displaced geometry from its two-millennia primacy in mathematics. While Leibniz and Wolff were both rationalists, central to Leibniz's metaphysics was the principle of sufficient reason (PSR), which Wolff considered self-evident and derived from the principle of contradiction (POC). Thus, PSR lost a definitive axiomatic place in his metaphysics. The theory of matter saw a major separation. Leibniz founded his metaphysics on monads, or later in his career geometric points of force, that are infinite in number. Wolff posited a finite number of monads as interacting primitive corpuscles, essentially atoms. Since Leibniz's monads are windowless, an ideal pre-established harmony set by God is required to connect them. Wolff's monads or atomic elements interact and dynamically influence each other. In physics textbooks three of the writers whom Wolff influenced were Ludwig Philipp Thümmig (1690–1728), the Russian Mikhail Vasilyevich Lomonosov (1711–65), and the Prussian minister and teacher Jean-Henri Samuel Formey (1711–91), who are noted later in this chapter.

For the early Petersburg Academy Wolff had an important role besides teaching the Russian students sent to him and publishing his own books. He advised the academy on what books, textbooks, and scientific instruments for experiments in physics, chemistry, astronomy, and medicine to bring from Europe to St. Petersburg. The result is the so-called 'Scientific Collection'.

1.3 Physics textbooks in Europe and North America

After physics during the seventeenth century surpassed that of Aristotle and the scholastics, alongside, at its end, with the writings of Rohault and Newton becoming known, eighteenth-century textbooks in Europe and the North American English colonies became more scientific. A prominent example is the work of the Dutch mathematician, natural philosopher, and lawyer Willem Jacob Storm van 's Gravesande (1688–1742), who studied at Leiden and the Hague and was, in 1717, appointed a member of the mathematics and astronomy faculty at the University of Leiden, in part recommended by Newton. 's Gravesande, an expert in the theories of heat and elasticity, invented the heliostat in 1719 and his ring to prove the expansion of bodies and air by heat.

Despite its title, 's Gravesande's two-volume textbook *Physices Elementa Mathematica, Experimentis Confirmata sive Introductio ad Philosophiam Newtonianam* (Mathematical Elements of Natural Philosophy Confirm'd by Experiments with an

Introduction to Newtonian Philosophy), published in 1719–21 and 1723, consists almost entirely of descriptions of experiments. Though he found the cause of gravity a mystery, 's Gravesande defended and applied Newton's mechanics against Cartesian critics. Not limited to Newton's mechanics with its laws of motion and gravitational attraction, this book also addresses parts of chapters to simple machines, astronomy—especially Newton on celestial motions—lenses and prisms in optics, properties of fluids (hydrostatics), sound (acoustics) and chemistry. It studied air (pneumatics), fire, electricity, and magnetism, all subjects of growing interest and factual knowledge during the Enlightenment search to reduce inaccuracies and errors as well as to achieve greater precision approaching exactness. Theoretical findings in these fields were at an early stage. 's Gravesande's textbook was one of the first treatises to confine itself almost entirely to the field of inanimate nature.

The textbook *Physices Elementa Mathematica* and 's Gravesande's laboratory drew scholars from across Europe, including Voltaire (1694–1778) and Samuel König (1712–57), and were essential to making the University of Leiden a citadel of Newtonian science. The text was accepted for use in many colleges and universities across Europe. 's Gravesande's next textbook, *Philosophia Newtonianae Institutianae Institutiones,* published in London in 1726, 1737, and 1747 drew principally upon Newton's *Principia* and *Opticks*.

The French-born English natural philosopher, cleric, and freemason John (Jean) Theophilus Desagulier (1683–1744) was another popularizer and promoter of Newtonian science across Europe. He lived in England from 1685 and was educated at Oxford University. In 1714 Isaac Newton, the president of the Royal Society of London, invited him to be his experimental assistant and demonstrator. Soon thereafter Desagulier was elected a fellow of that society. He wrote *Course in Experimental Philosophy*, published in 1725, and articles for the Royal Society's *Philosophical Transactions,* and he gave public lectures on mechanics, astronomy, hydrostatics, and optics. All of these followed the experimental methodology and were based largely on principles and calculations from Book 2 of the second edition of Newton's *Principia* in 1713 and its third edition in 1719. In the debate with Cartesian critics at the Paris Academy, Desagulier defended Newton on the shape of Earth as a melon flattened at the poles against the Cartesian elongated lemon.

Desagulier's study of electricity led to his discovery that electric current flows freely in some materials, like metals, which he called conductors in his *Dissertation Concerning Electricity*, printed in 1742. In that treatise he found that current does not travel freely through some materials like amber and glass, for which he introduced the term insulator. He proved the connection between kinetic energy and the velocity of an object. His principal contribution to science teaching was his invention of a planetarium, or orrery, to explain the motion in our Copernican heliocentric solar system.

The Dutch mathematician and natural philosopher Pieter (Petrus) van Musschenbroek (1692–1761) was the third leading scholar in physics education in Europe and North America and a principal figure in the transmission of Newton's science to the Continent in the early eighteenth century. Musschenbroek studied medicine and afterward philosophy at the University of Leiden under 's Gravesande and his advisor Hermann Boerhaave (1668–1738). He attended lectures of Desagulier and Newton in England in 1717. He taught mathematics and philosophy at the University of Duisberg from 1719 to 1723 and was promoted to professor of both at Utrecht, in 1723, and to professor of astronomy, in 1726, and to professor of mathematics at Leiden in 1739. He was early interested in scientific instruments. His father had made microscopes and telescopes. He examined capillary action, cohesion, magnetism, and electrostatic phenomena. In 1746 Musschenbroek together with colleagues in Holland invented the electric condenser, which Abbé Jean-Antoine Nollet (1700–70) in his books called the Leiden jar. Nollet showed its effect through 180 soldiers holding hands at Versailles.

Musschenbroek's two-volume *Epitome Elementorum Physico-Mathematicorum,* first published in 1726, with the later titles, after alterations, of *Introductio ad Philosophiam Naturalem* (Introduction to Natural Philosophy) in 1729 and *Institutiones physicae*, was his first major piece of writing. It made him one of the most influential European scholars in the natural sciences. He studied magnetism, capillary action, and cohesion. This was followed by his textbook *Elementa Physica* (Elements of Physics), published in 1734. Here he called for 'accurate observations and careful experiments', especially, for example, as concerned to magnetic attraction and to fine nature (the magnetic Musschenbroek's law). This book can be regarded as the first systematic exposition in a physics course. It gave an example of the expansion of iron from heating and described the electrical phenomenon of the Leiden jar, which collected a powerful electrical force in the bottom of the bottle. Musschenbroek also included other problems in optics, electricity, electrostatic phenomena, and heat theories. He was elected a Fellow of the Royal Society of London in 1734. After his announced creation of the Leiden jar, he was elected in 1746 an ordinary foreign member of the Royal Prussian Academy of Sciences and *Belle Lettres* in Berlin and the next year of the Royal Swedish Academy of Sciences. He was named a foreign associate member of the Petersburg Imperial Academy of Sciences in Russia in 1754.

While physics received a substantial experimental foundation in the eighteenth century, it was not yet an independent area of science, especially in college education in Europe. As a rule, physics was taught by mathematicians. For example, 's Gravesande and Musschenbroek were both professors of mathematics at Leiden. Nevertheless, three physics texts had appeared in the early eighteenth century: 's Gravesande's *Mathematical Bases of Physics Proved Experimentally*, 1720, Desagulier's *Course of Experimental Philosophy*, 1725, and Musschenbroek's *Elements of Physics,* 1734.

These three textbooks contained significant descriptions of experimental apparatuses and mechanisms, which were used to prove laws of physics and show their applications. During this period physics changed from a metaphysical Aristotelian subject, not at all like that taught in medieval universities, to a full-fledged science. Likely the inventions in electricity and their theoretical explanations by Desagulier and Musschenbroek were known to Benjamin Franklin and contributed to his success in experimental electricity. Franklin performed his famous kite experiment and wrote *Experiments and Observations on Electricity* in 1754.

A brief history of physics textbooks used in North America during the eighteenth-century Enlightenment follows[1]. It begins with textbooks employed at Harvard, Yale, Princeton, and William and Mary colleges. The teaching of physics might be divided into two parts. The first period began with the opening of Harvard University in 1638 and extended to the beginning of the downfall of Aristotelian physics which was in progress before 1687. The second, a transitional period, during which physics was evolving into a separate modern science, culminated around 1740.

The first physics taught at Yale was Aristotelian. The less metaphysical works were Leclerc's *Physica* and Rohault's *System of Philosophy and Natural Philosophy.* Like Harvard and Yale, William and Mary taught Aristotelian physics until 1736. The University of Pennsylvania and some colleges established shortly thereafter taught scientific physics from their openings. So did Princeton, which began in 1747.

The changes in astronomy at Harvard before 1659 marked the beginning of the end of the Aristotelian period. Gradually the works of Copernicus, Galileo Galilei, and Johannes Kepler were introduced and brought a consequent rejection of the Ptolemaic geocentric system. In physics Charles Morton's semi-scientific *Compendium Physicae* was already in use. This treatise was not printed but copied by students. An extract from the manuscript of a *Compendium* made in 1699 is shown in McCarthy[2]. Yale moved away from Aristotelian physics in the 1720s, William and Mary in 1736.

But Aristotelian science was not entirely forgotten. The transitional period combined the methods and principles of both the old physics and the Newtonian new physics. Some sections of the textbooks were more scientific, as they were based on experiments, observations, and mathematical principles, whereas other sections were Aristotelian. Rohault's *System of Philosophy and Natural Philosophy,* translated into English in London by John Samuel Clarke in 1723, for example, includes opinions of authorities, not unlike the practice in medieval and pre-Newtonian books. But the section on geometrical optics differs from the old traditions; it offers many diagrams and illustrations of reflection in mirrors and refraction in lenses. The astronomical section is also scientific, deriving some principles mathematically.

Another indicator that in the early eighteenth century physics was becoming a separate science was the frequent mention of lab ('Philosophical') apparatuses. Experimental apparatus could be found at William and Mary[3]. Pedagogically this meant that the teaching of natural philosophy was shifting from opinion endorsed by authorities to observations and demonstrations. This eventually led to the development of laboratory-based teaching later in the nineteenth century.

The transitional period ended in Harvard and Yale before 1740. Harvard used 's Gravesande's textbook before 1737. Its editions, translated by Desagulier from Latin to English, later employed in Yale, were published in London in 1726, 1737,

[1] McCarthy J J 1985 Physics in American Colleges before 1750 *Physics History* ed M N Phillips (College Park, MD: AAPT) 163–7.
[2] Ibid.
[3] Ibid.

and 1747. These editions explained experiments properly, giving much attention to astronomy, mechanics, and optics that were supported by experiments, and improving observations, especially in astronomy with better telescopes.

After 1750, Benjamin Franklin (1706–90) is a well-known, early advocate of science education in the American colonies. American colonial secondary schools did not offer science courses until the founding of the first academy in 1749 in Philadelphia. This 'spilling' of science teaching down to secondary schools evolved through the academy movement.

Through a series of discoveries and his famous kite experiment, Benjamin Franklin showed that electricity was not just a plaything but a natural force, like gravity. His kite experiment involved flying a kite during a storm with personal safeguards to prove that lightning is electricity. Franklin's work was to illustrate the Baconian and Enlightenment ideal that pure, theoretical science could have enormous practical importance and consequences. His invention of the lightning rod with pointed, upright rods of iron drastically reduced the threat of fire to churches and other tall buildings. What Franklin modestly described as his 'electrical amusement' made him a famous man of science and physicist. As the French Royal Minister of Finance Anne Robert Jacques Turgot (1727–81) would say to the kite flyer from Philadelphia, 'He snatched lightning from the sky and scepter from tyrants.'

The years from 1749 through 1757 were the most eventful period of Franklin's life. In addition to his research in the sciences, Franklin was thinking about ways to improve the schooling in his adopted town of Philadelphia. As a first step, in 1749 he published an article that outlined in detail his ideas on why such a school was needed and how to organize and fund it. And what would it teach? 'Those things that are likely to be most useful,' he explained, including astronomy, physics, mathematics, grammar, history, and geography. Unlike the other traditional colonial colleges, such as Harvard, Yale, Princeton, and William and Mary, the new school should be nonsectarian, the first such college in the American colonies. Franklin's proposals found enough support among prosperous Philadelphians that money was quickly raised to establish the school. The new academy, the Academy and College of Philadelphia, opened its doors early in 1751. In 1791 the college assumed the name it still bears: the University of Pennsylvania.

While the textbooks of Rohault, 's Gravesande, Desaguliers, and van Musschenbroek, reviewed above, were used traditionally in England, Denmark, Holland, Sweden and other countries, as well as in North America, Christian Wolff remained influential in natural philosophy in north German lands, especially Brandenburg-Prussia and Hanover, during the early and mid-eighteenth century. Wolff's colleague Ludwig Thümmig was largely responsible for this. Wolff had written a textbook for teaching physics and natural philosophy at Marburg University. From 1713 to 1725 Wolff had also published the seven volumes of his *Vernünftige Gedanken* (*Rational Thoughts*), including *Vernünftige Gedanken von der Wirkung der Natur*. But these were abstruse and tough to follow. Thümmig ignored them and the complicated theoretical and philosophical chapters of the textbook by Wolff for Marburg University. He shortened that textbook, methodically reworked chapters, added advice on practical experiments, made useful comments on special practical parts, and published this compilation in Latin as an independent title, his two-volume *Phisicae Institutiones Philosophiae Wolffianae in Ursus Academicos Adornatae*, which was published in Frankfurt and Leipzig in 1725 and 1726. Thümmig's short, easy to understand, and simply stated textbook was to be widely adopted in European, particularly German, educational programs. It was the first German textbook in physics. Its more readable account of Wolff's natural philosophy was popular. Thümmig wrote in Latin, because Wolff's new German terms were confusing. Wolff was building the German language for scholarly studies. Thümmig's first volume attempted to explain regular and strange phenomena in nature. It was perhaps Wolff who suggested that he address cosmology before psychology, and divide psychology into its empirical and rational branches.

Ludwig Philipp Thümmig (1697–1728) was one of the first and closest followers of Christian Wolff. After receiving his master's degree in 1721, Thümmig was an adjunct in philosophy at the University of Halle, where he mainly lectured on mathematics and physics. Also, in 1721, he was named a foreign or external member of the Royal Prussian Society of Sciences in Berlin. Thümmig followed his mentor Wolff, who was expelled from Halle, to Marburg in 1723. The next year, on Wolff's recommendation, Thümmig was named ordinary professor of philosophy and mathematics at the Collegium Carolinum in Kassel. He wished to expand further on his studies of Wolff's philosophy beyond the *Phisicae Institutiones Philosophiae Wolffianae*, but could not, for he died in 1728 at the age of 30.

To 1740 no national textbook in physics existed in Russia. While the two most notable physics texts used in St. Petersburg at the Academic University were Newtonian, the future outstanding Russian polymath, chemist, geographer, mineralogist, natural philosopher, physicist, artist, sculptor, and poet Mikhail Vasilyevich Lomonosov would make connections to Wolffian science, but opposed its monadic doctrine (figure 1.7). He together with two other Russians studied under Wolff at Marburg from 1736 to 1739. The three were sent personally by the Petersburg Imperial Academy of Sciences to learn mainly about Wolff's ideas in the sciences. Lomonosov became a follower of Wolff, who supported his studies in the sciences and languages, German and Latin. Lomonosov attended and made careful notes on all of Wolff's lectures, experiments, and treatises on the sciences and mathematics. He returned to St. Petersburg in 1741 and became an academic adjunct to the Petersburg Academy from 1742 to 1745. He began teaching physics and chemistry at the Academic University of the Petersburg Academy. He understood that success in teaching depended not only on the teacher's pedagogical level, but also the art of identifying interesting physical experiments and having quality textbooks written for students.

In 1745 no national physics textbooks could be found in the library of the Petersburg Academy. Only Lomonosov's single copy of Thümmig's *Phisicae Institutiones Philosophiae Wolffianae* that he had taken from Marburg was available. He decided to translate part of the first volume of Thümmig's textbook of 1725 from Latin to Russian and to publish it at the Petersburg Academy. His was the authorized translation. But it was also more. Lomonosov decided to shorten some

Figure 1.7. Mikhail Vasilyevich Lomonosov.
Source: http://www.kunstkamera.ru/images/floor/3_XIV_01b.j.

sections and to expand the experimental part significantly by developing his own demonstration techniques and experiments with fluids, air, light, magnets, and optics. He introduced new scientific terminology, remarked on especially difficult sections of Thümmig, and added comments on discoveries in physics since Thümmig wrote in 1725. He wrote a new preface. His work reflects the high level of physical knowledge of Lomonosov and his skill in translation from Latin.

The title of this Wolffian text is *276 Zametok po fizike i korpuskulyarnoy filosofii* (*276 Notes on Corpuscular Philosophy and Physics*). Its shorter title in Russian is *Wolfianskaya Experimentalnaya Fysyka* (*Wolffian Experimental Physics*) (figure 1.8). It was published in 1746 with 600 copies in the first printing and 1200 the next year. This book became the first physics textbook or physics manual in Russia. It was edited twice by different scholars in an enlarged version in 1760. It was a new translation prepared from Latin by professional translators with advice from Lomonosov. The new book had the title in Russian of *Wolffian Theoretical Physics.* While Lomonosov stressed the sixth chapter of Thümmig, the new book was from the seventh. A third edition appeared in 1765. Each new series printed 1200 books. On Euler's recommendation, Lomonosov's article on the elastic force of air was published in the *New Commentarii* of the Petersburg Academy in 1748, and

Figure 1.8. Wolffian experimental physics, 1760.
Source: http://vmest.ru/nuda/v-techenie-20-let-on-obuchal-studentov-fizike-himii-naturaleno/5.jpg.

another paper on electricity in 1756. Both articles influenced Euler in his preparation of the *Letters to a German Princess.* Lomonosov also reorganized the Petersburg Academy.

When his textbook was published in 1746, Lomonosov began lecturing on physics and chemistry at the Academic University not in Latin but Russian. Despite troubles with the Petersburg Academy's administrators, who briefly imprisoned him, Lomonosov became in 1746 with the help of Euler an ordinary member of the academy and afterward the rector of the Academic University. He was assisted in preparing experiments and demonstrations by the unfortunate Georg Wilhelm Richmann (1711–53), the head of the Physical Cabinet at the Petersburg Academy. Euler, who recommended Lomonosov's papers on the air and heat and cold for publication, supported him to become an ordinary member of the Petersburg Academy in 1746. So did Wolff. Richmann died tragically in a lightning experiment similar to Benjamin Franklin's. For a time, his death also halted research on lightning at the royal science academies in Paris and Berlin. The Physical Cabinet of the academy had increased its collection of instruments and devices to over 800 by 1745, and these were available to Lomonosov for preparing and presenting his demonstrations and experiments[4].

At first Lomonosov's textbook, *Wolffian Experimental Physics,* was only used at the Academic University, but it was soon widely adopted for courses in many schools and universities, including Moscow University, founded by Lomonosov in 1755. For more than 30 years after it appeared, no new manuals were written in Russian. The first was published in 1785 by Mikhail Golovin, the nephew and follower of Lomonosov. It had three editions, 1785, 1787, and 1797, with 300 books printed for every edition. Still, Lomonosov's *Wolffian Experimental Physics*

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[4] De Clercq P 2002 Scientific Instruments from Holland for Tsar Peter the Great and the Academy of Sciences in St. Petersburg *Proc. XVIII Scientific Instruments Symposium* (Moscow, 2002) pp 12–26.

remained as a famous Russian memorial treatise and was successfully used at schools and universities in Russia for the next hundred years.

Euler was in Berlin in 1746, when Lomonosov's textbook was published. Its second edition, printed in 1760 and probably read by Euler, turned out to be a useful introduction to his *Letters to a German Princess* in 1768. Euler had done much of his research in physics during his first stay in St. Petersburg from 1727 to 1741. This will be covered below in the third chapter. Although the *Letters to a German Princess* was written in French, it continued the Russian line of scientific textbooks.

In Prussia Jean Formey (1711–97) was the secretary of the philosophy department of the Berlin Academy from 1744 and the historiographer from 1745 and perpetual secretary of the academy, on the recommendation of Maupertuis, from 1748. He held the post of secretary so long that Voltaire quipped that it was indeed perpetual. Formey attempted to popularize and explain Wolffian philosophy. He wrote *La belle Wolfienne*, in 6 volumes, published from 1741–53. It began as a philosophical romance, a form that was dropped by the fourth volume. The main goal of this work, written in French, was to educate women. The Wolffian position with the opposition of Euler faced challenges in the 1750 and early 60s. A speech to celebrate King Frederick II's birthday in 1766, 'Reconciliation of the Philosophies of Leibniz and Newton', expressed a position not yet achieved. The debate over whether Newton or Leibniz deserved priority for the invention of differential calculus continued.

From 1770 to 1772, Formey published in two volumes a concise version of Wolff's physics from Thümmig titled *Abrégé de Physique.* It dealt with both experimental and theoretical physics. Euler had departed for Russia four years earlier, so his opposition did not have to be faced. Formey had been the pastor of the church that Euler attended in Berlin, and he was the father-in-law of Euler's eldest son, Johann Albrecht. From 1770 to 1773, the *Abrégé de Physique* was translated from French into German.

Chapter 2

The two princesses and the *Letters*

2.1 The two princesses: errors, lineage, and biographies

From the death of his two royal pupils (around 1810) to 2002, two widespread mistakes existed concerning to whom Euler had sent the letters. One was that the princess was an imaginary reader. This fits into the Enlightenment tradition of 'gallant' literature of anonymous private letters to an imaginary reader. An example is the Marquise Émilie du Châtelet's book *Lettres à mon fils* (Letters to my son, 1743). These letters were published anonymously, a custom of the time partly to avoid charges of arrogance. The 'son' was actually known: Mme du Châtelet (1706–49) was a lover and intellectual companion of Voltaire (1694–1778) and wrote her *magnum opus, Institutions de Physique* (Foundations of Physics, 1740), which she changed to become the revised *Institutions Physique*, 1742. It offered a metaphysical basis for Isaac Newton's dynamics. For 200 years no one could figure out who the certain 'German Princess' was. The other misconception was that there was only one princess. This account considers both errors and their resolution in 2002. It reviews the exact salutation given by Euler to the addressee: simply 'Madame'. To this Condorcet added in the preface to his 'Eulogy of Euler' in 1783, 'la princesse d'Anhalt-Dessau, nièce du roi de Prusse[1].' But no one identified the princess.

Language presented a difficulty. The French indefinite article 'une' on the title page contributed to two problems. It can be translated into English, German or Russian as the numerical 'one' or indefinite 'some' or 'a'. In Russian the title of the book was initially translated as *Letters to some [indefinite] Princess*, which was a common trope in the literature of the time. It raised doubts about the recipient of Euler's letters. Was it a real or an imaginary person? The gallant tradition in Enlightenment literature allowed for the latter. During the Enlightenment, the sending of anonymous letters to an imaginary reader was popular. In none of the

[1] Musielak D 2014 'Euler and the German Princess', arXiv preprint (arXiv:1406.7417).

doi:10.1088/2053-2571/aae6d2ch2

234 letters were the names of the addressees supplied. In a letter of 1782 and again in 1783, in his eulogy for Euler, Condorcet indicated in general who proposed writing the instructional letters and who the recipient was. This actually implied two princesses, but names were not supplied. Not until Condorcet's account was explained and justified in detail did the imaginary literary interpretation end.

The other mistake was that there was only a single princess. Researchers looked at the French linguistics of the words 'une Princesse d'Allemagne', and not 'une Princesse allemagne'. The first version of her title suggests that the princess belonged to the Prussian king's dynasty, being 'of the royal blood' and not simply 'some' German princess, 'une Princesse allemagne'. The title 'Princess of Germany' is precise and indicates the recipient is of royal blood, but Germany, while the name was used at the time, was not yet a single political entity. The Germans had been divided for two centuries by the Peace of Westphalia, in 1645. It would be another century until the Franco-Prussian War of 1870 and 1871 and the unification of Germany under Otto von Bismarck. But the core of Germany, Brandenburg-Prussia, existed. German scholars discovered the difference in to whom the letters were sent[2].

The central names in the genealogy of the princesses start with Prince Frederick Henry of Brandenburg-Schwedt and Margrave, 1771–88, who was a 'prince of royal blood' and Prince of Prussia. Frederick had jailed him for several weeks for the disorder in his regiment. Frederick II, the Great, (1712–86) referred to him in the French manner as '*mon cousin*', a traditional courtesy in Prussia at the king's court but accurate in this case. Frederick II and the margrave were half-first cousins. The margrave was a patron of the arts, especially the theatre. He was not interested in his regiment in the military, and Frederick II thought little of him.

Prussian King Frederick II, the Great (1712–86), and Prince Frederick Henry (1709–88) of Brandenburg were half-second cousins. They were not personally close. The family connection between them began with Frederick-William (1620–88), the Great Elector (in German, 'Kurfuerst', a person who belonged to the college of several electors, who voted for the Holy Roman Emperor in Vienna.) Since 1640 he had been the last Elector of Brandenburg. He was the paternal great-grandfather of both Frederick II and Frederick Henry.

The oldest son of the Great Elector Frederick-William came from his first marriage to Princess Louise Henriette of Nassau-Orange (1627–67). He was the Prince Elector (Kurfuerst) Frederick III (1657–1713) of Brandenburg from 1668 to 1701. In 1701, aided by the legal and historical writings of Gottfried Leibniz, he was elevated to be the first King in Prussia (Hohenzollern dynasty). As King of Prussia

[2] Seven names of faculty at the Vavilov History of Science and Technology Institute of the Russian Academy of Sciences, St. Petersburg division prepared the 2002 edition of the *Letters to a German Princess*. Four of these individuals and their articles are Judith Kopelevich, 'History of the Creation of "Letters" and their Addressees', pp 535–54; Nina Nevskaya, 'Saint-Petersburg Sources and Origins of Euler's Physical and Philosophical Concepts', pp 555–609; Elena Ojhigova, 'Editions of "Letters…" and Their Assessment', pp 610–6; and Jacob Smorodinsky, '"Letters to a German Princess…" of L Euler and Science of the XVIII Century', pp 617–710.

he was Frederick I: he reigned from 1701 to 1713. He belonged to the first generation after the Great Elector.

The second oldest son of the Great Elector Frederick-William came from his second marriage to Princess Dorothea-Sophie of Holstein-Gluecksburg (1636–89). He was Phillip-William (1669–1711), a half-brother of King Frederick I. He belonged to the first generation after the Great Elector.

From the second generation after the Great Elector, two lines began: the Hohenzollern royal line (line 1) and the Margraves of Brandenburg-Schwedt line (line 2). Phillip-William (the first person from line 2) became a general, he was Margrave and first owner of Brandenburg-Schwedt land.

The third generation followed with the Royal line 1 continues with King of Prussia Frederick-William I (1688–1740), who was King from 1713, and known as the 'soldier King'. In line 2 Prince Phillip (1690–1771) was Margrave of Brandenburg-Schwedt from 1711. Both were third-generation grandsons of Great Elector Frederick-William and half-first cousins.

The fourth generation had, in royal line 1, King Frederick II, the Great (1712–86), King of Prussia since 1740, and in line 2, Prince Frederick Henry (1709–88), Margrave of Brandenburg-Schwedt from 1771. Both were fourth generation, King Frederick II and Prince Frederick Henry (Euler's friend), great-grandsons of the Great Elector (Kurfuerst) of Brandenburg Frederick-William I and half-second cousins.

In Berlin Frederick Henry had attended several sittings of the Royal Prussian Academy of Sciences and *Belles Lettres* from 1748 to 1765 and had become Euler's friend. He stopped going to the academy shortly before Euler returned to Russia. Frederick Henry was the father of the two princesses and had introduced them to Euler at one of the meetings of the Berlin Academy (figure 2.1).

They were, then, half-second cousins of Frederick II. Their mother Marie Leopoldine (1716–86) was the daughter of the outstanding General Field

Figure 2.1. Friedrich August Calau, painting of the building of the Berlin Academy of Sciences and Arts, Unter den Linden, mid-eighteenth century.
Source: http://www.bbaw.de/forschung/berlinerklassik/uebersicht.

Marshall Prince Leopold I (1676–1747), a military mentor to Frederick II, known as 'the old Dessauer'. Born princess of Anhalt-Dessau, she became princess of Brandenburg-Schwedt upon her marriage, but was not 'of royal blood'. After the birth of her second daughter, she lived separately from the family for reasons given below.

The two German princesses taught in the *Letters* were Frederike Charlotte Leopoldine Louise, born 15 years earlier in 1745 in the town of Schwedt, north of Berlin and not far from the Oder River, in a line of the Prussian royal family, and her sister, Louise Henriette Wilhelmine, who in 1760 was ten. The parents, Frederick Henry of Brandenburg-Schwedt and Marie Leopoldine of Anhalt-Dessau were wed in 1739, but the marriage was violently quarrelsome. After the second birth, Marie Leopoldine had *post partum* depression and perhaps never recovered her sanity. She was banned from living at the palace in Kolobzerg Fortress in Silesia on the southern Baltic, remaining there for the rest of her life. Frederick II wanted to obtain Schwedt, perhaps a reason to keep the couple apart to assure that there were no male heirs. The mother was not 'of royal blood'.

After her parents' breakup, their daughter Frederike Charlotte was sent to the imperial Herford Abbey (Bielefeld, Westphalia), which belonged to the Holy Roman Empire, apparently to continue to be educated there (figure 2.2). Earlier a tutor and governess must have helped teach her. She also undertook studies at the royal Prussian court. In the eighteenth century, noble women were sent to abbeys to receive their education and sent only on special occasions to their homes and families. Some noble or prosperous family women who were not to be married also

Figure 2.2. Friederike Charlotte Leopoldine Louise of Brandenburg-Schwedt.
Source: https://arxiv.org/ftp/arxiv/papers/1406/1406.7417.pdf.

might live as nuns at a convent, which was a place of refuge as well as learning. Frederike Charlotte never married and was not from a male heir or marriage of Anhalt-Dessau. In 1755 she was named the coadjutor of the Herford Abbey, which made her the future successor to the princess abbess, which occurred in 1764, two years after Euler completed writing the *Letters to a German Princess*. His instruction must have helped her to prepare to run the Herford Abbey. Because of her position in the clergy, he could not cite her as the addressee of his letters. While the abbey was Lutheran, Princess Abbess Frederike Charlotte, like her female predecessors and Euler, was a Calvinist. She administered well and kindly the abbey as a noble court until 1802, when it was secularized and seized by Prussia, but she received a pension. In 1806 at age 61, she fled the First French Republican army of Napoleon to Altona (near Hamburg). The abbey had survived the scourge of smallpox. Frederike Charlotte died on 23 January 1808 and was buried in the collegiate church at Herford.

Louise Henriette Wilhelmine, born in 1750, had the title of princess of Brandenburg-Schwedt (figure 2.3). Only after her marriage on 17 July 1767 to her cousin Prince Leopold (III) Friedrich Franz von Anhalt-Dessau (1740–1817) did she become the princess or duchess of Anhalt-Dessau and later also margravine. Her father became the last margrave in 1771, a border military commander who defended the boundary areas of provinces in his region of the Holy Roman Empire. From her father, Louise Henriette was already of 'royal blood'. The

Figure 2.3. Louise Henriette Wilhelmine of Brandenburg-Schwedt.
Source: https://commons.wikimedia.org/wiki/File:Luise_von_Anhalt-Dessau.jpg.

search for the specific addressee of the *Letters* concluded in 2002 with the determination that Louise Henriette was that person. German scholars discovered the difference between the titles of the two sisters, the older princess of Brandenburg-Schwedt and born from her mother of Anhalt-Dessau, and the younger sister, born princess of Brandenburg-Schwedt and only later becoming princess (*fuerstin*) of Anhalt-Dessau because of her marriage. From her father she was 'of royal blood' and a 'niece of the king of Prussia' as well. Condorcet in his 'Éloge' of Euler in 1783 noted[3] that the princess of Anhalt-Dessau and niece of Frederick II requested some lessons in physics from Euler. It was long believed that this was Friederike Charlotte, but she was not the princess of Anhalt-Dessau; Louise Henriette was. She and Prince Leopold III were well educated, and she was a gifted painter of portraits. They had two children, a daughter who died in infancy, and a son, Frederick. Louise Henriette visited England in 1775, and later Switzerland and Italian cities. She lived with her husband in Dessau and then in the Woerlitz palace in Saxony. They had the palace constructed with a large landscaped park. They worked to create a museum of beautiful paintings and a library of antique, medieval, and modern books. Louise Henriette spent her last years in the Lisium Castle in Bavaria and kept a salon that held meetings with a circle of friends, who included Johann Wolfgang von Goethe and Alexander von Humboldt; several such salons headed by women were famous in Enlightenment literary society across Europe. The French name for these women was *female savants*. Louise Henriette died on 21 December 1811 in Dessau and was buried in the cemetery, now with St. Bartholomew Church.

2.2 The *Letters*: creation, publication and translation

The request to instruct the daughters in the sciences most likely came from their father, Frederick Henry, who was a patron of the arts, especially the theatre. He and Euler, whom he had met at Berlin Academy sittings, loved music. Frederick Henry, who occasionally visited the Berlin Academy from 1748, became friends with Euler. Frederick Henry brought his daughters to the Berlin Academy and introduced them to Euler, who visited the father's stately mansion in the center of Berlin, the Margrave's *Wellersche* palace, also called the *Prinzessinenpalais,* which the father had purchased in 1755. There the two heard musical performances and possibly played duets. Euler's instrument was the clavier, which aside from chess was his only hobby. After he became the last margrave in 1771, Frederick Henry built an operetta theater for 400 in the orangery of his castle in Schwedt 1777.

In 1783 Condorcet's 'Éloge' of Euler asserted that the princess had been the first to recommend the instruction[4]. Whoever suggested to Euler the instructional letters, they helped prepare Frederike Charlotte to rule as the Princess Abbess of Imperial Herford Abbey and for what developed into her sister's salon. This might also be the reason for the greater range of subjects. Euler at 53 was already

[3] Nicolas de Condorcet 1786 Éloge de Léonard Euler (1783) (Eulogy of Leonhard Euler) *History of the Royal Academy of Sciences* (Paris) pp 37–68.
[4] Ibid.

celebrated for his scientific research and teaching. That he found time for the instruction with all of his other work, including essentially being the acting president of the Berlin Academy, testifies to his commitment to teaching. There might be an additional reason for his being asked: his deep religious devotion, as reflected in the letters on religion. He was a Calvinist and regular member of the Huguenot *Französiche Friedrichstadtkirche* (French Reformed Church) in Berlin. He was known to read and discuss passages from the Bible each evening to his whole household, family and servants, often followed by a sermon, mainly as his pastor father had done in teaching youngsters.

Euler's first letter states that the hope of having the honor to instruct 'your highness […] in person' with 'my lessons in geometry (applied mathematics)' was growing increasingly distant. The notion that he began with a few physics lessons in Berlin in 1759 or even early 1760 at Frederick Henry's recently acquired residence seems unlikely. Euler had already taught a few sons of the nobility with Frederick's approval, and Russian students who resided at his home. He also had a *Tafelrunde* (Roundtable), a weekly luncheon meeting at his home with about a dozen colleagues and students. Roundtables were a common practice beginning with the king at his palace Sanssouci. But the time of instruction in person in 1759 or early 1760 runs counter to the locations on both sides, unless Frederick Henry delayed his departure to Magdeburg.

This was a time of turmoil during the Seven Years' War with brief, separate occupations of Berlin by Russian and Austrian troops in October of 1760. Assuming that the princesses did not leave Berlin until 1760, the personal instruction was possible. During the Seven Years War from 1756 to 1763 the Prussian monarch Frederick II, the Great had moved the royal Prussian court from Potsdam and Berlin 150 kilometers south-west to Magdeburg. Many nobles also left the city. The queen followed the king in 1759. This movement forced Euler to give his personal instruction by correspondence. He had remained in Berlin with his younger children, with his mother and a tutor at his country estate in Charlottenburg, nearby west of Berlin. He would often take walks to visit with them. In 1760 Cossack troops looted his Charlottenburg estate. In the letters Euler wrote with engaging insight and a clarity of explanation about the natural world, two characteristics of all of his published writings. For the *Letters*, he had to use terms that an educated teenager could understand. Both princesses supported the 234 letter course to its conclusion.

After writing what became the first section on general science, Euler posed difficult letters in section two on philosophy and religion with issues relating to the origin of evil, the soul, prayer, liberty, and the search for true happiness. A major change in the subjects of the letters away from those of section two came in May 1761, as part of a personal meeting between Euler and the princesses along with their father. In May Euler visited them in Magdeburg for a few weeks. It was during a trip in which he took his son Karl Johann to study medicine at the University of Halle. The drive in the wagon on that journey must have been fascinating, for Karl was talkative, while Euler was restrained but lively in conversations. Even with Prussia at war and enemy troops on its soil, Euler was not fearful about travel. After the close of section two, on modes of syllogisms and truth in philosophy, Euler started section three with an examination of the nature of matter. In Magdeburg he gave a few lessons, and Princess Frederike Charlotte informed him that she could no longer completely understand his letters.

Wolff's monads, Euler's main subject for the nine letters in May, was controversial in Prussia. During 1747, they had been heatedly and widely debated all across Berlin and Prussia, and were the subject for the annual prize of the reorganized Royal Prussian Academy of Sciences and *Belles Lettres* in Berlin that year. The German learned journals and Wolffian philosophers at universities, particularly Halle and Leipzig, had criticized Euler and the academy for their supposed partiality and injustice in granting the prize at a meeting with noble dignitaries for a paper against monads. Euler had struck first in this debate about them by writing an anonymous brochure, *Gedancken von den Elementen der Cörper, in welchen das Lehr-Gebäude von den einfachen Dingen und Monaden geprüft, und das wahre Wesen der Cörper endteckt wird* (Thoughts on the elements of bodies, in which the doctrine of simple things and monads is examined and the true essence of bodies is discovered, 1746 (E81)), which was known to be his. He now wrote that the new Royal Prussian Academy President, Pierre-Louis Moreau de Maupertuis (1698–1759), had named a committee headed by Euler himself to select the best paper. One troubling issue in 1761 was the concept of divisibility *ad infinitum.* Euler found to be wrong the Wolffian rejection of divisibility *ad infinitum* in bodies, or extension, and asked how many monads are needed to start an extension[5]. He asserted that an infinity of new classes with smaller magnitudes could be found. He saw the monadic doctrine of Wolff 'an extravagance into which the spirit of philosophizing may run[6]'. After May Euler promised to limit himself to questions in the physical sciences, and he did. His search for generalization in the sciences, under way in the first two sections of letters, clearly developed in section three.

The contacts between Euler and Frederick Henry continued after Euler returned to St. Petersburg in 1766. In 1767 Euler wrote the margrave from Russia about the publication soon of the first two volumes of the *Letters to a German Princess in* St. Petersburg, which occurred in 1768. Euler sent both volumes by mail to the margrave in Berlin. The last letter from Euler came in 1773. The third volume of the *Letters* had appeared in 1772. He promised to send six copies to Berlin. The last contact came in 1780 when the margrave visited the Imperial Court in St. Petersburg on a secret diplomatic mission about the first partition of Poland among Imperial Russia, Prussia, and Imperial Austria. They had not seen each other since Euler's departure from Berlin in 1766. Euler was ill and in bed. Frederick Henry sat next to him and held his hand for several hours. Frederick Henry could not converse about mathematics. The two spoke instead about the history and literature of ancient Rome and Greece. Most of all they spoke of Wolff's philosophy. Euler remained a major critic of Wolff's rational philosophy in physics. His opposition to Wolff's

[5] Letters, II, 10.
[6] Letters, II, 15.

philosophy continued to center on the monadic doctrine. Wolff's philosophy went against Newtonian science, which Euler defended, except for its optics. Euler, who found a number of errors in Wolff's scientific writings, still apparently highly regarded him as a talented teacher.

Euler and Frederick Henry would never see each other again. After Euler died in 1783, the margrave of Brandenburg-Schwedt sent Princess Ekaterina Dashkova (1743–1810), the director of the Petersburg Academy (1786–1796), his condolences. With the help of the academy's secretary, Johann Albrecht Euler, Leonhard Euler's eldest son, she answered him.

For the *Letters to a German Princess* Euler employed French, the second language of Europe. French was adopted at continental royal courts by the nobility and at the Berlin Academy. It also was the language of the Paris Academy, the leading center of scientific research in Europe, and the French Republic of Letters. Latin, the language of learning, would have had a limited audience. Euler's goal was to reach a wider readership with sound information about fundamentals of the physical sciences that addresses its scope, somewhat as Galileo had reached more readers outside Latin-reading scholars by publishing in the Italian vernacular his *Dialogo sopra i due massimi sistemi del mondo* (The Dialogue Concerning the Two Chief World Systems, 1632), comparing the Copernican heliocentric system with the traditional Ptolemaic-Apollonian geocentric model. Hypothetically it favored the Copernican model and ignored the construct of Tycho Brahe, which was influential at the time.

In the years after 1762, in Berlin, Euler had edited the *Letters to a German Princess,* hoping to publish them in their entirety. His and Frederick Henry's troubles with Frederick II probably accounted for their not having yet been printed. Frederick II wanted Schwedt, a territory north-east of Berlin held by Frederick Henry. He was also upset with the financial records of the Berlin Academy, which Euler oversaw. Upon his return to Russia in 1766, Euler was nearly blind and his *Letters* might have been lost in the trunks containing his unpublished writings. Occupied with the publication of multivolume books on lunar theory, lenses and optics, and integral calculus, Euler did not at first recover the individual *Letters*. Jacob Stählin (1709–85), the conference secretary of the Petersburg Imperial Academy, discovered them in a search of the trunks filled with the unpublished writings by Euler then took them directly to Count Vladimir Orlov, the director of the Petersburg Academy from 1766 to 1774, to arrange for their publication. The letters, supported by Catherine II, were to be published in either two or three volumes. In 1768 the Petersburg Academy published the first two volumes of the *Letters to a German Princess* in French. The third and final volume appeared in St. Petersburg in 1772 in Russian and in Frankfurt in 1774 in German.

Euler wrote the *Letters to a German Princess* in Berlin at the summit of his career. According to Condorcet, it was the first time a person of Euler's stature had written such a work. Its publication was part of the revolution that the printing press was making possible in the transmission of knowledge. Major figures in the scientific community usually did not issue popularizations, and these were often criticized as being inferior for a lack of command of materials. If they did write one, that effort usually happened at the start of their career or late in it. But Euler's letters were sent from 1760 to 1762 and edited in the following few years, when he was active in publication of numerous articles and completing many manuscripts for forthcoming books: his landmark work. the 775-page *Theoria Motuum Lunae* (On the theory of lunar motion in analytical form, 1772), his *Dioptricae* (three volumes on the properties of lenses, the general theory of optics, and optical instruments, 1769–71) and *Institutionum Calculi Integralis* (three volumes on the foundation of integral calculus and with the analytical form of the calculus of variations, 1768–70, to be printed in St. Petersburg).

The *Letters to a German Princess* is a work distinguished by its high quality and clarity of insights presented in the simpler, popular style of his writing. It thereby continued and improved upon the level for popularization of the sciences set by Bernard Fontenelle (1657–1757) in his *Entretiens sur la Pluralité des Mondes* (Conversations on the Plurality of Worlds, 1686). The *Mondes,* as it was known, gives evening discussions between an astronomer and a marquise. The book supported the Copernican heliocentric astronomy as most likely true, with the second and third nightly presentations describing the findings of Galileo and Kepler, and its fourth and fifth explaining, along with Cartesian physics, the relatively new Cartesian vortex cosmology, claiming whirlpools of an elastic ether existed in the heavens. Euler also considered adding to his writing important new topics, such as intelligent life on the Moon and the planets. As the full title shows, the *Letters to a German Princess* were intended for the instruction of young women in the sciences. It offered an enlarged plan for their education. Besides sewing, music, or art, they could also possibly learn about the phases of the Moon, the tides, or the effects of electricity. It is amazing that Euler was able to set aside time on Saturdays and Tuesdays to compose one or sometimes more of his letters.

Euler knew directly that three other writers of his time addressed possible women readers in the sciences. In popularity the *Letters to a German Princess* quickly surpassed even Fontenelle. The *Mondes* had reached a large readership by making the sciences more interesting and understandable. In 1726/7 the Russian noble *philosophe* Antiokh Kantemir (1708–44), who had studied at the Academic University in St. Petersburg, translated it into Russian, but could not get it published there. From 1728 to his departure from Russia in 1733 Daniel Bernoulli led the support for its publication. During the 1730s Euler, too, advocated its publication, but until 1740 church censors, holding to their opposition to the heliocentric theory, suppressed it. Euler did not sharply criticize the censors, but compared the dispute to debates in the sciences and mathematics. In response to Fontenelle, Count Francesco Algarotti (1712–64) wrote *Newtonianismo per la dame* (*Newtonianism for the Lady*, 1734). Algarotti's book depicted the rise of Newtonian science against Fontenelle's Cartesian views. His work, mostly on optics, was meant more for amusement than for instruction. At Kantemir's urging, Algarotti in 1739 visited Czarina Anne (1693–1740) in St. Petersburg to present her a copy of his book. During the visit Euler first met Algarotti. Possibly they saw each other later in Berlin.

In religion Euler stood in contrast to leading Enlightenment scholars regarding the continuing struggle between faith in written revelation in the Scriptures and reason. This explains his not being called *a philosophe* even with his extraordinary and transforming achievements. Euler wrote on religion on only two occasions. The first was his brochure 'Gedancken von den Elementen der Cörper …' (Thoughts on the Elements of Bodies …, 1746, E81) in part against Wolffians and freethinkers, who questioned the literal truth of the Bible. He labeled them rabble, troublemakers. His second effort was the section on philosophy and religion in the *Letters to a German Princess.* Both were considered poor by French *philosophes* and Wolffian philosophers at German universities.

A devout Calvinist, Euler defended core religious and spiritual positions. In the debate over the literal interpretation of Scriptures, Euler considered them trustworthy. Galileo had said that two books, the Scriptures and the Book of Nature, were being investigated. Euler compared religion to science, in that it also has contradictions and paradoxes. Still, both were accepted. Euler replied to philosophers' objections to the usefulness of prayer. He maintained that prayers were not determined by God in a pre-established harmony. He rejected Leibniz's principle for religious not scientific reasons. Euler found it mechanist in outcome. He believed that the Universe is not a machine (letter 97). The pre-established harmony was 'destructive of human liberty' (letters 91, 93, and 94). Prayers could be free depending on the circumstances surrounding them. Not even God could block their liberty. Further, he thought the soul is the principal organ in the body and resides in the brain; the union between the mind and body that occurs in humans and animals is wonderful but remains a mystery. Euler discussed life, the knowledge and properties of bodies established by experiments(letters 90, 92 and 93); the liberty of intelligent beings (85 and 91), the pursuit amid evils of true happiness ('a certain disposition', that is, virtue, to attain a perfect union with the Supreme Being, letters 103 and 104). These ideas would reappear in a little over a decade in the new United States 'Declaration of Independence', 1776.

The *Letters to a German Princess* contains information concerning Euler's mature position in major disputes in the history of science. They relate Euler's views in the competition between Cartesians and Newtonians centering at the Paris Academy on such matters as the shape of Earth (E48), the tides (E62–7), and the extent of the application of Newton's gravity (E45–61 and E68). In the *Principia mathematica,* Newton had indicated the need for further such studies. Euler differed with Newton on the nature of light and the ultimate nature of matter. Newton held light to be corpuscular, while for Euler it was a pulse, a wave (E17–20). That it could be both a particle and a wave was not yet accepted. Newton depicted the ultimate matter as hard, impenetrable, passive, and indivisible atoms. Euler reviewed the conservation of *vis viva* polemic and disagreed with Newton on indivisibility. He posited point atoms and held inertia and impenetrability to be basic characteristics of them.

Euler was at the center of the intense dispute between Pierre-Louis Moreau de Maupertuis (1698–1759), the president of the Berlin Academy, and Johann Samuel König (1712–57). Maupertuis maintained that he had first formulated and published the famous principle of least action and that it completed Newton's laws. During a visit to Maupertuis and the Berlin Academy in October 1749, König set off a priority dispute by claiming that he had an alleged fragment of a letter from Leibniz to Jakob Hermann in Basel that had a statement of the principle in 1707. (Actually Euler had first published the principle in his *Methodus inveniendi lineas curvas maximi minimive proprietate gaudentes (Method of finding curves that show some property of maximum or minimum,* numbered, E65), but credited Maupertuis with priority.) In the dispute Wolffians at the academy supported the claim for Leibniz against Maupertuis and questioned the generality of the principle. After an investigation, which included Frederick II requesting searches in Basel and collections of Leibniz's papers that did not produce the letter, members present at a meeting of the Berlin Academy in 1751 voted unanimously for Maupertuis. Those members opposed to Maupertuis did not attend. Euler read the judgment of the academy and developed the differential equations that apply to the principle of least action, which Maupertuis could not do.

Making translations of the letters from the French original has posed challenges. Translators must know the subjects of the original and its author, which requires an understanding of different grammars, in part the conjugation of verbs, root meanings and other words in their vocabulary, possible colloquial phrases, the points of view of their author, and their social and cultural contexts. Difficulties with these and alterations to the original text made over time can hinder and frustrate translations. While translations of the *Letters to a German Princess* convey Euler's thought in general, they were not without some faults. Euler was uneasy with his command of French learned in Berlin, and when he returned to St. Petersburg worked to improve the translating service at its Imperial Academy of Sciences. The initial publication of its three volumes (volumes one and two in 1768, and volume three in 1772) was made during his lifetime. The volumes were beautifully done. Euler's sight was worsening. He was growing nearly blind, so each morning his son Johann Albrecht read him the mail, papers, and likely parts of his coming publications. Between 1768 and 1774 the astronomer and student of Euler Stepan Rumovskij (1734–1812) made the translation from French to Russian. It was the first of five Russian editions by 1808. Those wishing to know Euler's exact views should consult the original Petersburg edition and, for example, not Condorcet's French edition that has omissions and changes.

Criticisms of the language and religious portions, although not the scientific matter, of Euler's French original came in 1787 from the French philosopher of the Enlightenment, historian, mathematician, and educational reformer Marie Jean Antoine Nicolas Caritat, Marquis de Condorcet (1743–94), known as Nicolas de Condorcet[7], the secretary of the Paris Academy and since 1776 a member '*Honoris Causa*' of the Petersburg Academy. Assisted by the French mathematician Sylvestre François Lacroix (1765–1843), Condorcet made an important edition in French with the printing of volume 1 in 1787, volume 2 in 1788, and volume 3 in 1789. Condorcet wrote the preface, added notes, and initially omitted letters with 'any moral, spiritual, or religious views'; the second French edition in 1795 restored these letters

[7] Condorcet was a member of the Royal Academy of Sciences in Paris and the French Academy. In mathematics he worked on probability.

on religion, which Condorcet viewed as 'anathema' to the teaching of science and reason. These showed that Euler was a devout reform Christian, in his case a devout Calvinist after he left Basel. Condorcet used the three volumes of the St. Petersburg edition in French by Euler. Condorcet's edition, in its more elegant Parisian French, was the first to make the *Letters to a German Princess* famous across Europe. These translations were published during difficult times in France amid years of crisis and the beginning of the great French Revolution. As the translators worked, these were difficult and chaotic times in Paris. Perhaps seeking to put the *Letters to a German Princess* into the better Parisian French, Condorcet made several editorial changes in language, even though Euler had written the letters in French, added notes, and at first left out the frequent courtly references to 'Your Highness', which he found 'tiresome'.

Condorcet's edition was the one that most translators, including Hunter, employed, and the second French edition restored the omitted letters. Those missing letters had been published in an edition of the first volume of the *Letters to a German Princess* in 1770 in Leipzig and Mitau (Jelgava, Latvia), then a German town and the capital of Courland (figure 2.4). Hunter noted that he, unlike Condorcet, had included them.

The chief criticism of later translations was that they altered the original by adding scientific information not known in Euler's time. He still accepted that in the heavens the stars were fixed and only comets moved, but he did not give the list of the planets that usually appears in the translations. They have Earth together with six major and four minor other planets. But the asteroids Ceres, Palas, Juno, and Vesta were not to be discovered for about a half century after Euler's death, and at home with his soon to be successor at the Petersburg Academy, Anders Johan Lexell (1740–84), he discussed computations for the orbit of Herschel's recent finding of Georgium Sidus (Uranus) on the last day of his life. Other criticisms of editions and translations of the *Letters to a German Princess* were that they deleted parts for political purposes or excised words from a time past.

The Scottish minister Henry Hunter (1741–1802) made the English translation given here. It was the first and arguably the only rendering in English. Hunter had received his Doctor of Divinity degree in 1771 from the University of Edinburgh. From 1771 he preached at the London Wall church and was the chaplain to the Scots Corporation in London. Skilled in languages, he began his translations with the Swiss writer Johann Kaspar Lavater's *Essays on Physiognomy* (1775–8). When it was well received, Hunter looked for another work to translate. He considered an essay in German by Euler on electricity. But with the popularity of Euler's *Letters to a German Princess* established, Hunter decided to translate it entirely, and did so in two volumes, published in 1795, from Condorcet's French edition. Hunter added the letters on religion that Condorcet had not. A minister, he believed that Condorcet had 'thought it proper to suppress' them. But Hunter agreed that the frequent reference to 'Your Highness' was 'an unnecessary waste of time'. He deleted it after the first reference. His chief goal was to follow the lead of Euler in educating young women, in his case British.

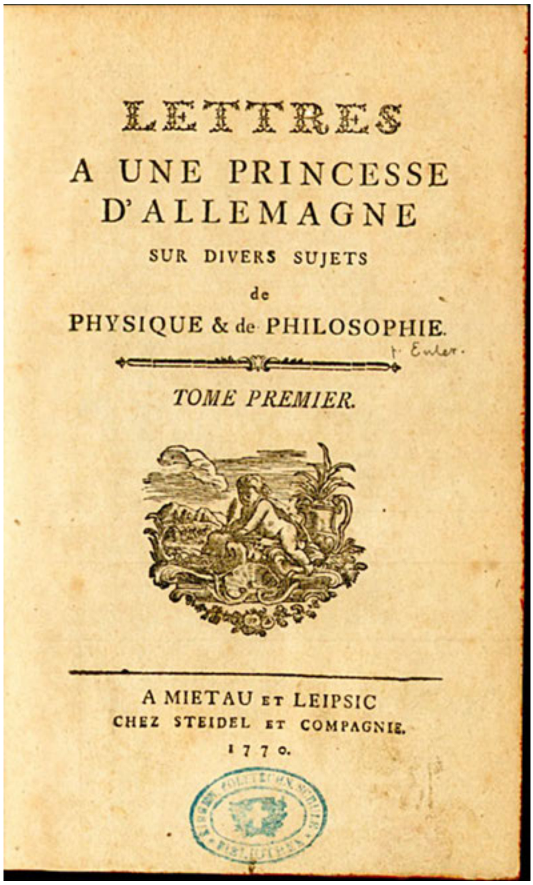

Figure 2.4. Frontispiece of *Lettres à une Princesse d'Allemagne sur divers sujets de Physique & de Philosophie* (1770).
Source: https://arxiv.org/ftp/arxiv/papers/1406/1406.7417.pdf.

In 1823 the Scottish physicist David Brewster (1781–1868), Fellow of the Royal Society, known for his research in physical optics, added a preface, a life of Euler, together with notes to Hunter's English translation of the *Letters to a German Princess*, that he called 'the most popular work that ever was written' in the sciences. In addition, Euler's theory 'that light consists of undulations of an ethereal medium[8]' was gaining particular interest at the time.

[8] Leonhard Euler 1840 *Letters of Euler on Different Subjects in Natural Philosophy* (New York: Harper and Brothers). David Brewster added the preface, notes, and a life of Euler; preface, p 12.

Chapter 3

Euler: life, research, and teaching

By 1760 Leonhard Euler was the foremost mathematician, theoretical physicist, and theoretical astronomer in Europe and imperial Russia. He was the director of the mathematics department of the Berlin Academy, a member of the Petersburg Academy (figure 3.1) and a foreign member of the Paris Academy, as well as Fellow of the Royal Society of London. His reputation as a celebrity mathematician reached across Europe from St. Petersburg to London and extended over the Atlantic to the English colonies in North America. He attained a degree of celebrity that stemmed from his genius in both theory and application, otherwise achieved in the sciences solely by Galileo Galilei, Isaac Newton, and Albert Einstein. In Euler's time, the only person better known was Voltaire.

Driven by a passion for mathematics and natural science, a near boundless energy, a phenomenal memory, a commitment to building a strong and independent base for the sciences beyond the older patronage, and a defense of reformed Christianity, Euler made extensive and profound contributions across almost the entire range of pure and applied mathematics during the Enlightenment era. The core of his research was in differential calculus and celestial mechanics, which he made the sciences *par excellence* of the eighteenth century. Euler was the principal founder of two major branches of calculus, the calculus of variations and that of differential equations, where he made hundreds of bold discoveries. Among his most famous formulas are $V - E + F = 2$ for the vertices, edges, and faces of a complex polyhedron, and the now so-called Euler identity, $e^{i\pi} + 1 = 0$, that connects the five most important mathematical constants and the three fundamental transcendental numbers (e, i, and π), and has important applications in the sciences. Though it has been called 'beautiful' and 'the most remarkable formula in mathematics', Euler never wrote it as given here. In pure mathematics he created in geometry Euler angles, the Euler triangle, and the Euler circle; in analysis the Euler number, the Euler method for approximating differential equations, and the Euler–Mascheroni transcendental constant, gamma = 0.577 215 66… ; in number theory the Euler

doi:10.1088/2053-2571/aae6d2ch3

Figure 3.1. At the Kunstkamera, the location of the early Petersburg Academy. Source: https://themathematicaltourist.wordpress.com/category/russia/page/2/#jp-carousel-490.

indicator, Euler's pentagonal formula for partition; in topology the Euler–Poincaré characteristic; for natural logarithms he made the base e = 2.718 281 8… standard. He also pioneered the differential geometry of surfaces. Through his many writings he made standard more notation by adopting such symbols as Σ for summation, π for *pi*, the circumference of a circle divided by its diameter, in trigonometry the abbreviations sin, cos, and sec for sine, cosine, and secant, and for triangles ABC opposite angles of the vertices by abc; and he increased the number of symbols by providing new ones, such as replacing Johann Bernoulli's f of x with the symbols $f(x)$ for a function of x, and providing i for the square root of -1 used in imaginary numbers and the exponential number e as the base of natural logarithms—all of these taking decades to gain acceptance. He was deeply drawn to number theory for its connections with other fields, its aesthetics, the appeal of its abstraction, and Fermat's conjectures, and he made significant contributions to it. These accomplishments in pure mathematics alone place Euler in the company of Archimedes, Isaac Newton, and Carl Friedrich Gauss as one of the four greatest mathematicians in history. Today there are dozens of formulas, equations, theorems, and objects in all branches of mathematics that carry the name of Euler.

Working in cooperation with, as well as occasional rivalry, Jean d'Alembert, Alexis Clairaut, Daniel Bernoulli, Tobias Mayer, and Colin Maclaurin, and later with Joseph-Louis Lagrange, Euler was the principal figure in transforming the older exact sciences of mechanics, astronomy, and optics into the modern mathematical sciences based on differential calculus. Euler start with this in print with the *Mechanica* (1736). In mechanics Euler, not Newton, formulated most of the fundamental differential equations before William Rowan Hamilton. Fluid mechanics has Euler equations. Euler was the creator of continuum mechanics and the mechanics of elasticity. He also advanced the study of ballistics, cartography, dioptrics, hydraulics, hydrodynamics, ship theory, and music theory. His massive and fearless computations, his extraordinary applications of analysis and analogies, his near-unerring instinct, and the clarity of his writing made him more than any other geometer since Claudius Ptolemy, in second-century Alexandria, dominant in all branches of what became the mathematical sciences.

Euler won the influential annual prizes of the Paris Academy an unmatched dozen times and led in making the Petersburg and Berlin Academies into prominent European research centers. He was also the most prolific mathematician and mathematical scientist in history. All but twenty of Euler's 868 articles and books are listed chronologically in the inventory made by Swedish mathematician Gustaf Eneström and usually referred to as the *Eneström Index,* assembled from 1910 to 1913. If correspondence is removed, the total number is about 850. Of these, 756 were published in Euler's lifetime. He predicted that it would take twenty years to publish all the work he was leaving behind; it actually took nearly fifty, with the last publication in 1830. Euler's writings are listed here as a capital E followed by its number in the order of its appearance, thus E1 is his first publication and E36 the thirty-sixth[1]. His *Opera Omnia* (Collected Works) consists of four series. Of his research articles and books, 810 fill the first three series with 74 volumes of 300–600 pages each with little repetition. Two volumes of series II, 26 and 27, on perturbation theory in astronomy are still missing. The last series, IVA, of his correspondence and twelve notebooks, which total some 4000 pages, is nearing completion with the *Repertorium* IVA 1 and six more volumes in eight parts, IVA 2–7, in print. Another volume is scheduled for 2020. The rest of the Euler correspondence and remaining parts of his manuscript heritage will be on the online platform Bernoulli–Euler Online (BEOL). Until recently, no detailed biography of Euler existed. Emil Fellmann, Rüdiger Thiele, and Philippe Henry had completed shorter biographies[2]. The first comprehensive biography was written by Ronald Calinger and published in 2016.

3.1 The Swiss years, 1707–27

Euler was born on 15 April 1707 (n.s.) in Basel in the Swiss Confederation. He was the eldest child of the Basler Evangelical Reformed minister Paul III Euler and his wife Margaretha (*née* Brucker). The next year Paul III was named pastor of Riehen, a village located about 5.4 km northeast of Basel, near the river Wiese, a tributary of the Rhine. In 1708 the family moved to Riehen. The father remained there until his death in March 1745. The village had a temperate, roughly sub-Mediterranean climate, with rich vegetation. It was known for the white blossoms of its cherry trees,

[1] Original publications can be found at http://eulerarchive.maa.org/ using the same reference numbers.
[2] For full title information, see the selected bibliography.

its colorful plum trees, and the gold and red leaves on a variety of grapes in the vineyards. Euler grew up with two younger sisters.

Euler's father, who had studied under Jakob Bernoulli, began his son's education in mathematics not with geometry but with the German cossist (algebraist) Michael Stifel's enlarged second edition (1553) of Christoph Rudolff's *Coss* (1525), the first comprehensive text on algebra in German. Coss roughly means 'something unknown'. It still solved equations, most of the second degree, largely in verbal form but had introduced some new notation. At Euler's age, Stifel's book was extremely difficult.

Leonhard did not spend much of his youth in Riehen. At about the age of eight, he was sent to live with his maternal grandmother in Basel. He now observed boats—mainly ferry boats, freighters, and canoes—at the nearby dock and on the Rhine. Euler was moved to Basel to attend its Gymnasium, a Latin school that was in a poor state and by a vote of the citizens of Basel did not teach mathematics. Like most parents, his hired a tutor, a minister, Johann Burckhardt, who sided with Johann I Bernoulli in fights over the calculus with British geometers and natural philosophers. Euler's father, his maternal grandfather, and Burckhardt were all ministers, so religion was often in his life. Likely Burkhardt and Euler discussed Copernicus, Descartes, and Newton. Euler was a talented child whose biographers agree was open, cheerful, and sociable.

In October 1720 Leonhard registered at the University of Basel in the philosophical faculty. He was thirteen. To enter a university at that age was not unusual for the time. Through hard work, his photographic memory and his brilliant mind, Euler excelled in his studies. He displayed his photographic memory by reciting by heart long passages from the ninety-five hundred verses of Virgil's *Aeneid.* It took two years to receive his *prima laurea*, roughly his bachelor's degree. He was fifteen. His undergraduate thesis was on moderation, *De temperantia.* He also gave a speech in Latin titled 'Declamatio: De arithmetica et geometria' (Rhetoric: On arithmetic and geometry), which commended the superiority of geometry. The philosophical faculty imparted a general education and the preparatory part of the university that was like secondary schools today.

In the summer of 1723 Euler passed the examination for a master's degree for a paper and lecture in Latin comparing the natural philosophies of Descartes and Newton. About this time as his son's interest turned more to mathematics and natural philosophy, Euler's father obliged him in October to register for theology to prepare to become a rural pastor. Probably with the assistance of his friend Johann II Bernoulli, Euler was able to negotiate a Saturday private tutorial with the difficult Johann I Bernoulli to read major and classical works in the sciences by Copernicus, Kepler, Rohault, Newton, and others and remove obstacles or difficulties in solving their mathematical problems. Euler found the work exhilarating. Nearly untiring, he ended with few obstacles. Perhaps Johann I Bernoulli, who now saw Euler's extraordinary genius, traveled to Riehen in 1725 with him on a visit to his father to ask that Leonhard switch to mathematics. The father agreed.

Few jobs existed for mathematicians or places to publish their articles and books. Almost no scientific journals, except for *Acta Eruditorum,* existed in central Europe.

In 1725 Bernoulli recommended Euler for a physics professorship at the University of Basel, but hiring was done by lottery, and the young Euler was not selected. In the competition for the position, he wrote a paper titled 'Dissertatio Physica de Sono', which showed him to be an original thinker able to synthesize logically consistent elements from divergent scientific traditions to construct a general theory. 'De Sono' was to become a classic. Around 1725 Euler had begun to outline in notebooks an ambitious research program for himself. The first, or Basel notebook, includes a proposal to write a substantial treatise on mechanics and gave three sections for a new mathematical theory of music composition.

When his friend Daniel Bernoulli left in 1725 to be one of the first members of the new Petersburg Academy, Euler asked to be recommended for the first open position there. One opened in 1725 in medicine and Daniel Bernoulli's recommendation was accepted. Euler began to take medical courses for the post but left Basel in April 1727 after he failed to get the physics position at the university. That year Euler came in third in a competition for the annual prize of the Paris Academy, the *Prix de Paris*, on arranging the number and height of masts of ships and the shape of sails to maximize speed. These were the top scientific awards of the century. Although Euler had not yet seen ocean ships on the seas, he did what others had not: he formulated and solved the required second-order differential equation. His paper reflected a powerful intuition in physics[3].

3.2 At the new Petersburg Academy: pathbreaking groundwork, 1727–41

Leaving Basel in April 1727, Euler went down the Rhine River to Frankfurt, crossed over Hanover and Brandenburg to Lübeck, and sailed along the Baltic coast to St. Petersburg. He arrived in May, a month and a half after the death of Isaac Newton. Russian Empress Catherine I (1684–1727) died the week before Euler arrived. He shared housing with other foreign natural philosophers, mathematicians, and physicians. Princess Praskovia, Peter I's sister-in-law, supplied furnished quarters for all single, young, male members of the academy. The early academicians, except for officials, were Swiss, German, or French. Euler worked with two other Baslers: Jakob Hermann (1668–1733), particularly on their mistaken solution of the brachistochrone problem, the curve of quickest descent by a weighted particle in a vacuum, the cycloid, before 1730, and Daniel Bernoulli (1700–82) mainly on his *Hydrodynamica,* published in 1738. The two had occasional arguments, which the diffident Euler tried to avoid. Hermann, who was designated as Nestor, the wise counselor, returned to Basel in 1731 and Bernoulli in 1733. On May 27, three days after his arrival, at a reception of the academy's president Lorenz von Blumentrost (1692–1755), Euler met the academy's secretary, Prussian jurist, and amateur mathematician Christian Goldbach (1690–1764). Euler succeeded Bernoulli as the prestigious professor of mathematics with a higher salary, though less than

[3] Calinger R S 2016 *Leonhard Euler: Mathematical Genius in the Enlightenment* (Princeton, NJ: Princeton University Press), ch 1.

Figure 3.2. Leonhard Euler by Vasilij Sokolov, 1737.
Source: http://eulerarchive.maa.org/portraits/portraits.html.

Bernoulli's. In early January of 1734 (n.s.), he married Katherina Gsell (1707–73), the daughter of the artist Georg Gsell. The father was a friend of the academy's despotic administrator Johann Schumacher; Leonhard and Katharina may have met at parties at his house. The Eulers would go on to have 13 children, only five of whom survived to adulthood. This survival rate was not unusual for the time. The Eulers' first child, Johann Albrecht, born in 1734, lived to 1800. Of the four children who followed to 1740, three daughters died as infants[4].

At the Petersburg Academy, Euler prepared the groundwork for his teaching and his research (figure 3.2)[5]. All members of the academy were required to teach. Euler taught mathematics, mechanics, physics, and astronomy at the Academic Gymnasium, where he was a member of its examination board, and served at the Academic University. In 1732 the French astronomer Joseph Nicolas Delisle (1688–1768) taught astronomy. Teaching was often done in collaboration. Delisle taught Euler how to make and record careful, professional astronomical observations. A representative and equipment from the Physical Cabinet assisted with experiments. Euler was known to devise experiments of his own for classes. At the Academic University, Newtonian texts were employed in teaching physics, especially Willem 's Gravesande's two-volume Newtonian textbook *Physices Elementa Mathematica*…

[4] Ibid., ch 3.

[5] See Nevskaya N 2002 Saint-Petersburg Sources and Origins of Euler's Physical and Philosophical Concepts *Leonard Euler: Letters to a German Princess*, pp 555–609.

(Mathematical Elements of Natural Philosophy…), published in 1719–21 and 1723, which consists almost entirely of descriptions of experiments. The Academy's library also had Pieter Musschenbroek's textbook *Elementa Physica* (Elements of Physics, 1734), which likewise stressed experiments.

Euler's research ranged across a wide spectrum in the sciences from number theory and music theory to astronomy and ballistics. His articles for the academy's journal, *Commentarii,* deal chiefly with differential calculus and rational mechanics. In number theory Christian Goldbach encouraged his research.

Earlier, others had shown that Fermat's conjecture, that all $2^{2^n} + 1$ are prime, holds for $n = 1$–4, but could not solve it for $n = 5$. Euler proved that it is wrong for $n = 5$ or $2^{32} + 1 = 4\ 294\ 967\ 297 = 641 \times 6\ 700\ 417$. His discovery that composite Fermat numbers must have divisors of the form $2^{n+1}k + 1$ made it easier to find the divisor 641 for 2^6 when $k = 10$. Euler's results appeared in volume 7 of the *Commentarii* for 1734/35, which was not published until 1743. His summation circulated at first by mail. His achievement was recognized, but his summation was criticized for a lack of the rigor that he would not attain until 1742.

A more important accomplishment was Euler's computation of the exact sum of the Basel Problem, the infinite series of the reciprocals of square integers, $1 + 1/2^2 + 1/3^2 + \ldots$. Now it is called the zeta(2) series. It had been investigated for seventy-five years. James Stirling, Nikolaus Bernoulli, Daniel Bernoulli and Euler himself had computed ever closer approximations; but the exact solution had escaped even Leibniz. Computing π at the time, Euler solved the Basel Problem in four different ways. In December 1734 he found the exact sum to be $\pi^2/6$. This was considered a remarkable triumph for him, the greatest achievement in mathematics since Gottfried Leibniz had computed $\pi/4 = 1/1 - 1/3 + 1/5 - 1/7 + \ldots$.[6] Euler used Leibniz's series in his computations. His results appeared in volume 7 of the *Commentarii*, in '*De summis serierum reciprocarum*', pp 73–86. Euler's reputation was rising.

In the mid-1730s the academy charged Euler with many tasks. He served on the commission for weights and measures, helped test fire pumps, saws, and scales, ordered the ink and paper for the academic printing press, submitted expert reviews of candidates along with possible technology projects, inspired by Johannes Segner's water wheel worked on developing hydraulic machines, wrote articles for the local paper, the St. Petersburg *Vedomosti* (*Gazette*) and in supplements to the journal *Primechaniya k Vedomost*yam (*Notes to the Gazette*) to introduce the general public to the new concepts and developments in the sciences, such as gravitational attraction, and in technology. He presented open lectures at the academy on logic and mathematics for the general public. When he became professor of mathematics in 1733, he temporarily lessened his studies in physics and astronomy. But the main assignment of Euler was to learn from Delisle and assist him in cartography and geodesy. The chief state project of the early academy and the most heavily funded was the Second Kamchatka (or Great Northern) Expedition, conducted from 1734

[6] Sanders C E 2007 Euler's solution of the Basel Problem—the longer story *Euler at 300: An Appreciation* ed R E Bradley, L A D'Antonio (Washington D.C.: MAA) pp 105–19.

to 1743, led mostly by Vitus Bering (1681–1741). Its objective was to help prepare the first accurate map of the Russian Empire. By late 1734 Euler had moved from student of Delisle to colleague, independently making astronomical observations needed to construct meridian tables. From 1735 he directed the geography department, concentrating on mapmaking.

Even with these many assignments at the academy, Euler's research proceeded extremely rapidly. In mathematics he made pioneering advances in differential equations, variational calculus, elementary and differential geometry, and integrations of algebraic functions. He wrote on average five articles a year. With the departures of Jakob Hermann and Daniel and Johann II Bernoulli, who belonged to the so-called 'dynasty' of great Swiss mathematicians, no others, except for Christian Goldbach, understood Euler's research. Goldbach had returned from Moscow in 1732 and lived in a private house on Vasilievski Island, where Euler also lived from 1734 to 1740, a short walk from Goldbach. Besides seeing each other at the academy conferences on Tuesday and Friday at 4:00 pm, they regularly visited each other's residence. In August 1735 the academy accepted Goldbach's recommendation that Euler circulate the papers for his research just for a mathematical conference to be attended by Goldbach, the physicist Georg Wolfgang Krafft, the astronomer Christian Nikolaus Winsheim, and occasionally Delisle. The *Protokoly,* the minutes of the academy, show that nobody else at the academy now took part in mathematical research. Volume 8 of the *Commentarii* for 1736, for example, has 12 papers on mathematics with one by Daniel Bernoulli and 11 by Euler, totalling 159 pages.

Two years after Euler exactly summed the Basel Problem, the principal reason for his growing reputation across Europe appeared in his 980-page *Mechanica sive Motus Scientia analytice* (Mechanics of the science of motion set forth analytically, E15, E16), his first major book. In letters Johann Bernoulli had urged him to put aside his work on music and complete the *Mechanica,* which was printed in two volumes as a supplement to the *Commentarii* for 1736. Besides translating into calculus much of Newton's *Principia*, it broke decisively with the geometric format of mechanics, as Euler became the first systematically to apply differential equations to the science of motion. It was part of his ambitious program to compute the motion of bodies that are elastic, fluid, flexible and rigid. In what would have been exhausting for many, he took twenty years to complete it. Except for Benjamin Robins, who faulted the *Mechanica's* lack of an experimental basis and 'blind submission to computation', most reviews praised it. Johann I Bernoulli hailed the *Mechanica* as a landmark book in physics; Formey lauded its use of the analytic method; Maupertuis called it an 'excellent publication'; and its review in the Parisian *Mémoires de Trevoux* in 1740 credited Euler with founding modern mechanics[7,8].

From 1735 to 1740 Euler suffered from two serious health problems. Both involved high fevers and bad headaches. In 1735 he had to calculate a confirmation

[7] Calinger R S 1996 Leonhard Euler: the First St. Petersburg Years (1727–41) *Historia Mathematica* **23** 121–66.

[8] Calinger R S 2016 *Leonhard Euler: Mathematical Genius in the Enlightenment* (Princeton, NJ: Princeton University Press), ch 4.

of the midday correction for determining the latitude of St. Petersburg. Euler completed the task in a short time that before had been time consuming. Nicholas Fuss's eulogy of Euler contends that exhausting work here weakened Euler's health but did not yet affect his sight. He had invented a new procedure for the computation that took less time. In the summer of 1738 he had a near-fatal fever. An infection caused an abscess in the right eye. From the elderly Euler, Fuss reported that there had been a complete loss of sight in the right eye. But portraits of Euler suggest a gradual weakening of sight. Harsh weather and his extensive study of maps, along later with a cataract, brought a deterioration of sight in his right eye and led toward blindness.

During the years from 1736 to 1741, Euler wrote three books after the *Mechanica.* The first, the *Scientia Navalis* (E110, E111), on shipbuilding and navigation, was finished in 1738 but not published until 1749; the *Tentamen novae theoriae musicae ex ceretissimis harmoniae principiis dilucide exposita* (*An attempt at a new theory of music composition exposed in full clarity according to the most certain principles of harmony*, E33), which sought to make music theory part of mathematics, was published in 1739. This had little influence. A major criticism was that its mathematical computations were too advanced for musicians, and for mathematicians it was too musical. Euler continued his study of acoustics and presented his figure for the speed of sound. Newton had it as 1020 feet per second, and Euler was pleased to have a higher speed than Newton's, 1134 feet per second, that was to differ only slightly from the actual value of 1125 feet per second (300 m/s). The speed of light, the fastest known speed though not then exactly determined, also fascinated Euler. For the Academic Gymnasium he prepared the two-part *Einleitung zur Rechenkunst* (*An Introduction to Arithmetic,* E17), which was printed in 1738 and 1740. The explanations and selection of topics in the *Einleitung,* including a long presentation on currency conversions for his noble students, showed Euler to be a talented instructor. This book may partly account for the absence of parts of its basic mathematics from the *Letters to a German Princess.*

The diffusion, criticism, articulation and extension of Newtonian dynamics were the main developments in eighteenth century science. At the Paris Academy the dominant Cartesians had in the 1720s heatedly debated the rising young Newtonians. The challenge was to demonstrate whether Descartes' or Newton's science was superior. The first test was finding the true shape of the Earth. Cartesian science, assuming whirlpools of ether in the heavens, predicted an elongated spindle, while in Book 3, Proposition 19, of the *Principia mathematica* Newton computed using his dynamics an oblate spheroid flattened at the poles. Is Earth elongated or flattened? At the Paris Academy criticism grew of past geodetic measurements and those of arcs of meridians in the north and south of France that were not far enough apart to indicate Earth's shape, and a rejuvenation of interest in mapmaking and the desire of craftsmen to test new surveying equipment prompted the Paris Academy to support an expedition to Lapland from 1736 to 1737 under Pierre-Louis Moreau de Maupertuis (1698–1759) and to Peru (today Ecuador) under Charles-Marie de la Condamine (1701–74). In seven articles, generally titled 'Von der Gestalt der Erden' (On the Shape of Earth, E46) in 1737 in the *Vedomosti* newspaper, Euler employed telescopic observations, assumed a more diminished flattening at the poles than had Newton, and noted results of the Lapland expedition to arrive at the more correct orange shape. While Maupertuis from his expedition supported the Newtonian shape and was hailed as a hero as 'Sir Isaac Maupertuis', bad weather and insects affected his measurements, which were later shown to be poor. Euler awaited the forthcoming pendulum studies and measures from Charles-Marie de la Condamine (1701–74) and Pierre Bouguer (1698–1758) in Ecuador. La Condamine returned to France with his results in 1744. The course of comets was another crucial item under debate. There was contradictory data, so Euler asked for more observations with improved telescopes and new differential equations. In the Newtonian–Cartesian polemic, the return of Halley's comet in 1759, the lunar theory of Clairaut and Euler with second-order approximations that gave the motion of the Moon at apogee, and the discovery of Uranus would all provide further definitive support for Newton.

Making pathbreaking advances in mathematics, Euler, in perfecting computational methods, discovered an interesting interpolation between the partial sums of harmonic series and sequences of factorial numbers 1, 2, 6, 24, … that enabled him to invent his constant, or what was later called the gamma function, that has extensive connections in the sciences. He computed gamma to equal $\lim_{n\to\infty} (1/1 + 1/2 + 1/3 + \ldots + 1/n - \ln n) = 0.577\,215\,664\ldots$. It is an important real constant and stands alongside the transcendental numbers π and e. But to the present we do not know whether gamma is rational, algebraic, or transcendental. G. H. Hardy had promised to turn over his Savilian professorship at Oxford to anyone who could prove that gamma is algebraic.

By developing three elementary classes of transcendental functions, exponential, logarithmic, and trigonometric, Euler carried out a preparatory stage for his *Introductio in Analysin Infinitorum* (E101, E102). Another of his significant computations is of the exponential function, e^x, in which the exponent is the variable and the derivative is the same as the original. He discovered that $e^x = \lim_{n\to\infty}$ of the binomial expansion $(1 + 1/n)$ and the series $e = 1 + 1/2! + 1/3! + \ldots$ that gives $e = 2.718\,281\,828$, which is accurate to nine decimal places. He also computed it as a continued fraction. He had not yet established e as a natural base for logarithms, but his massive computations gave logarithms a greater place in calculus. In trigonometry Euler transformed the Ptolemaic chords and half chords into ratios of numbers. This made the trigonometric lines into functions. In 1737 Euler discovered the cardinal formula of analytical trigonometry, in modern notation $e^{ix} = \cos x + i \sin x$.

Besides work on his books, Euler presented in 1736 his anti-Wolffian views in the sciences. While generally taciturn and not given to argument, he now publicly criticized Wolff for the first time, finding *Cosmologia Generalis* to be filled with errors and in Wolff's latest edition of his *Elementa Matheoseos Universae* questioned the theory of positive and negative infinity.

From 1736 to 1739 Euler continued to investigate number theory, especially Fermat's conjectures. Number theory was destined to fill four large volumes of his *Opera Omnia.* In 1736 Euler discovered from a reading of Diophantus the 'outstanding' little theorem that if p is a prime number and does not divide integer a, then p divides $a^{p-1} - 1$, proving this by induction, but his article did not appear in print until 1741 (E54). He did not realize that Fermat had earlier offered a proof. In 1738 he addressed Fermat's Last, or Great, Theorem: for n greater than 2, there are no positive integers x, y, and z, such that $x^n + y^n$ would $= z^n$. He proved it for $n = 4$. He wrote to France urging further searches for Fermat's writings. Euler's study of prime numbers and their distribution moved him toward the prime number theorem (PNT), which was still a conjecture not clearly stated and without a proof. His major discovery was that the asymptotic distribution of the primes among positive numbers is related to natural logarithms. In modern notation, as later stated, $\pi(N) \sim N/\ln N$ gives a good approximation of the number of primes less than or equal to N. The gap between consecutive positive integers is roughly $\ln N$. In 1736 he also resolved the Königsberg Bridge Problem. Königsberg was in East Prussia on the Pregel River. Euler showed it to be impossible to cross the five bridges without repeating any of the journey. While Euler's paper on this, '*Solutio Problematis*', contains no graphs, it is considered an early work in graph theory. That article did not appear in the *Commentarii* until 1741. In 1737 he studied the nature of space and time. He found both to be absolute. He agreed with Newton. They provided a frame for his laws of mechanics.

In another forum, Euler's reputation was quickly growing for eminence in the sciences in Europe. He shared the *Prix de Paris* in 1738 on the nature and properties of fire. Of the four Aristotelian elements, earth, air, fire, and water, fire was considered the most volatile. Along with heat, electricity, and magnetism, it was a crucial subject for research and figured in many studies of combustion, including the phlogiston theory. This was before the discovery of oxygen. In 1738 two people earned the *accessit*, Madame du Châtelet and Voltaire. Euler shared the prize in 1740 on tides and won the *acccesit* in 1739 and 1741. In even years the prize went to a paper on a theoretical topic selected from astronomy, matter theory, optics, and mechanics that had a handsome total award of 2500 *livres*. In odd years prizes were for practical papers on shipbuilding, loading, and navigation with an award of 2000 *livres*. Euler won in both categories. Papers were submitted without the author's name and only a phrase connected with the author's name in a sealed envelope.

In the summer of 1740, the Eulers seemed settled in St. Petersburg. Euler's sixteen-year-old brother Heinrich had arrived in 1735 and was studying art. In July 1740 a second son, Karl Johann, was born and survived. But already at the end of May a new monarch, Frederick II, who ascended the Prussian throne at age twenty-eight, was determined to bring Euler to Berlin and renovate the Royal Brandenburg Society of Sciences. In St. Petersburg the departure of Delisle to join the second Kamchatka Expedition worried Euler, for it left him to examine maps, a task that harmed his vision. When Empress Anna died late in 1740, life became dangerous for foreigners in St. Petersburg. When asked to cast a horoscope for the two-month-old Ivan VI, Euler passed the honor to Georg Krafft, who was known as the court astrologer.

3.3 At the renovated Berlin Academy: the summit of his career, 1741–66

At first Johann Schumacher blocked Euler's release from the academy and Russia, but relented. Euler first rejected what amounted to a salary cut from Frederick II but then accepted an equal amount along with moving funds. After a rough three-week passage on the Baltic, the Euler family arrived in July 1741 in Stettin, which conferred honors on him, and left for Berlin, which they reached on 22 July after a further three-day trip. The First Silesian War (1741–4) made funds from the government scarce. The family had to live on credit the first year in Berlin. Euler's brother Heinrich soon left for Paris and Rome to study art. After that he returned to Basel. At first the family moved near the Potsdam Bridge[9]. For New Years Day 1742, the Queen Mother Sophie Dorothea (1687–1757), the daughter of George I of Great Britain and who had known Leibniz, invited Euler to a party at her salon for polite conversation with Calvinist and Lutheran clergy, nobles, and French intellectuals. When he uttered only monosyllables, Sophie Dorothea asked him why he did not join the discussions. He responded, 'Madame … I have just come from a country [Czarist Russia] where a man's words can get him hanged[10].' This had in fact just happened to an academician in St. Petersburg. In 1742 Euler purchased a house with a large garden on what would become 20 and 21 Behrenstrasse near to the planned science academy and new observatory. He negotiated to be exempt from the quartering of troops. It took time to make repairs, so the Eulers did not move into the house until September 1743. Euler lived there until his departure from Berlin in 1766 (figure 3.3). The house, which still exists, is today across from the Comic Opera. After several modifications, except for the façade, it is now the residence of the Bavarian representative.

In Berlin, the family grew to seven. Joining their two sons were Katherine Helene, born in 1741, Christoph in 1743, and Charlotte the next year. These three survived to be adults. Euler enjoyed taking them to the zoo, especially to watch the bear cubs play, and to puppet shows. He was known to laugh heartily at the puppets. In the evening he read Scriptures to the children.

Euler remained active in teaching. To 1756 the Petersburg Academy sent several Russian students, who lived in his house. He was their advisor and tutor. With the approval of the king, he gave lessons in mathematics, astronomy, and physics to sons of the nobility. Among these was Karl Eugen, Duke of Württemberg (1728–93), the son of the Duchess of Württemberg, at their residence. Later Karl Eugen was to build palaces in the Darmstadt region and became the patron of the German dramatist and poet Friedrich Schiller (1759–1805). Euler also enjoyed playing the clavier alone or in duets and invited composers to give recitals at his house. In Berlin Euler was addicted to pipe tobacco. He preferred the 'good tobacco' from Virginia, which he ordered from England.

[9] Calinger R S 2016 *Leonhard Euler: Mathematical Genius in the Enlightenment* (Princeton, NJ: Princeton University Press), ch 6, p 187.
[10] Ibid., ch 6, p 187.

Figure 3.3. Leonard Euler by Jakob Handmann, 1756.
Source: http://eulerarchive.maa.org/portraits/portraits.html.

Neither the move to Berlin, teaching assignments, nor family and health concerns slowed Euler, who continued making of a host of discoveries in his extensive research and computations. In 1742 and 1743 he pursued his nearly exhaustive calculations preparatory to his two-volume *Introductio in Analysin Infinitorum* (E101 and E102). These computations were done in stages. Euler made calculations until blocked. When Euler or someone else discovered a way to resolve this methodical blockage, he resumed the computations.

His calculation $e = 2.718\,281\,845\,904$ was less precise than before, and he approached what in modern symbols is $e^{i\pi} = -1$. Euler defined the natural logarithm as the inverse of the exponential function and $\log(-1) = i\pi$ to be an imaginary number. In correspondence he and Jean d'Alembert debated whether it was imaginary. The comet of February and March 1742 heightened Euler's interest in orbits and celestial mechanics in general. In 1742 Delisle wrote to him about difficulties in determining the actual course of the comet that March. Euler's text, *Theoria Motuum Planetarum et Cometarum* (Theory of the motions of planets and Comets, E166), published in 1744, gives the first differential equation for computing each point in the orbits of Earth and Mars.

In 1742 the Royal Brandenburg Society welcomed the 'famous' Mr Euler. He pressed for founding a rejuvenated and expanded Royal Prussian Academy of Sciences. He addressed the issue of the source of income from the almanac monopoly. By refining computation methods, Euler increased taxes collected from

10 000 to 13 000 *Reichsthaler*, but not his projected 16 000. The king disparaged this abstract proposal that would bring debits rather than revenue.

Euler was not alone in being displeased with the slowness of progress in forming the new science academy. Count Samuel von Schmettau founded the *Nouvelle Société Litteraire* in 1743. The king did not want to lose leadership. On his birthday in 1744 the old and new *sociétés* were joined to form the new Academy of Sciences and *Belles Lettres*. Euler hoped to be president, but being frank, independent, and bourgeois he was not acceptable to the king, who wanted a French intellectual, noble, and witty in conversation with a European reputation. That person was Pierre-Louis Moreau de Maupertuis. The king even selected someone besides Euler to be director of the new academy's mathematics department. Because of Maupertuis's illness and trips to Saint Malo, Euler was the acting president of the Berlin Academy from 1753 and from 1759 after the death of Maupertuis.

The first of Euler's landmark books in Berlin, the 320-page *Methodus Inveniendi Lineas Curvas Maximi Minimive Proprietate Gaudentes, sive Solution Problematis Isoperimetrici Latissimo Sensu Accepti* (The method of finding curves that show some property of maximum or minimum, E65), published in 1744, represents an early stage of the classical calculus of variations, determining minimal and maximal lengths of curves, still treated mainly geometrically. It removed the ad hoc approach to problems and called for general solutions. Euler gave his differential equation, the first necessary condition for an extremum. At the end of chapter three, Euler gave the most elegant solution of the brachistochrone problem to date. He added two appendices. The first is a tract on the mathematical theory of elasticity. The second introduces the principle of least action. Constantin Carathéodory called the *Methodus inveniendi* 'one of the most beautiful mathematical works ever written[11].'

Apparently it was Euler, not Frederick II, who proposed the translation into German of Benjamin Robins' *New Principles of Gunnery* (1742). The result with added annotations and appendices ran to 720 pages, almost five times longer than the original. The *Neue Grundsätze der Artillerie* of 1745 (E77) began to transform the collection of separate rules into the initial scientific treatise on gunnery. Euler started to devise the first accurate differential equations for ballistic motion in the atmosphere.

In 1746 Euler worked on the vibrating string problem, which had challenged Bernoulli in the 1730s. Euler soon developed differential equations for strings with fixed ends. This problem was later addressed by d'Alembert and Lagrange. In 1747 Euler presented in the article '*De numeris amicabilis*' (E100) an algorithm for generating amicable or friendly numbers. Each number is the sum of the proper divisors of the other. In antiquity only two had been known, 220 and 284. Euler gave 27 new pairs[12].

In 1748, roughly seventy-five years after its invention, calculus lacked a framework with systematically arranged fundamental concepts. In a June 1744 letter to

[11] Calinger R S 2016 *Leonhard Euler: Mathematical Genius in the Enlightenment* (Princeton, NJ: Princeton University Press), ch 7, p 225.
[12] Ibid., ch 8, p 270.

Goldbach, Euler said that he was writing a 'prodromus in *analysin infinitorum*', a precalculus textbook. It defines and treats algebraic and transcendental functions which are evolving. The *Introductio in analysin infinitorum* is the first text of his trilogy on calculus. It has lengthy computations, with π in Book I to 127 decimal places and e in Book II to 23 decimal places. It uses the symbol π, lx for ln x, $f(x)$, and abbreviations for trigonometric functions, sin., cos., tan., cot., sec., and cosec. The second book in his calculus trilogy is *Institutiones calculi differentialis* (*Foundations of Differential Calculus*; E212, 1755).

At the academy two major disputes arose. The subject for the first in the 1747 annual prize competition was Wolff's monadic doctrine. Members were not supposed to submit papers for this, but Euler broke the rule. He wrote in French a pamphlet 'Thoughts on the Elements of Bodies' (E81) that had as an essential property impenetrability from Newton and infinite divisibility from Euclid and Leibniz, both of which Wolff rejected. Euler headed the prize committee that selected a modest paper criticizing Wolff. This angered Wolffian professors at universities, who charged him with suppressing the truth and a lack of impartiality. Euler had won this competition, but lost another in 1748 in trying to stop the election to the academy of the French physician, philosopher, hedonist, and materialist Julien Offray de la Mettrie (1709–51) to the *Belles-Lettres* department of the academy[13].

In the late 1740s a break was emerging between d'Alembert and Euler. They had argued over negative logarithms. D'Alembert believed they were real; Euler found this 'not fully correct' and offered counterexamples. He rejected $\log(-x) = \log x$ for positive numbers. He believed that negative logarithms were imaginary. When d'Alembert's health posed a difficulty, they briefly stopped the dispute. But the main point in their disagreement was fluid mechanics. In 1749 d'Alembert submitted an awkward paper on that topic for the Berlin annual prize. Euler believed it did not deserve the award. D'Alembert was informed of Euler's review of this paper.

In 1749 the Paris Academy announced its annual prize on the motion of the moon's apogee, a three-body problem with first-order approximations of differential equations to confirm the exactness of Newtonian celestial mechanics or to add a small correction.

This was a central problem for determining geographical coordinates in map-making and for navigational position at sea. Separately Clairaut and d'Alembert found initially that Newton's attraction alone could not account for the motion of lunar apogee. Clairaut briefly added an inverse fourth power to calculations. Euler was not satisfied with these results. They came at a time when the rivalry pitting Euler against Clairaut and d'Alembert was growing. In a sharp reversal the next year Clairaut found using Newton's attraction complete agreement on lunar motion. He worked with a second-order approximation but kept his computations secret. To get these Euler urged the Petersburg Academy to put as the topic for its first annual prize, whether lunar inequalities follow from Newton's laws. Schumacher and the

[13] Ibid., ch 9, p 297.

academicians agreed. As a member of the prize committee, Euler would see Clairaut's calculations. Once he did so, he refined Clairaut's equations[14].

In 1746 Euler received a job offer from the Petersburg Academy and in 1748 another to be Johann I Bernoulli's successor in Basel. He had kept in close contact with the Petersburg Academy and knew that it had lost members. He declined both offers, arguing that mathematicians are past their prime at thirty, and he was nearly forty. This view of mathematicians has endured.

At the Berlin Academy a further dispute erupted with the Wolffians, known as the Maupertuis–König affair, destined to be the foremost scientific dispute of the century. Maupertuis claimed priority for the principle of least action and its universality. In 1749 the Swiss mathematician Johann Samuel König (1712–57), a new member of the Berlin Academy, credited Leibniz with priority basing his argument on a letter of 1707 to Jakob Hermann. After a search in Leipzig, Basel, and other areas holding Leibniz manuscripts produced no letter, Euler pronounced the judgment of the academy. Defending the honors of a great man, it found that the letter was a fraud. At the academy Johann Bernhard Merian (1723–1807) provided the strongest support for Euler in this dispute. Critics claimed that this judgment was a setback for freedom of the press and reflected elitism. Euler alone was able to devise differential equations for the principle of least action and prove its universality. In 1752, with Voltaire visiting Frederick II, the debate turned from a scientific to a literary controversy. Voltaire, who was competing for royal favor, wrote separate pieces that were collected into a pamphlet *Diatribe du Docteur Akakia, médecin du Pape* (*Story of Doctor Akakia, physician to the Pope*) that ridiculed and pilloried Maupertuis. Voltaire praised Euler but presented him as Maupertuis's lieutenant-general. The pamphlet became very popular and harmed Maupertuis's reputation. Frederick II had it burned in public, and he wrote an anonymous pamphlet in French, entitled 'A letter of a Berlin academician' defending the academy.

In 1750 Euler's brother died, and his mother accepted his invitation to move from Basel to Berlin. Euler and his wife met her at a relative's home in Frankfurt. In 1753 he purchased for her an estate in Charlottenburg, near Berlin. It had residences of nobles and the monarch. Euler sent the younger children and a tutor to live with his mother. The estate was about a mile from the family house in Berlin. For relaxation, exercises, thinking in solitude, and visits with his mother and children; Euler often took walks to Charlottenburg.

Among Euler's tasks were state projects. Frederick II assigned him to grading the brine in Schönbeck salt mines, to aid in building dams and bridges in eastern Friesland, and to lead in planning the leveling of the Finow Canal between the Oder and Havel Rivers, which he did by changing the pressure in its many locks. He was also assigned to raising the height of fountains for the new palace of Sansouci and designing a lottery to increase state income. Euler won the lottery once. Wars had devastated the Prussian economy. As time passed, the king increased the number of state projects[15].

[14] Calinger R S 2016 *Leonhard Euler: Mathematical Genius in the Enlightenment* (Princeton, NJ: Princeton University Press), ch 11, p. 377.
[15] Ibid., ch 12, pp 429–31.

In the years just before 1756 the physical cause of electricity was a frequent topic of Euler. In 1752 he obtained a copy of Benjamin Franklin's *Experiments and Observations on Electricity* in French translation. He was saddened to learn in July 1753 of the death of Georg Richmann in St. Petersburg, while Richmann was making measurements related to Franklin's kite experiment on lightning. The project had not taken the necessary precautions.

Another major project of Euler during the 1750s was to improve lenses for telescopes and other optical equipment. In England the Royal Society believed that he was criticizing Newton. Actually he was attempting to develop differential equations for his ideas. He and John Dollond, who corrected Newton's optical experiments, competed as they worked to reduce chromatic dispersion.

After Euler failed in 1753 to be elected a foreign member of the Paris Academy, he was named in 1755 an associate member. Probably Clairaut helped with his appointment. From June 1754, Euler corresponded with Joseph-Louis Lagrange (1736–1813). They began with Lagrange's rich ideas in the calculus of variations and expanded to primary numbers. Euler withheld his publications on the calculus of variations, so Lagrange would receive due credit for his discoveries.

In May 1755 Euler and Johann Tobias Mayer (1712–62) decided to enter the competition for the 20 000 sterling prize of the British parliament for producing accurate enough lunar tables to find longitude at sea to within a half degree. Euler believed that Mayer had refined the equations given in his Paris prize paper of 1748 and had better observations from the latest telescopes that produced lunar tables that were superior and could provide this. Thus the prize should go to him. Euler was also secretly trying to recruit Mayer for the Berlin Academy. Euler said that D'Alembert's claim to have such lunar tables Euler was just bragging. The British could not test Mayer's tables and longitudinal method because of the Seven Years War (1756–63), and in 1762 he died. The next year his wife sent to London Mayer's final manuscript with corrected tables of lunar and solar motion. The prize was awarded in 1765, when the English clockmaker John Harrison (1693–1776) invented the first naval chronometer giving longitude, and it was tested at sea. It had better results of longitude than those found the year before by the lunar method. The British parliament had wanted an English winner. In 1765 Harrison received half of the parliamentary award, 10 000 pounds. Mayer's wife was given 300 pounds. Euler was surprised and delighted to receive a gift of 3 pounds from the Board of Longitude of the British parliament. Although the marine chronometer was widely acquired and employed, for more than a century the British Nautical *Almanacks* contained Mayer's lunar tables computed by Euler's method and equations. In his second lunar theory, developed continuously from 1764 to 1772, Euler developed lunar tables better than the semi-empirical ones of Mayer and Clairaut that were clearer and easier to use.

The year 1760 had brought a new prize and war losses. Euler wrote papers for his sons. His son Karl, a physician, won the Paris prize on the average variable motion of planets. That year was roughly the midpoint of the Seven Year War. It was going badly for Frederick II. In October 1760 the Russians and later the Austrians entered Berlin. Many had fled the city before the Russian arrival, but not Euler. The Russian general Count Gottlob Totleben had promised safety for Euler's estate. But when the Prussian troops withdrew, the Cossacks advanced rapidly before the general's assurance reached them. Totleben exclaimed that he had not come to make war on the sciences. He reimbursed Euler, who submitted bills for all losses. Empress Elizabeth (1709–1762) paid him a second time by sending him an additional 4000 rubles.

By 1761 relations with Frederick II had so deteriorated that Euler seriously contemplated leaving Berlin. He was cosmopolitan and not bound by early national interests. A letter of May 17 to Gerhard Müller, the secretary of the Petersburg Academy, declared that he had sold the Charlottenburg estate, so no reason existed to remain in Berlin. In 1763 he was alarmed that Jean d'Alembert would be offered the presidency of the Berlin Academy and accept, but he declined, choosing to remain in Paris. Instead d'Alembert recommended Euler for the post, which was not granted, and a raise, which was done. From his salary, prize monies, and investments, Euler was wealthy for a man of the middle class. D'Alembert asked to meet with Euler and was astounded by his colossal memory, his extraordinary knowledge by heart of computation formulas, and the clarity of his logic. The two became good friends. By correspondence d'Alembert became 'the secret president' of the Berlin Academy. From the close of the Seven Years' War in 1763–6, Euler remained at the Berlin Academy, even as his position deteriorated. Frederick II assumed the presidency and took away from Euler the selection of new members. He had opposed the selection of Gotthold Lessing, famous German writer, as a member. Euler also lost control of finances. The king established an economic commission. A nasty quarrel erupted over the sale of almanacs, which was managed by David Köhler. Euler defended Köhler, who actually pocketed some of the tax monies collected. To raise income for the state, Euler proposed a new lottery and a porcelain industry.

From 1762 to 1766, Euler was working on four books besides his articles. He was writing three multivolume books that were not published until he was in St. Petersburg. For the first, the *Letters to a German Princess* (E343, E344, and E417), he gathered and edited the individual letters and wanted them to be a textbook. It was completed and ready for publication before he left Berlin, but funds did not exist for the printing and Frederick II did not support it. After the Seven Years' War the Berlin Academy had only limited funds. In August 1766, Euler submitted to the academy his monumental three volume *Institutionum Calculi Integralis* (E342, E366, and E385) that completes his trilogy on calculus. In 1765 Euler submitted an article on dioptrics. His *Dioptricae* (E367, E386, and E404), published from 1769 to 1771, is a didactic work that contains the general theory of optics and lenses for telescopes and microscopes. On the title page Euler indicated that he was assisted in his computations by his sons Johann Albrecht and Karl together with Wolfgang Krafft and Joannis Lexell. From 1764 Euler worked continuously on his 775 page landmark text in three lengthy volumes, *Theoria Motuum Lunae, Nova Methodo Pertractata* (Theory of lunar motion, treated by means of a new method, E418). It was the most advanced lunar theory of the time.

In 1765 one text was published in Berlin. It was Euler's *Theoria Motus Corporum Solidorum* (E289, E418). This landmark book is the final piece in his thirty-year study of mechanics. Euler had previously provided differential equations for fluid, elastic, and flexible bodies. This work stresses rotational motion, which was new. The *Theoria Motus Corporum Solidorum* designed the mathematical algorithms and methods for dealing with the motion of rigid or hard bodies. It is often called Euler's *Second Mechanics*.

In 1765 Euler no longer attended meetings with the Berlin Academy. Early the next year two requests for permission to leave were met with silence. When Russian Catherine II (1728–1796) learned of Euler's interest in returning to Russia, she was determined to have a distinguished science academy and ordered the Russian ambassador to Berlin to negotiate with him and grant him whatever he wanted. He was to receive an annual salary of 3000 gold rubles, roughly twice the first offer, a widow's pension of 1000 gold rubles for his wife, a professorship in the Petersburg Academy with a salary of 1000 gold rubles for Johann Albrecht, and positions for his younger sons. The academy was also to help recruit about seven young scholars to help with Euler's research, especially computations. They became the Euler circle.

3.4 Return to the Petersburg Academy: a vigorous autumn, 1766–83

In 1766 Euler's first two requests for release from the Berlin Academy received no reply from Frederick II. But Catherine had already had her ambassador negotiate with Euler. Frederick II did not want to antagonize his 'dear sister'. In February 1766 permission was granted after a third letter. On 29 May the Euler family, except for the youngest son, and servants left. A small group, including Frederick Henry and his two daughters, came to say farewell. The family traveled first to Warsaw, where Euler received honors. On 25 June they arrived in St. Petersburg. Euler was fifty-three. Catherine II generously sent him 10 800 rubles to purchase a two-storey house with furniture on Vasilievski Island on the Great Neva River. Thus began another very busy period in Euler's life, in which he wrote 415 articles and books, or roughly half of his total publications (figure 3.4).

In May 1771 a great fire broke out on Vasilievski Island behind the Petersburg Academy. About 550 wooden and a few brick houses were lost. A Basler handyman, Peter Grimm, put a ladder to the second floor of the Euler house and saved Euler, nearly blind from a cataract, by carrying him over his shoulder. Euler wanted to stay and save his papers. Catherine II covered the loss in the fire by giving him 6000 rubles to reconstruct the house and buy furniture. In September 1771 Euler hoped to have his vision restored. Euler had followed new operations in Paris on the subject of cataracts. An eye surgeon, Baron Jakob Michael von Wenzel (1724–90), had developed early cataract surgery. He was brought to St. Petersburg. There was rejoicing when after an operation Euler's vision was restored. But within a month he was again nearly blind and in occasional pain. He said that with the loss of sight he had 'one less distraction'.

In 1773 Euler asked Daniel Bernoulli to recommend an assistant from Basel. Bernoulli made an excellent choice, Nicholas Fuss (1755–1826), who served as Euler's

Figure 3.4. Leonhard Euler by Joseph Darbes (1778).
Source: http://eulerarchive.maa.org/portraits/portraits.html.

personal secretary. He became important in the Euler circle, which included Johann Albrecht Euler, Mikhail Evseevic Golovin, Anders Johann Lexell, and Stepan Rumovskij. They worked in the study of Euler's house with a large table in the middle of the room. Euler would exercise by walking around the table when they were not present. From Euler's rubbing his hands on it during his walks, its edges were shiny. In September of 1773 French writer Denis Diderot (1713–1784), the editor of the multivolume *Encyclopédie* and critic of religion, arrived for a visit. In November he spoke at the academy. Euler, who chaired the session, treated him quite properly. There was no argument at the academy, as is sometimes mistakenly thought. There was, however, one later with Franz Aepinus, a German physics professor (1724–1802). In November 1773 Euler's wife Katharina died. Euler refused to follow the custom of the elderlys being cared for by their children. At Christmas 1775 he proposed to a friend of his wife, but his sons blocked this. He next quietly proposed in 1776 to his wife's half-sister, Salome Abigail, *née* Gsell. They married in July.

In January 1783 Princess Ekaterina Romanovna Dashkova (1743–1810) was named by Empress Catherine II to be director of the Petersburg Academy[16]. She

[16] Calinger R S and Polyakhova E N 2007 Princess Dashkova, Euler, and the Russian Academy of Sciences *Leonhard Euler: Life, Work and Legacy* ed R E Bradley and C E Sandifer (London: Elsevier), pp 75–95.

initially asked to decline the offer but knew that Catherine II would insist[17]. Princess Dashkova met with the academicians and announced that she would be available for discussions of issues. She asked Euler to be on his arm as she entered the academy. He was honored and agreed. He had not been visiting the academy since 1777. Princess Dashkova describes the inauguration scene in detail in her *Memoirs*[18]. Princess Dashkova was to end the mismanagement of the academy and raise more funds for it. In 1783 she was named a member of Benjamin Franklin's American Philosophical Society in Philadelphia by Franklin himself; she was the only woman member for the next eighty years. In 2018, the American Philosophical Society in Philadelphia issued a greeting to the participants of the XXIII International Dashkova Readings, dedicated to the 275th anniversary of the birth of Princess Dashkova: 'The society proudly notes that incomparable Princess Dashkova invited in 1789 to the Philosophical Society, was the first and remains the only woman so honored for another 80 years … Clearly Benjamin Franklin recognized Princess Dashkova as an intellectual equal. Today we would say she was an extraordinary—possibly unparalleled—woman of her time[19].' In 1789 Dashkova reciprocated by making him the first American member of the Russian Academy[20].

Princess Dashkova initiated construction of the new building of the Russian Academy of Sciences next to the Kunstkamera. It was built in 1790s after Euler's death. Euler's scientific legacy continues to be researched and developed in the Russian Academy of Sciences and the Berlin Academy of Sciences. Euler's scientific legacy continues to be researched and developed in the Russian Academy of Sciences (figure 3.5) and the Berlin Academy of Sciences (figure 3.6).

In his last years Euler remained active in fields that included number theory with studies of pentagonal numbers and the prime number theorem, rational mechanics that includes his proving in three dimensions the vectorial character of moments in physics, the perfection of his treatment of the buckling of columns in 1780, and perturbations in the orbits of Jupiter and Saturn in astronomy, together with hydrodynamics, elasticity and acoustics. In April 1773 his last book was published, *Théorie Complete de la Construction et de Maneuver des Vaisseaux, Mise la Portée des (!) Ceux, qui s'Appliquant à la Navigation* (Complete theory of the construction and maneuver of ships brought into the reach of everyone involved in navigation, E426). It was his second ship theory. Euler also served on the academy's technological committee. He was convinced that the improvement of telescopes and microscopes was critical to the advancement of the sciences. He reviewed proposals, including three from Ivan Petrovich Kulibin (1735–1818), for a one-arch stone and wooden bridge across the Neva river. Since Kulibin, the engineer and

[17] Woronzoff-Dashkoff A I 2008 *Dashkova: A Life of Influence and Exile* (Philadelphia, PA: American Philosophical Society).

[18] Dashkova E 1995 *The Memoirs of Princess Dashkova: Russia in the Time of Catherine the Great* ed K Fitzlyon (Durham, N.C.: Duke University Press).

[19] Tychinina L 2018 *Catherine the Great and Princess Dashkova: View from XXI Century*.

[20] Wilford J N 2006 'Russian Princess Stands With Franklin as Comrade of the Enlightenment' *New York Times* (March 14, 2006) https://www.nytimes.com/2006/03/14/science/14prin.html.

Russian inventor, had a workshop close to the Euler's house, as long as it was possible Euler would often walk there.

To 1783 Euler continued in his study his complex computations with his circle. On 18 September 1783 Lexell came to have lunch with Euler and work on problems

Figure 3.5. The building of the Russian Academy of Sciences, Photograph by Carlos Dorce. Source: https://themathematicaltourist.wordpress.com/2012/12/30/the-russian-academy-of-sciences/#jp-carousel-490.

Figure 3.6. The Berlin Academy of Sciences and Arts in the twentieth century. Source: http://blogs.plymouth.ac.uk/artsinstitute/wp-content/uploads/sites/60/2016/06/akademie_der_wissenschaften.jpg.

Figure 3.7. Leonhard Euler's bust at the Kunstkamera by Jean-Dominique Rachette (1785). Source: http://www.eduspb.com/node/1535.

of balloon aerostatics from the ascents of the Montgolfier brothers in Paris and computing the path of Herschel's planet, Uranus. Euler told Lexell that he was finally totally blind. After lunch Euler felt faint and took a nap. At about 5:00 pm he was playing with a grandson whom he had tutored earlier. He asked his wife for a second cup of tea. Suddenly his pipe fell from his hand. When he bent to pick it up he clasped his hands to his forehead and uttered '*Ich sterbe*' ('I am dying'). He suffered a stroke and lost consciousness. After several unsuccessful efforts to revive him, he was pronounced dead at 11:00 pm. The death of the great Euler was news across Europe. It was reported by the four major science academies in London, Paris, Berlin, and St. Petersburg as well as in Basel, Lisbon, Munich, Stockholm, and Turin. Euler's death was treated as a notable public loss. He was buried in the Evangelical Lutheran section of the Smolensk Cemetery on the Vasilievsky Island. Condorcet and Fuss delivered the two major eulogies for him.

At the Petersburg Academy Princess Dashkova demanded that a half-length marble bust be made by a French sculptor, Jean-Dominique Rachette (figure 3.7). Its officers installed that work in the library hall at the Kunstkamera, the main building of the academy at that time. This scene was depicted in the silhouette by

Johann Friedrich Anthing, a German artist. The second figure on the left holding the bust is Euler's eldest son Johann Albrecht Euler (figure 3.8).

St. Petersburg had such heavy rains that by the early nineteenth century, not even Fuss, who participated in the funeral, could find the modest gravestone of Euler. It was rediscovered in 1832. The decision was made to move Euler's grave to the

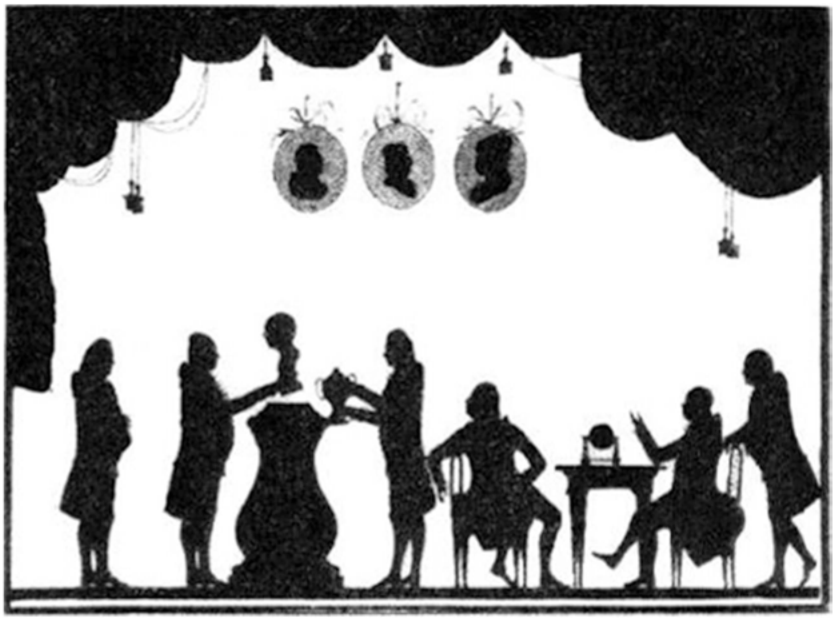

Figure 3.8. Silhouette by Anting, 1784.
Source: http://az.lib.ru/img/l/litwinowa_e_f/text_1892_eiler/text_1892_eiler-2.jpg.

Figure 3.9. Euler's gravestone, Lazarev Cemetery, St. Petersburg.
Source: http://amorbidfascination.blogspot.com/2010/03/lazarev-cemetery-st-petersburg.html.

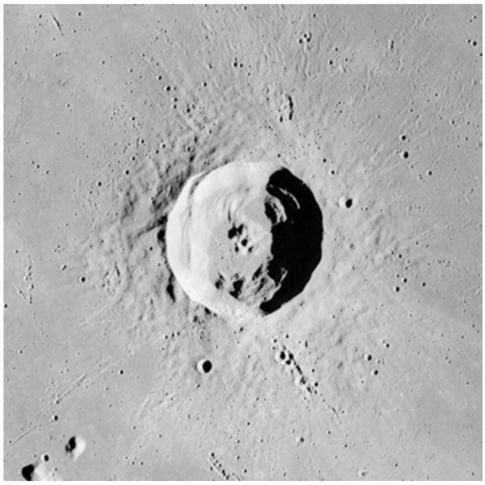

Figure 3.10. Leonhard Euler lunar crater.
Source: https://en.wikipedia.org/wiki/Euler_(crater)#/media/File:Euler_crater_AS17-M-2922.jpg.

Lazarev Cemetery of the Alexander Nevskiy Monastery (Lavra) located near St. Petersburg's main street, Nevskiy Prospect. A simple monument (figure 3.9), which still exists, was constructed out of pink Finnish granite with the inscription *Leonhardo Eulero*, *Academia Petropolitana* 20 meters away from Lomonosov's tombstone.

In 1973, a stony asteroid from the inner regions of the asteroid belt, approximately 17 kilometers in diameter, was named 2002 Euler to honor Euler's contributions to the developments of science and mathematics. Euler's name was also given to a 28 km lunar crater, Leonhard Euler (figure 3.10).

Chapter 4

Selected letters from Volume 1 (letters 1–79)[1]

In this and the next chapter, we have selected some of the most important topics of Euler's letters, which may be of interest to physics educators or those with interest in history and philosophy of science. The excerpts from the *Letters* included in this chapter are taken from an 1802 publication by Murray and Highley, J. Cutrell, Vernor and Hood, Longman and Rees, Wynn and Scholey, G. Cawthorn, J. Harding, and J. Mawman. Please see the References for better quality modern publications of the *Letters*.

4.1 Sound

Euler frames his ideas on sound around his theory of music, the topics for which he was passionate. He reformulates the ordering of musical intervals on a new mathematical basis and introduces a new criterion based on pleasure. To explain his ideas, he treats an octave as a simple interval, and its numerical expression can be written as a simple proportion.

[1] https://archive.org/details/letterseulertoa00eulegoog.

doi:10.1088/2053-2571/aae6d2ch4

4.1.1 Letter 3. Of sound and its velocity

LETTER III.

Of Sound, and it's Velocity.

THE elucidations of the different degrees of velocity, which I have had the honour to lay before your Highneſs, carry me forward to the examination of ſound, or noiſe in general. It muſt be remarked, that a certain portion of time always intervenes before ſound can reach our ears, and that this time is longer in proportion to our diſtance from the place where the ſound is produced; a ſecond of time being requiſite to convey ſound 1000 feet.

When a cannon is fired, thoſe who are at a diſtance do not hear the report for ſome time after they have ſeen the flaſh. Thoſe who are a mile, or 24,000 feet diſtant, hear not the report till 24 ſeconds after they ſaw the flame. You muſt no doubt have frequently remarked, that the noiſe of thunder reaches not the ear for ſome time after the lightning: and it is by this we are enabled to calculate our diſtance from the place where the thunder is generated. If, for example, we obſerve that 20 ſeconds intervene between the flaſh and the thunder-clap, we may conclude that the ſeat of the thunder is 20,000 feet diſtant, allowing 1000 feet of diſtance for every ſecond of time. This primary property leads us to inquire, In what ſound conſiſts? Whether it's nature is ſimilar to that of ſmell, that is, whether ſound iſſues from the body which produces it, as ſmell is

emitted

emitted from the flower, by filling the air with subtile exhalations, proper to affect our senſe of ſmelling. This opinion was formerly entertained, but it is now demonſtrated, that from a bell ſtruck nothing proceeds that is conveyed to our ear, and that the body which produces ſound loſes no part of it's ſubſtance. When we look upon a bell that is ſtruck, or the ſtring of an inſtrument when touched, we perceive that theſe bodies are then in a ſtate of trembling, or agitation, by which all their parts are affected; and that all bodies, ſuſceptible of ſuch an agitation of their parts, likewiſe produce ſound. Theſe ſhakings or vibrations are viſible in the ſtring of an inſtrument when it is not too ſmall; the tenſe ſtring A C B paſſes alternately into the ſituation A M B and A N B. *(See plate I. fig. 1. in which I have repreſented theſe vibrations much more obvious to ſenſe than they are in fact.)* It muſt be further obſerved, that theſe vibrations put the adjacent air into a ſimilar vibration, which is ſucceſſively communicated to the more remote parts of the air, till it come at length to ſtrike our organ of hearing. It is the air, then, which receives theſe vibrations, and which tranſmits the ſound to our ear. Hence it is evident, that the perception of ſound is nothing elſe but the impreſſion made on our ear by the concuſſion of the air, communicated to us through the organ of hearing; and when we hear the ſound of a ſtring touched, our ear receives from the air as many ſtrokes as the ſtring performs vibrations in the ſame time. Thus, if the ſtring performs 100 vibrations in a ſecond,

cond, the ear likewife receives 100 ftrokes in the fame time; and the perception of thefe ftrokes is what we call found. When thefe ftrokes fucceed each other uniformly, or when their intervals are all equal, the found is regular, and fuch as is requifite to mufic. But when the ftrokes fucceed unequally, or when their intervals are unequal among themfelves, an irregular noife, incompatible with mufic, is the refult. On confidering fomewhat more attentively the mufical founds, whofe vibrations take place equally, I remark firft, that when the vibrations, as well as the ftrokes impreffed on the ear, are more or lefs ftrong, no other difference of found refults from it, but that of ftronger or weaker, which produces the diftinction, termed by muficians, *fortè & piano*. But there is a difference much more effential, when the vibrations are more or lefs rapid, that is, when more or fewer of them are performed in a fecond. When one ftring makes 100 vibrations in a fecond, and another ftring makes 200 vibrations in the fame time, their founds are effentially different; the former is lower or more flat, and the other higher or more fharp. Such is the real difference between the flat and fharp founds, on which all mufic hinges, and which teaches how to combine founds different in refpect of flatnefs and fharpnefs, but in fuch a manner as to produce an agreeable harmony. In the flat founds there are fewer vibrations in the fame time than in the fharp founds; and every key of the harpfichord contains a certain and determinate number of vibrations, which are completed in a fecond.

Thus

Thus the note marked by the letter C,* makes nearly 100 vibrations in a ſecond; and the note marked $\overset{\equiv}{c}$ makes 1600 vibrations in the ſame ſpace of time. A ſtring which vibrates 100 times in a ſecond, will give preciſely the note C; and if it vibrated only 50 times, the note would be lower or more flat. But with regard to our ear, there are certain limits beyond which ſound is no longer perceptible. It would appear that we are incapable of determining either the ſound of a ſtring which makes leſs than 30 vibrations in a ſecond, becauſe it is too low; or that of a ſtring which would make more than 7552 in a ſecond, becauſe ſuch a note would be too high.

26th April, 1760.

* The note C is that which is produced by touching the thickeſt ſtring of a violoncello; the note $\overset{\equiv}{c}$ is the fourth octave of the firſt; accordingly, theſe two notes, repreſented by the uſual method of pricking muſic, are

Mr. Euler marks the progreſſion of octaves thus:

	1ſt octave,	2d octave,	3d octave,	4th octave.
C, or *ut*.	c.	$\bar{c}$	$\overset{=}{c}$	$\overset{\equiv}{c}$

and in like manner for the other notes of the gamut; D. E. F. G. A. B. or *re, mi, fa, ſol, la, ſi.*

In writing the chromatic ſcale, he employs the following ſigns:

C.	Cs,	D,	Ds,	E,	F,	Fs,	G,	Gs,	A,	B,	H,	c
ut,	*ut*♯,	*re,*	*re*♯,	*mi,*	*fa,*	*fa*♯,	*ſol,*	*ſol*♯,	*la,*	*ſi*♭,	*ſi*♮,	*ut.*

4.1.2 Letter 5. Of unison and octaves

LETTER V.

Of Unifon and Octaves.

YOUR Highnefs has by this time remarked, that the accord which muficians call an octave, ftrikes the ear in a manner fo decided, that the flighteft deviation is eafily perceptible. Thus, having touched the Key marked F, that marked f, which is an octave higher, is eafily attuned to it, by the judgment of the ear only. If the ftring which is to produce this note be ever fo little too high or too low, the ear is inftantly offended, and nothing is eafier than to put the two keys perfectly in tune. Thus we obferve, that in finging the voice flides eafily from one note to another, which is juft an octave higher or lower. But were it required to pafs immediately from the note F to the note d, for example, an ordinary finger might eafily fall into a miftake, unlefs affifted by an inftrument. Having fixed the note F, it is almoft impoffible all at once to make the tranfition to the note d. What then is the reafon of this

this difference, that it is so easy to make note f harmonize with note F, and so difficult to make note d accord with it? The reason is evident from the remarks already made: it is this, that note F and note f make an octave, and that the number of vibrations of note f is precisely double that of note F. In order to have the perception of this accord, you have only to consider the proportion of one to two, which, as it instantly strikes the eye by the representation of the dots I formerly employed, affects the ear in a similar manner. You will easily comprehend, then, that the more simple any proportion is, or expressed by small numbers, the more distinctly it presents itself to the understanding, and conveys to it a sentiment of satisfaction.* Architects likewise carefully attend to this maxim, as they uniformly employ in their works proportions as simple as circumstances permit. They usually make the height of doors and windows double the breadth, and endeavour to employ throughout proportions capable of being expressed by small numbers, because this is obvious and grateful to the understanding. The same thing holds good in music: accords are pleasing only in so far as the mind perceives the relation subsisting between the sounds, and this relation is so much more

* In order to have a clear conception of what follows, it must be recollected, that the terms *relation* and *ratio* are synonimous, and that the author is here considering geometrical proportion, which consists in the number of times that the first term is contained in the second.—*F. E.*

eaſily perceptible, as it is expreſſed by ſmall numbers. Now, next to the relation of equality, which denotes two ſounds in uniſon, the ratio of two to one is undoubtedly the moſt ſimple, and it is this which furniſhes the accord of an octave: hence it is evident, that this accord poſſeſſes many advantages above every other conſonance. Having thus explained the accord, or interval of two notes denominated by muſicians an octave, let us conſider ſeveral notes, as F, f, $\bar{f}$, $\bar{\bar{f}}$, $\bar{\bar{\bar{f}}}$, each of which is an octave higher than the one immediately preceding: ſince then the interval of F from f, of f from $\bar{f}$, of $\bar{f}$ from $\bar{\bar{f}}$, of $\bar{\bar{f}}$ from $\bar{\bar{\bar{f}}}$, is an octave, the interval of F to $\bar{f}$ will be a double octave, that of F to $\bar{\bar{f}}$, a triple octave, and that of F to $\bar{\bar{\bar{f}}}$, a quadruple octave. Now, while note F makes one vibration, note f makes two, note $\bar{f}$ makes four, note $\bar{\bar{f}}$ makes eight, and note $\bar{\bar{\bar{f}}}$ makes ſixteen: hence we ſee, that as an octave correſponds in the relation of 1 to 2, a double octave muſt be in the ratio of 1 to 4, a triple in that of 1 to 8, and a quadruple in that of 1 to 16. And the ratio of 1 to 4, not being ſo ſimple as that of 1 to 2, for it does not ſo readily ſtrike the eye, a double octave is not ſo eaſily perceptible to the ear as a ſingle; a triple is ſtill leſs perceptible, and a quadruple ſtill much leſs ſo. When, therefore, in tuning a harpſichord, you have fixed the note F, it is not ſo eaſy to attune the double octave $\bar{f}$ as the ſingle f; it is ſtill more difficult to attune the triple octave $\bar{\bar{f}}$ and the quadruple $\bar{\bar{\bar{f}}}$ without riſing through the intermediate octaves.

Theſe

Thefe accords are likewife comprehended in the term confonance; and as that of unifon is moft fimple, they may be arranged according to the following gradations:

I. Degree, unifon, indicated by the relation of 1 to 1.
II. Degree, the immediate octave, in the ratio of 1 to 2.
III. Degree, the double octave, in that of 1 to 4.
IV. Degree, the triple octave, in that of 1 to 8.
V. Degree, the quadruple octave, in that of 1 to 16.
VI. Degree, the quintuple octave, in that of 1 to 52.

And fo on, as long as found is perceptible. Such are the accords denominated confonances, to the knowledge of which we have been thus far conducted; but hitherto we know nothing of the other fpecies of confonance, and ftill lefs of the diffonances employed in mufic. Before I proceed to the explication of thefe, I muft add one remark refpecting the name octave, given to the interval of two notes, the one of which contains twice the vibrations contained in the other. You fee the reafon of it in the principal ftops of the harpfichord, which rife by feven degrees before you arrive at the octave C, D, E, F, G, A, B, c, fo that ftop c is the eighth, reckoning C the firft. And this divifion depends on a certain feries of mufical intervals, the nature of which fhall be unfolded in the following letters.

3*d May*, 1760.

4.2 Composition of atmosphere

In letter 15 Euler describes what happens to air when it is exposed to heat or cold, and outlines the connection between elasticity of air and its density. The concept of ideal gas was not known yet, but Euler's ideas became foundations of Charles' law.

4.2.1 Letter 15. Changes produced in the atmosphere by heat and cold

LETTER XV.

Changes produced in the Atmoſphere by Heat and Cold.

HEAT and cold produce the ſame effect on air, as on every other body. Air is rarefied by heat, and condenſed by cold. From what I have ſaid of the elaſticity of air, you eaſily perceive, that a certain quantity of this fluid is not determined to occupy only a certain ſpace, as all other bodies are; but by

* There are three kinds of thermometers in uſe at preſent, that of Reaumur is adopted in France, Switzerland, and Italy; that of Celſius in Sweden and Denmark. In both of theſe, the ſcale commences at the freezing point; but the interval, between that and the boiling point, is divided, in the former, into 80 parts, and the latter, into 100. Farenheit's thermometer is uſed in Britain and Holland; the freezing point is marked on it 32, and the boiling 212, the interval containing 180 degrees. The freezing point is very nearly permanent, but the boiling point depends on the preſſure of the atmoſphere, and near the ſurface of the earth it varies one degree and ſix-tenths for every inch of variation in the height of the barometer. Water has been heated in a cloſe veſſel to ſuch a degree, as to melt lead and tin; and in the receiver of an air-pump, it may be converted into vapour, at the ordinary temperature of the air. Hence the reaſon why water boils ſo quickly on the ſummit of lofty mountains. The boiling point would be at 172° on the heights of the Andes.—*E. E.*

E 4 it's

it's nature, it has a perpetual tendency to dilate, and actually does expand itſelf, as long as it meets no obſtacle.

This property of air is denominated elaſticity. When this fluid is confined in a veſſel, it makes efforts in every direction to burſt it; and theſe efforts are greater or leſs in proportion to it's condenſation. Hence we come to this concluſion, that the elaſticity of air is in exact proportion to it's denſity; ſo that when it's denſity is doubled, it's elaſticity is likewiſe doubled; and that, in general, a certain degree of elaſticity correſponds to a proportional degree of denſity. It muſt be remarked, however, that this takes place no longer than while the air preſerves the ſame degree of heat. Whenever it becomes hotter, it acquires greater power of expanſion than what correſponded to it's denſity; and cold produces the oppoſite effect, by diminiſhing it's expanſive power. In order then to determine the elaſticity of a maſs of air, it is not ſufficient to know it's denſity; you muſt likewiſe know it's degree of heat. In order to ſet this in a clear light. Let us ſuppoſe two chambers cloſely ſhut on all ſides, but united by a door of communication; and that the heat in both is equal. In order to this the air in both chambers muſt have the ſame degree of denſity. For were the air more denſe, and conſequently more elaſtic, in the one than in the other, part of it would eſcape from the one, and force it's way into the other, till the denſity in both were the ſame. But let us ſuppoſe that one of the chambers has become hotter than the other, the air thereby acquiring

acquiring a greater elasticity, would of course force itself into the other, and reduce that which it found there into a smaller space, till the elasticity in both chambers was brought to the same degree. During this change there will be a current of air, through the door, from the chamber which is more, into that which is less heated; and when the equilibrium is restored, the air will be more rarefied in the warm apartment, and more condensed in the cold; and yet the elasticity of both will be the same. From this it clearly follows, that two masses of air of different density, may have the same elasticity, when the one is hotter than the other; and this circumstance taken into consideration, it may happen, that with the same degree of density, they may be endowed with different degrees of elasticity.

What I have said of two chambers may be applied to two countries; and hence it may be concluded, that when one country becomes warmer than the other, there must of necessity be a current of air from the one to the other: and from this results the wind.

Here, then, is one fruitful source of winds, though there are perhaps others, which consist in the different degrees of heat, which prevail in different regions of the earth; and it is demonstrable, that the whole air which surrounds the earth could not be in a state of rest, unless that, universally, at equal heights, there were found the same degree, not only of density, but likewise of heat. And should it happen that there were no wind over the whole surface of the earth, it might with certainty be concluded, that the

the air would likewiſe be every where equally denſe and warm at equal heights. Now as this never happens, there muſt of neceſſity always be winds, at leaſt in ſome regions. But theſe winds are, for the moſt part, to be met with only on the ſurface of the earth; and the higher you riſe, the leſs violent winds are. Winds are hardly perceptible at the ſummit of very high mountains;* there perpetual tranquillity reigns; from which it is impoſſible to doubt, that at conſiderable elevations, the air is always in a ſtate of reſt. Hence it follows, that in regions remarkably elevated, there univerſally prevails all over the earth, the ſame degree of denſity and heat; for were it hotter in one place than in another, the air could not be in a ſtate of reſt. And, as there is no wind in theſe elevated regions, it muſt neceſſarily follow, that the degree of heat there muſt be univerſally and always the ſame; which is a very ſurprizing paradox, conſidering the great variations of heat and cold which we feel on the ſurface of the earth, during the courſe of a year, and even of one day; without taking into the account the difference of climate, that is, the intolerable heats felt under the equator, and the

* This does not appear perfectly exact. A perpetual current of wind, from eaſt to weſt, muſt be produced by the motion of the earth's rotation. It reſults, likewiſe, from M. d'Alembert's theory of winds. Beſides, the attraction of the moon, which is capable of raiſing the waters of the globe, undoubtedly communicates ſome motion to the atmoſphere. Here, then, we have ſuperior currents.

When aëroſtation is carried to perfection, it will, perhaps, procure us ſatisfying information reſpecting this article of meteorology.—*F. E.*

dreadful

dreadful cold which ever prevails toward the poles of the earth. Experience itſelf, however, confirms the truth of this aſtoniſhing fact. The ſnow and ice remain equally, ſummer and winter, on the mountains of Switzerland, and are equally unchangeable on the Cordeliers, lofty mountains of Peru, ſituated under the very equator, and where there perpetually reigns, neverthelefs, a cold as exceſſive as that of the polar regions. The height of theſe mountains is not a German mile,* or 24,000 feet. From this it may be, with confidence, concluded, that were it poſſible for us to aſcend to the height of 24,000 feet, above the earth, we ſhould always and univerſally meet with the ſame degree of cold, and that cold exceſſively ſevere.† We ſhould remark there no ſenſible difference during either ſummer or winter, under the equator, or near the poles. At this height, and ſtill higher, the ſtate of the atmoſphere is univerſally, and at all ſeaſons, the ſame; and the variations of heat and cold take place near the ſurface of the earth alone. It is only in theſe inferior regions, that the effect of the rays of the ſun becomes perceptible. You have, undoubtedly, ſome curioſity to know the reaſon of this. It ſhall be the ſubject of the following letter.

31ſt May, 1760.

* About 4 3-5ths miles, Engliſh.

† M. Charles, in his aërial voyage of the 1ſt Dec. 1783, felt this change of temperature in a very ſenſible manner; for then, on the ſurface of the earth, the fluid in the thermometer ſtood at 7° above the freezing point, and after about 10 minutes of aſcenſion, it had fallen to 5° below it.—*F. E.*

4.3 Nature of light

Similarly to sound, Euler treats light as a vibration in the ether. His strong opposition to Newton's corpuscular optics did not mean that Euler was anti-Newtonian, but that he found fault with Newton's optics, particularly with reflection and refraction. Indeed, Newton's optics could not to explain these two phenomena.

In terms of application of ether to the light theory as the contradiction to Newton's corpuscular light theory, it was the conceptual prediction of the wave theory of light presented as certain vibrations of ether.

4.3.1 Letter 17. Of light, and the systems of Descartes and Newton

LETTER XVII.

Of Light, and the Syſtems of Deſcartes and Newton.

HAVING ſpoken of the rays of the ſun, which are the focus of all the heat and light that we enjoy, you will undoubtedly aſk, What are theſe rays? This is beyond queſtion one of the moſt important inquiries in phyſics, as from it an infinite number

ſolution of the fact. It is indifferent what portion of the air firſt receives the heat; the effect depends entirely on the nature of it's diſtribution. If the atmoſphere were of an uniform denſity throughout, the heat would at all heights be likewiſe the ſame. But as the denſity varies according to the altitude, the diſtribution of heat is affected by that circumſtance, and follows a certain correſponding law. I would gladly develope the principles from which this theory is deduced, but the popular nature of the preſent treatiſe forbids all abſtract diſcuſſion. I ſhall therefore content myſelf with giving a table of the diminution of heat at different altitudes.

Altitude in feet.	Diminution of heat, in degrees, of Farenheit,
3,000	12°
6,000	$24\frac{1}{2}$
9,000	38
12,000	53
15,000	$68\frac{1}{2}$
18,000	$86\frac{1}{2}$
21,000	$94\frac{1}{2}$

The diminution of heat, on the aſcent, is not quite ſo great in extenſive continents; for the intercourſe between the rare and the denſe portions of the atmoſphere is, in this caſe, neceſſarily ſlow, and the heat, which is principally formed at the ſurface, will only be partially diſperſed.

number of phenomena is derived. Every thing that refpects light, and that renders objects vifible, is clofely connected with this inquiry. The ancient philofophers feem to have taken little intereft in the folution of it. They contented themfelves with faying that the fun is endowed with the quality of fhining, of giving heat and light. But is it not worth while to inquire, Wherein does this quality confift? Do certain portions, inconceivably fmall, of the fun himfelf, or of his fubftance, come down to

It is a common miftake to fuppofe, that the fame heat obtains, at a certain depth, in every part of the globe. The fact is, that heat, originally derived from the fun, is communicated very flowly to the matter below the furface, which, therefore, does not feel the viciffitude of feafons, but retains the average temperature of the climate for many ages. Hence the utility of examining the heat of fprings, which is the fame with that of the fubftances through which they flow. The following table exhibits the average heat of places on the level of the fea, computed by the celebrated aftronomer, Profeffor Meyer, for every five degrees of latitude.

Latitude.	Average Temperature.	Latitude.	Average Temperature
0	— — 84°	50	— — 53½°
5	— — 83½	55	— — 49
10	— — 82½	60	— — 45
15	— — 80½	65	— — 41½
20	— — 78	70	— — 38
25	— — 74½	75	— — 35½
30	— — 71	80	— — 33½
35	— — 67	85	— — 32½
40	— — 62½	90	— — 32
45	— — 58		

By comparing this table with the preceding, it is eafy to difcover, for any latitude, the altitude of the curve of congelation, or where the average temperature is 32°.—*E. E.*

us?

us? Or is the tranſmiſſion ſimilar to the ſound of a bell, which the ear receives? though no part of the ſubſtance of the bell be ſeparated from it, as I obſerved in explaining the propagation and perception of ſound.

Deſcartes, the firſt of modern philoſophers, maintained this laſt opinion, and having filled the whole univerſe with a ſubtile matter compoſed of ſmall globules, which he calls the ſecond element, he ſuppoſes that the ſun is in a ſtate of continual agitation, which he tranſmits to theſe globules, and pretends that they again communicate their motion in an inſtant to every part of the univerſe. But ſince it has been diſcovered that the rays of the ſun do not reach us inſtantaneouſly, and that they take eight minutes to fly through that immenſe diſtance,* the opinion of *Deſcartes*, which laboured beſide under other difficulties, has been given up.

The great *Newton* afterwards embraced the former ſyſtem, and maintained that the luminous rays are really ſeparated from the body of the ſun, and the particles of light thence emitted with that inconceivable velocity which brings them down to us in about eight minutes. This opinion, which is that of moſt modern philoſophers, particularly the Engliſh, is

* This important fact was diſcovered toward the end of the laſt century by Roemer, a learned Dane, of the ancient Academy of Sciences. It was an inequality of the ſatellites of Jupiter which led him to it. The cauſe of this aberration, diſcovered by Bradley in 1728, inconteſtably demonſtrates the ſame phenomenon.—*F. E.*

F 2 called

called *the ſyſtem of emanation ;* it being imagined that rays emanate from the ſun and other luminous bodies, as water emanates or ſprings from a fountain.

This opinion appears at firſt ſight very bold, and irreconcileable to reaſon. For were the ſun emitting continually, and in all directions, ſuch floods of luminous matter, with a velocity ſo prodigious, he muſt ſpeedily be exhauſted, or at leaſt ſome alteration muſt, after the lapſe of ſo many ages, be perceptible. This, however, is contradicted by obſervation. It cannot be a matter of doubt, that a fountain which ſhould emit ſtreams of water in all directions, would be exhauſted in proportion to the velocity of the emiſſion ; much more the ſun, whoſe rays are emitted with a velocity ſo inconceivable. Let the particles of which rays of light are formed be ſuppoſed as ſubtile as you pleaſe, nothing will be gained : the ſyſtem will ever remain equally untenable. It cannot be affirmed that this emanation is not made in all directions : for, wherever you are placed, the whole ſun is viſible, which proves inconteſtably, that rays from every point of the ſun are emitted toward the ſpot which you occupy. The caſe is very different from that of a fountain, which ſhould emit ſtreams of water in all directions. For one point in the fountain could furniſh only one ſtream directed to a particular ſpot, but every point of the ſun's ſurface muſt emit an infinite number, diffuſing themſelves in all directions. This circumſtance alone infinitely increaſes the expenditure of luminous matter, which the ſun would have to make.

Another

Another difficulty, and which appears equally infuperable, is, that the fun is not the only body which emits rays, but that all the ftars have the fame quality: and as every where the rays of the fun muft be croffing the rays of the ftars, their collifion muft be violent in the extreme. How muft their direction be changed by fuch collifion! This collifion muft take place with refpect to all luminous bodies, vifible at the fame time. Each, however, appears diftinctly, without fuffering the flighteft derangement from any other: a certain proof that many rays may pafs through the fame point, without difturbing each other, which feems irreconcileable to the fyftem of emanation. Let two fountains be fet a playing upon each other, and you will immediately perceive their different ftreams difturbed and confounded: it muft of confequence be concluded, that the motion of the rays of light is very effentially different from that of a *jet d'eau*, and in general from all fubftances forcibly emitted.

Confidering afterwards tranfparent bodies through which rays are freely tranfmitted in all directions, the fupporters of this fyftem are under the neceffity of affirming that thefe bodies contain pores, difpofed in ftraight lines, which iffue from every point of the furface, and proceed in all directions; it being inconceivable how there could be any line through which a ray of the fun might be tranfmitted with fuch amazing velocity, and even without the flighteft collifion. Here then are bodies wonderfully po-

F 3 rous,

rous, which have the appearance, neverthelefs, of being extremely folid.

Finally, in order to enjoy vifion, the rays muft enter into the eye, and penetrate it's fubftance with the fame velocity. All thefe difficulties, taken together, will, I doubt not, fufficiently convince you, that the fyftem of emanation has in no refpect a foundation in nature; and you will certainly be aftonifhed that it could have been conceived by fo great a man, and embraced by fo many enlightened philofophers. But it is long fince Cicero remarked, that nothing fo abfurd can be imagined as to find no fupporter among philofophers. For my part, I am too little a philofopher to adopt the opinion in queftion.

7th June, 1760.

4.3.2 Letter 20. Of the propagation of light

LETTER XX.

Of the Propagation of Light.

THE propagation of light in the ether is produced in a manner fimilar to that of found in the air; and juft as the vibrations occafioned in the particles of air conftitutes found, in like manner the vibration of of the particles of ether conſtitutes light or luminous rays; ſo that *light is nothing elſe but an agitation or concuſſion of the particles of ether*, which is every where to be found on account of it's extreme ſubtilty, in virtue of which it penetrates all bodies.

Theſe bodies, however, modify the rays of light in many different ways, by tranſmitting or ſtopping the propagation of the concuſſions. Of this I ſhall treat at large in the ſequel. I confine myſelf at preſent to the propagation of rays in the ether itſelf, which fills the immenſe ſpace in which the heavenly bodies revolve. There the propagation takes place in perfect liberty. The firſt thing which here preſents itſelf to the mind is the prodigious velocity of the rays of light, which is about 900,000 times more rapid than that of ſound, though this laſt travels no leſs than 1000 feet in a ſecond.

This amazing velocity would be ſufficient of itſelf to overturn the ſyſtem of emanation; but in that which I am attempting to eſtabliſh, it is a natural conſequence, from the principles laid down, as I hope to demonſtrate. They are the ſame with thoſe on which is founded the propagation of ſound in the air, and this depends at once on it's denſity and elaſticity. It is evident that if the denſity of air were diminiſhed, ſound would be accelerated, and if the elaſticity of the air were increaſed, the ſame thing would happen. If the denſity of the air diminiſhed, and it's elaſticity increaſed at once, we ſhould have a two-fold reaſon for the increaſe of the velocity of ſound. Let us conceive, then, the denſity of the air diminiſhed,

diminiſhed, and it's elaſticity increaſed, till it's denſity and elaſticity became equal to thoſe of ether, and we ſhould then no longer be ſurpriſed that the velocity of ſound had become many thouſands of times greater than it actually is. For you will be pleaſed to remember, that, according to the firſt ideas we formed of ether, this fluid muſt be inconceivably rarer, and more elaſtic than air. Now both of theſe qualities equally contribute to accelerate the velocity of vibrations. From this explanation, the prodigious velocity of light is ſo far from preſenting any thing irreconcileable to reaſon, that it rather perfectly harmonizes with the principles laid down; and the parallel between light and ſound is in this reſpect ſo firmly eſtabliſhed, that we may confidently maintain, That if air ſhould become as ſubtile and as elaſtic as ether, the velocity of ſound would become as rapid as that of light.

The ſubtlity of ether, then, and it's great elaſticity, are the reaſon which we aſſign for the prodigious velocity of the motion of light; and ſo long as the ether preſerves this ſame degree of ſubtlity and elaſticity, this velocity muſt continue the ſame. Now it cannot be doubted that the ether has, through the whole univerſe, the ſame ſubtlity and the ſame elaſticity. For were the ether leſs elaſtic in one place than in another, it would force itſelf into it till the equilibrium was perfectly reſtored. The light of the ſtars, therefore, moves with as great velocity as that of the ſun; and as the ſtars are at a much greater diſtance from us than the ſun, a much greater quantity of time is requiſite to tranſmit their rays to us. However great the diſtance of the ſun may appear, whoſe rays, nevertheleſs, reach the ſurface of our globe in eight minutes, the fixed ſtar neareſt to us is at leaſt 400,000 times more diſtant than the ſun: a ray of light iſſuing from that ſtar will employ then 400,000 times eight minutes in travelling to us, that is 53,333 hours, or 2,222 days, or ſix years nearly.

It is then upwards of ſix years ſince the rays of light iſſued from that fixed ſtar, the leaſt remote and probably the moſt brilliant, in order to render it viſible to us, and theſe rays have employed a period ſo conſiderable to fly through the ſpace which ſeparates us from that ſtar. Were God juſt now to create a new fixed ſtar, at the ſame diſtance, it could not become viſible to us till more than ſix years had elapſed, as it's rays require that length of time to travel this diſtance. Had one been created at the beginning of the world a thouſand times more diſtant than that which I have mentioned, it could not yet be viſible to us, however brilliant, as 6000 years are not yet elapſed ſince the Creation. The firſt preacher of the court of Brunſwick, Mr. Jeruſalem, has happily introduced this thought in one of his ſermons; the paſſage runs thus:

" Raiſe your thoughts from the earth which you " inhabit to all the bodies of the vaſt univerſe, which " are ſo far above you: launch into the immenſity " of ſpace which intervenes between the moſt re- " mote which your eyes are able to diſcover, and " thoſe whoſe light, from the moment of creation

" till

"till now, has not as yet, perhaps, come down to us. "The immenſity of the kingdom of God juſtifies "this repreſentation." (Sermon *on the Heavens, and Eternal Beatitude.)*

I flatter myſelf that theſe reflections will excite a deſire of further inſtruction reſpecting the ſyſtem of light, from which is derived the theory of colours, and of viſion.

17th June, 1760.

4.3.3 Letter 19. A different system respecting the nature of rays and of light, proposed (ether)

LETTER XIX.

A different Syſtem reſpecting the Nature of Rays and of Light, propoſed.

YOU have ſeen that the ſyſtem of the emanation of the rays of light labours under inſuperable difficulties, and that the doctrine of a vacuum for the heavenly bodies to range in, is equally untenable; as the rays of light would completely fill it. Two things, then, muſt be admitted: firſt, the ſpace through which the heavenly bodies move is filled with a ſubtile matter; ſecondly, rays are not an actual emanation from the ſun and other luminous bodies, in virtue of which part of their ſubſtance is

" ing, in our reſearches into the phenomena of this viſible world, " which lies open to the examination of our ſenſes, how wretched " muſt we have been had God left us to ourſelves with reſpect to " things inviſible, and which concern our eternal ſalvation? On " this important article a Revelation was abſolutely neceſſary to " us; and we ought to avail ourſelves of it with the moſt pro- " found veneration. When it preſents to us things which may " appear inconceivable, we have but to reflect on the imperfection " of human underſtanding, which is ſo apt to be miſled, even as " to ſenſible objects. Whenever I hear a pretended Freethinker " inveighing againſt the truths of religion, and even ſneering at " it with the moſt arrogant ſelf-ſufficiency, I ſay to myſelf: poor " weak mortal, how inexpreſſibly more noble and ſublime are the " ſubjects which you treat ſo lightly, than thoſe reſpecting which " the great *Newton* was ſo groſsly miſtaken! I could wiſh your " Highneſs to keep this reflection ever in remembrance: occaſions " for making it occur but too frequently."—*E. E.*

violently

violently emitted from them, according to the doctrine of *Newton*.*

That ſubtile matter which fills the whole ſpace in which the heavenly bodies revolve, is called *Ether*. Of it's extreme ſubtilty no doubt can be entertained. In order to form an idea of it, we have only to attend to the nature of air, which, though extremely ſubtile, even on the ſurface of the earth, becomes more and more ſo as we aſcend; and entirely ceaſes, if I may uſe the expreſſion, when it comes to be loſt in the ether. The ether, then, is likewiſe a fluid as the air is, but incomparably finer and more ſubtile, as we are aſſured that the heavenly bodies revolve

* The materiality of light is ſupported by the moſt convincing proofs that phyſics can afford. The inflection, refraction, and reflection of it's rays, ſhew manifeſtly that, like other bodies, it is ſubject to attraction and repulſion; and the ſimple application of the doctrine of forces not only explains ſatisfactorily the phenomena, but aſſigns the preciſe effects with the moſt perfect accuracy. The difficulties which ſeem to attend the theory of emanation vaniſh on a cloſe inveſtigation. So vaſt is the tenuity of light, that it utterly exceeds the powers of conception. The moſt delicate inſtrument has never been certainly put in motion by the impulſe of the accumulated ſun-beams. Even on the moſt unfavourable ſuppoſition it appears from calculation that, in the ſpace of 385,130,000 Egyptian years (of 360 days) the ſun would loſe only the 1,217,420th of his bulk, from the continual efflux of light. On the ſame hypotheſis the force impreſſed upon the earth by each emiſſion is ſuch as would make it recede only the two billionth part of an inch in an hundred ſeconds, and it's effect, during a ſeries of ages, would therefore be altogether inſenſible. After ſtating numbers of a magnitude ſo enormous, it would be ſuperfluous to conſider the quantity of ſtroke which the eye receives.

freely

freely through it, without meeting any perceptible resiſtance. It is alſo without doubt poſſeſſed of elaſticity, by means of which it has a tendency to expand itſelf in all directions, and to penetrate into ſpaces where there would otherwiſe be a vacuum; ſo that if by ſome accident the ether were forced out of any ſpace, the ſurrounding fluid would inſtantly ruſh in and fill it again.

In virtue of this elaſticity, the ether is to be found not only in the regions which are above our atmoſphere, but it penetrates the atmoſphere univerſally, inſinuates itſelf by the pores of all bodies, and paſſes irreſiſtibly through them. Were you, by the help of the air-pump, to exhauſt the air from a receiver, you muſt not imagine that you have produced an abſolute vacuum; for the ether, forcing itſelf through the pores of the receiver, completely fills it in an inſtant. Having filled a glaſs tube of the proper length with mercury, and immerged it, when inverted, in the ciſtern, in order to make a barometer, it might be ſuppoſed that the part of the tube which is higher than the mercury is a vacuum, becauſe the air is completely excluded, as it cannot penetrate the pores of glaſs: but this vacuum which is apparent only, is undoubtedly ſupplied by the ether, inſinuating itſelf without the ſmalleſt difficulty.

It is by this ſubtilty and elaſticity of ether that I ſhall by and by explain to you the remarkable phenomena of electricity. It is even highly probable that ether has an elaſticity much ſuperior to that of air, and that many of the phenomena of nature are produced

produced by means of it. For my own part I have no doubt that the compreffion of the air in gunpowder is the effect of the elaftic power of ether. And as we know by experiment that the air in it is condenfed almoft 1000 times more than common air, and that in this ftate it's elafticity is likewife 1000 times greater, the elafticity of the ether muft in this cafe be fo too, and confequently 1000 times greater than that of common air. We fhall then have a juft idea of ether, in confidering it as a fluid in many refpects fimilar to air, with this difference, that ether is incomparably more fubtile and more elaftic.*

Having feen then that the air, by thefe very qualities, is in a proper ftate for receiving the agitations or fhakings of fonorous bodies, and to diffufe them in all directions, as we find in the propagation of found, it is very natural to fuppofe that ether may in the fame circumftances likewife receive agitations in the fame manner, and tranfmit them to the greateft diftances.† As the vibrations of the air produce *found*,

* This, perhaps, is what in modern times they denominate the matter of heat.—*F. E.*

† The hypothefis of an ether is a clumfy attempt to preclude the neceffity of admitting *action at a diftance*. It has been a received maxim, that caufe and effect muft exift in the fame place; but the leaft reflection will convince us that, were this principle true, there could never be any communication of motion. The difficulty is really the fame, to conceive action exerted at the diftance of the thoufandth part of an inch, as at that of a thoufand miles. The particles of matter are far from being in mutual contact, otherwife

found, What will be the effect of those of ether? You will undoubtedly guess at once *light*. It appears in truth abundantly certain, that light is with respect to ether, what sound is with respect to air; and that the rays of light are nothing else but the shakings or vibrations transmitted by the ether, as sound consists in the shakings or vibrations transmitted by the air.

The sun, then, loses nothing of his substance in this case, any more than a bell in vibrating; and, in adopting this system, there is no reason to apprehend that the mass of this orb should ever suffer any diminution. What I have said of the sun must also be extended to all luminous bodies, such as fire, a wax taper, a candle, &c.

It will, undoubtedly, be objected, that these terrestrial luminaries evidently waste, and that unless they are continually fed and kept up, they will be speedily extinguished; that consequently the sun must in time be wasted away, and that the parallel of a bell is not accurate. But it is to be considered, that these fires, besides their light, throw out smoke, and a great deal of exhalation, which must be carefully distinguished from the rays of light. Now the smoke and exhalation evidently occasion a considerable diminution, which must not be imputed

wise all bodies would have the same density, and be totally incapable of compression. Were the universe an absolute *plenum*, motion and animation would for ever cease. To ascribe to ether an extreme rarity, and at the same time to assert that it fills all space, and pervades all bodies, is a contradiction in terms. But the hypothesis is so big with absurdity, that it deserves not a particular examination. *See note, p.* 41.—*E. E.*

to

to the rays of light; for were it poſſible to ſeparate them from the ſmoke and other exhalations, the luminous quality alone would occaſion no expenditure. Mercury may, by means of art, be rendered luminous, as you have probably ſeen, and that without any diminution of it's ſubſtance, which proves that light alone produces no waſte of luminous bodies. Thus though the ſun illuminates the whole world by his rays, he loſes nothing of his own ſubſtance, his light being only the effect of a certain agitation, or violent concuſſion of his minute particles, communicated to the adjoining ether, and thence tranſmitted in all directions by means of this fluid to the remoteſt diſtances, as a bell when ſtruck communicates it's own agitation to the circumambient air. The more we conſider this parallel between ſonorous and luminous bodies, the more we ſhall find it conformable to nature, and juſtifiable by experience; whereas the more we attempt to reconcile the phenomena of nature to the ſyſtem of emanation, the more difficulties we encounter.

14th June, 1760.

4.3.4 Letter 28. Nature of colors in particular

LETTER XXXVIII.

Nature of Colours in particular.

THE ignorance which prevailed refpecting the true nature of colours, has occafioned frequent and violent difputes among philofophers; each of whom made an attempt to fhine, by maintaining a peculiar opinion on the fubject. The fyftem which made colours to refide in the bodies themfelves, appeared to them too vulgar and too little worthy of a philofopher, who ought always to foar above the multitude. Becaufe the clown imagines that one body

body is red, another blue, and another green, the philosopher could not distinguish himself better than by maintaining the contrary; and he accordingly affirms that there is nothing real in colours, and that there is nothing in bodies relative to them.

The Newtonians make colours to consist in rays only; which they distinguish into red, yellow, green, blue, and violet; and they tell us that a body appears of such and such a colour when it reflects rays of that species. Others, to whom this opinion seemed absurd, pretend that colours exist only in ourselves. This is an admirable way to conceal ignorance; the vulgar might otherwise believe that the scholar was not better acquainted with the nature of colours than themselves. But you will readily perceive that these affected refinements are mere cavil. Every simple colour (in order to distinguish from compound colours) depends on a certain number of vibrations, which are performed in a certain time; so that this number of vibrations, made in a second, determines the red colour, another the yellow, another the green, another the blue, and another the violet, which are the simple colours represented to us in the rainbow.

If, then, the particles of the surface of certain bodies are disposed in such a manner, that being agitated, they make in a second as many vibrations as are necessary to produce, for example, the red colour, I call such a body red, just as the clown does; and I see nothing like a reason for deviating from the common mode of expression. And rays which make

make ſuch a number of vibrations in a ſecond, may, with equal propriety be denominated red rays; and finally, when the optic nerve is affected by theſe ſame rays, and receives from them a number of impulſions, ſenſibly equal, in a ſecond, we receive the ſenſation of the red colour. Here every thing is clear; and I ſee no neceſſity for introducing dark and myſterious phraſes, which really mean nothing.

The parallel between ſound and light is ſo perfect, that it hits even in the minuteſt circumſtances. When I produced the phenomenon of a muſical chord, which may be excited into vibration by the reſonance only of certain ſounds, you will pleaſe to recollect, that the one which gives the uniſon of the chord in queſtion is the moſt proper to ſhake it, and that other ſounds affect it only in proportion as they are in conſonance with it. It is exactly the ſame as to light and colours; for the different colours correſpond to the different muſical ſounds.

In order to diſplay this phenomenon, which completely confirms my aſſertion, let a dark room be provided; make a ſmall aperture in one of the ſhutters; before which, at ſome diſtance, place a body of a certain colour, ſay a piece of red cloth, ſo that, when it is illumined, it's rays may enter by the aperture into the darkened room. The rays thus tranſmitted into the room will be red, all other light being excluded: and if you hold on the inſide of the room, oppoſite to the aperture, a piece of cloth of the ſame colour, it will be perfectly illumined, and its red colour appear very brilliant; but if you ſub-

I ſtitute

ſtitute in it's place a piece of green cloth, it will remain obſcure, and you will hardly ſee any thing of it's colour. If you place on the outſide, before the aperture, a piece of green cloth, that within the chamber will be perfectly illumined by the rays of the firſt, and it's green colour appear very lively. The ſame holds good as to all other colours; and I do not imagine that a more convincing demonſtration of the truth of my ſyſtem can be demanded.

We learn from it, that, in order to illuminate a body of a certain colour, it is neceſſary that the rays which fall upon it ſhould have the ſame colour; thoſe of a different colour not being capable of agitating the particles of that body. This is farther confirmed by a well known experiment. When the ſpirit of wine is ſet on fire in a room, you know that the flame of ſpirit of wine is blue, that it produces only blue rays, and that every perſon in the room appears very pale, their faces, though painted ever ſo deep, have the aſpect of death. The reaſon is evident; the blue rays, not being capable of exciting, or putting in motion the red colour of the face, you ſee on it only a feeble and bluiſh colour: but if one of the company is dreſſed in blue, ſuch dreſs will appear uncommonly brilliant. Now the rays of the ſun, thoſe of a wax taper, or of a common candle, illuminate all bodies almoſt equally; from whence it is concluded, that the rays of the ſun contain all colours at once, though he himſelf appears yellowiſh.

In truth, when you admit into a dark room the rays of all the ſimple colours, red, yellow, green,

 blue,

blue, and violet, in nearly equal quantities, and blend them, they reprefent a whitifh colour. The fame experiment is made with various powders, coloured in like manner; on being mixed together, a whitifh colour is the refult. Hence it is concluded that white is nothing lefs than a fimple colour; but that it is rather a compound of all the fimple colours; accordingly we fee that white is adapted to the reception of all colours. As to black, it is not properly a colour. Every body is black when it's particles are fuch that they can receive no motion of vibration, or when it cannot produce rays. The want of rays, therefore, produces the fenfation of that colour; and the more particles there are found in any body not fufceptible of any motion of vibration on it's furface, the more blackifh and obfcure it appears.

15th July, 1760.

4.3.5 Letter 31. Refraction of rays of different colors

LETTER XXXI.

Refraction of Rays of different Colours.

YOU have feen, that when a ray of light paffes obliquely from one tranfparent medium to another, it undergoes an inflection, which is called refraction, and that the refraction depends on the obliquity of the incidence, and the denfity of the mediums. I muft now call upon you to remark, That diverfity of colours occafions, likewife, a fmall variety in the refraction. This arifes, undoubtedly, from hence, that the rays which excite in us the fenfations of different colours, perform unequal numbers of vibrations in the fame times, and that they differ among themfelves, in the fame manner as fharper or flatter founds do. Thus, it is obfervable, that

that rays of red undergo the leaſt inflection or refraction; after them come the orange; the yellow, the green, the blue and the violet, follow in order; ſo that violet-coloured rays undergo the greateſt refraction; it being always underſtood, that the obliquity of the incidence, and the denſity of the mediums are the ſame. Hence, it is concluded, that rays of different colours have not the ſame refrangibility; that the red are the leaſt refrangible, and the violet-coloured the moſt ſo.

If then, P C (*plate* I. *fig.* 10.) is a ray paſſing, for example, from air into glaſs; the angle of incidence being P C E, the refracted ray will approach the perpendicular C F; and if the ray be red, the refracted ray will be in the direction C—*red*; if it be orange, the refracted ray will be C—*orange*, and ſo of the reſt, as may be ſeen in the figure. All theſe rays deviate from the line C Q, which is P C produced, toward the perpendicular C F; but the red ray deviates the leaſt from C Q, or undergoes the leaſt inflection, and the violet recedes the fartheſt from C Q, and undergoes the greateſt inflection.

Now, if P C is a ray of the ſun, it produces, at once, all the coloured rays indicated in the figure; and if a piece of white paper is placed to receive them, you will, in effect, ſee all theſe colours; hence, it is affirmed, that every ray of the ſun contains, at once, all the ſimple colours. The ſame thing happens, if P C is a ray of white, or if it proceeds from a white body. We ſee all the colours produced from it by refraction, whence it is concluded, that

that white is an aſſemblage of all the ſimple colours, as we ſhewed formerly. In truth, we have only to collect all theſe coloured rays into a ſingle point, and the colour of white will be the reſult.

It is thus we diſcover what are the ſimple colours. Refraction determines them inconteſtibly. In following the order which it preſents, they are theſe: 1. red, 2. orange, 3. yellow, 4. green, 5. blue, 6. violet. But it muſt not be imagined, that there are but ſix: for as difference of colours ariſes from the number of vibrations which rays perform in one and the ſame time, or rather the undulations which produce them: it is clear, that the intermediate numbers equally give ſimple colours.* But we want names, by which to deſign theſe colours; for be-

* This remark, that the number of primitive colours much exceeds ſix, is very juſt. The colours of the rainbow, or of the ſpectrum, formed by a priſm, paſs into each other by inſenſible ſhades, ſo that it is impoſſible to define their boundaries. There is reaſon to ſuſpect, that, even the great *Newton* was, in this inſtance, miſled, by a predilection for the number ſeven, which during many ages, has been regarded with a ſort of myſtical veneration. The correſpondence, which he obſerved, between the diviſions of the ſpectrum, and thoſe of the monochord, and which ſo many authors have ſince repeated, is wholly ideal; for the proportions, between the extent of the different colours are, in a great meaſure, determined by the peculiar quality of the refracting mediums. Thus a priſm of glaſs, in which alkali predominates, forms a ſpectrum, extremely unlike that formed by one of glaſs, compoſed principally of lead. Were a perſon to reckon only the moſt conſpicuous of the primitive colours, he would, moſt probably, ſelect the number ſix, for the indigo can hardly be diſtinguiſhed.—*E. E.*

tween

tween yellow and green, we evidently perceive intermediate colours, for which we have no ſeparate names.

In conformity to the ſame laws, are produced the colours viſible in the rainbow. The rays of the ſun, in paſſing through the drops of water which float through the air, are, by them, reflected and refracted, and the refraction decompounds them into the ſimple colours. You muſt, undoubtedly, have remarked, that theſe colours follow each other, in the ſame order, in the rainbow, the red, orange, yellow, green, blue, and violet; but we diſcover in it, alſo, all the intermediate colours, as ſhades of one colour to another, and had we more names to diſtinguiſh theſe degrees, we might find more of them from the one extremity to the other. A more copious language may, perhaps, enable another nation actually to reckon up a greater number of different colours; and another, it may be, cannot reckon up ſo many; if, for example, it wants a term to expreſs what we call orange. Some to theſe add purple, which we perceive at the extremity of the red, but which others comprehend under the ſame name with red.

C.	D.	E.	F.	G.	A.	B.
Purple.	Red.	Orange.	Yellow.	Green.	Blue.	Violet.

Theſe colours may be compared to the notes of an octave, as I have done here, becauſe the relations of

OF DIFFERENT COLOURS. 127

of colours, as well as thoſe of ſounds, may be expreſſed by numbers. There is even an appearance, that by ſtraining the violet a little more, you may come round to a new purple, juſt as in riſing from ſound to ſound, on going beyond B, you come round to *c*, which is the octave above C. And, as in muſic, we give to theſe two notes the ſame name, becauſe of their reſemblance, the ſame thing takes place in colours, which, after having riſen through the intervals of an octave, reſume the ſame names: or, if you will, two colours, like two ſounds, in which the number of vibrations in the one, is preciſely the double of the other, paſs for the ſame, and bear the ſame name.

On this principle it was, that father *Caſtel*, in France, contrived a ſpecies of muſic of colours. He conſtructed a harpſichord, of which every key diſplayed a ſubſtance of a certain colour, and he pretended, that this harpſichord, if ſkilfully touched, would preſent a moſt agreeable ſpectacle to the eye. He gave it the name of the *ocular* harpſichord, and you muſt, undoubtedly, have heard it talked of. For my part, painting rather ſeems to be that to the eye, which muſic is to the ear; and I greatly doubt, whether the repreſentation of ſeveral ſhreds of cloth, of different colours, could be very agreeable.

27th July, 1760.

4.4 Optics and optical instruments

In the *Letters,* Euler studied the optics of a human eye: considering the eye lens biconvex, he arrives at a geometric optics formula for the human eye. He also introduces a method to calculate the refraction coefficient for two media. Euler describes the human eye as an ideal optical object: its nature elegantly combines various media with different optical properties (refraction coefficients) so that the human eye, as an optic system, does not experience chromatic aberration. At that time, the issue of chromatic aberrations of optical instruments (microscopes, telescopes) presented a serious challenge, as it limited the quality of imaging of both macro and micro objects. Isaac Newton believed that chromatic aberrations are a natural characteristic of any optical device and are unavoidable. Euler, however, shows that an achromatic lens was quite possible to engineer by combining two lenses with different refraction coefficients. He includes detailed calculations of optical systems for microscopes and telescopes. Euler's achromatic lens was later manufactured by the English optician John Dollond in 1758. In the *Letters*, Euler continuously repeats the notion that no human engineer would ever be able to create such an ideal optical instrument as a human eye. He suggested to build an optical system, in which two convex–concave lenses with one common axis are facing each other and the space between them is filled with water.

Euler was wrong in his assumption that lenses in human and animal eyes are free of aberration defect. As it was shown later, many optical imperfections of a human eye are compensated by its biomechanics and the brain's ability to interpret signals from the eye nerve. Euler did not study the eye's biomechanical properties, which affect, in particular, the lens' curvature and its refractive property. However, Euler's fundamental work on optics and the popular explanation in the *Letters* of the physics behind the human eye as an optical system greatly influenced generations of mathematicians, physicists, and physiologists working on the development of the theory of the eye's biomechanics.

4.4.1 Letter 43. Farther continuation. Difference between the eye of an animal, and the artificial eye, or camera obscura

LETTER XLIII.

Farther Continuation. Aſtoniſhing Difference between the Eye of an Animal, and the artifical Eye, or camera obſcura.

THE principle on which the ſtructure of the eye is founded, is, in general, the ſame as that according to which I explained the repreſentation of objects on white paper by means of a convex lens. Both

Both of them muſt be reſolved into this, that all the rays, proceeding from one point of the object, are again collected in a ſingle point by refraction; and it ſeems of little importance whether this refraction is performed by a ſingle lens, or by the ſeveral tranſparent ſubſtances of which the eye is compoſed. It might even be inferred from thence, that a ſtructure more ſimple than that of the eye, by employing one ſingle tranſparent ſubſtance, would have been productive of the ſame advantages; which would amount to a very powerful objection againſt the wiſdom of the Creator, who has aſſuredly purſued the ſimpleſt road in the formation of all his works.

Perſons have not been wanting who, from not having attentively examined the advantages reſulting from the apparent complication, preſumed to cenſure this beautiful production of the Supreme Being with a levity worthy of cenſure. They have pretended it was in their power to produce a plan more ſimple for the ſtructure of the eye, becauſe they were ignorant of all the functions which that organ had to diſcharge. I ſhall examine this plan of theirs; and I hope to convince you, that it would be highly defective, and altogether unworthy of being put in competition with that which actually exiſts.

Such an eye, therefore, would be reduced to a ſimple convex lens, A B C D, (*plate* II. *fig.* 10.) which collects, in a point, all the rays coming from one and the ſame correſponding point in the object. But this is only near to the truth. The ſpherical form, given to the ſurfaces of a lens, is liable to this inconvenience,

inconvenience, that it does not completely collect in one and the ſame point the rays which paſs through it's centre, and thoſe which paſs through the extremities. There is always a ſmall difference, though almoſt imperceptible, in the experiments, by means of which we receive the image on a piece of white paper; but if this happened in the eye itſelf, it would render viſion very confuſed.

The perſons to whom I have been alluding, allege, that it may be poſſible to find another figure for the ſurfaces of the lens, which ſhall have the property of collecting anew all the rays iſſuing from the point O, in a point R, whether they paſs through the centre, or through the extremities. I admit that this may be poſſible; but ſuppoſing the lens to poſſeſs this property, with reſpect to the point O, at the fixed diſtance C O, it would not poſſeſs it at points at a greater or leſs diſtance from the lens; or, even admitting this to be poſſible, which it is not, the lens would moſt certainly loſe that property with regard to objects placed on one ſide, at T, for inſtance. Accordingly we ſee that when objects are repreſented on white paper, though ſuch as are directly before the lens, ſay at O, may be ſufficiently well expreſſed, thoſe which are obliquely ſituated, as at T, are always much disfigured, and very confuſedly expreſſed: and this is a defect which the moſt ingenious artiſt is incapable of rectifying.

But there is another and one not leſs conſiderable. In ſpeaking of rays of different colours, I remarked, that in paſſing from one tranſparent medium to another,

ther, they undergo a different refraction; that rays of a red colour undergo the least refraction, and violet-coloured rays the greatest. Hence, if the point O were red, and if it's rays, in passing through the lens A B, were collected at the point R, this would be the place of the red image. But if the point O were violet, the rays would be collected nearer to the lens, at V. Again, as white is an assemblage of all the simple colours, a white object, placed at O, would form several images at once, situated at different distances from the point O; the result of which would be, on the retina, a coloured spot that would greatly disturb the representation.

It is accordingly observable, that when in a dark room the external objects are represented on white paper, they appear bordered with the colours of the rainbow, and it is impossible to remedy this defect by employing only one transparent body. But it has been remarked, that this may be done by means of different transparent substances; but neither theory nor practice have hitherto been carried to the degree of perfection necessary to the execution of a structure which should remedy all these defects.* The human

* A similar defect has been remarked in the common telescope. Objects do not appear in it very clearly. You see, besides, at the circumference of the field which it encompasses, a mixture of colours, which is called *iris*. To remedy this inconveniency, achromatic telescopes have been constructed, whose object-glass, being composed of more than one lens of different densities, and which of consequence refract the rays differently, produce an effect analogous to that of the transparent substances of the eye, of which our Author has been treating.—*F. E.*

eye,

eye, however, labours under none of the imperfections which I have mentioned, nor many others to which the hypothetical eye we have been analyzing would be liable. What a fublime idea muft we form of Him who has furnifhed not only the whole human fpecies, but every animal, nay even the vileft infects, with an organ of fuch curious conftruction!*

* The object of the Tranflator being not only to difplay *Euler's* philofophy, but likewife to exhibit the man as defigned by his own pencil, he takes the liberty of prefenting the Englifh Reader with the conclufion of this letter, in the Author's own manner and words, tranfcribed from the original edition of this work. Though a French philofopher and ftatefman may feel afhamed of the alliance of fcience to religion, and endeavour to keep it out of fight, it would furely ill become us to follow the example. Let the Author exprefs his own fentiments in his own way.

" But the eye which the Creator has formed is fubject to no " one of all the imperfections under which the imaginary con- " ftruction of the freethinker labours. In this we difcover the " true reafon why infinite wifdom has employed feveral tranfpa- " rent fubftances in the formation of the eye: it is thereby fe- " cured againft all the defects which characterife every work of " man. What a noble fubject of contemplation! How pertinent " that queftion of the Pfalmift! *He who formed the eye, fhall he not* " *fee? and He who planted the ear, fhall He not hear?* The eye alone " being a mafter-piece that far tranfcends the human underftand- " ing, what an exalted idea muft we form of Him, who has be- " ftowed this wonderful gift, and that in the higheft perfection, " not on man only, but on the brute creation, nay, on the vileft " of infects!"—*E. E.*

[1] *19th Auguft*, 1760.

4.5 Gravity

When discussing gravity, Euler explains that when objects are said to be heavy, they *possess* gravity and he characterizes *falling down* as a consequent behavior of bodies that possess gravity. He also explains why the Moon neither falls down to Earth nor moves in a straight line. Letters 52 and 53 describe Newton's law of gravitation and the so-called great inequality of Jupiter and Saturn. Euler investigated this problem by studying the Sun–Jupiter–Saturn three-body problem. Can there be such a mutual gravitational interaction which leaves the Solar system stable? Euler proved that irregularities of the two planets' motion were due to their mutual attraction.

The problem was solved later, in the early nineteenth century: since Jupiter goes around the Sun five times while Saturn goes around the Sun twice, after every five orbits Jupiter receives the perturbation from Saturn, always in the same parts of orbit, the so-called resonant commensurability 5:2. After some time Jupiter will slow down and Saturn will speed up. The alternative changes of speed happen with periodicity of 929 years; the system is stable.

4.5.1 Letter 45. Of gravity, considered a general property of body

LETTER XLV.

Of Gravity, conſidered as a general Property of Body.

HAVING now treated of light, I proceed to the conſideration of a property common to all bodies, that of gravity. We find that all bodies, ſolid and fluid, fall downward, when they are not ſupported. I hold a ſtone in my hand; if I let it go, it falls to the ground, and would fall ſtill farther, were there an aperture in the earth. While I write, my paper would fall to the ground, were it not ſupported by the table. The ſame law applies to every

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body with which we are acquainted. There is not one that would not fall to the ground, if it were not fupported, or ftopped by the way.

The caufe of this phenomenon, or of this propenfity of all bodies, is denominated gravity. When it is faid, that bodies are heavy, or poffefs gravity, we mean, that they have a propenfity to fall downward, and actually would fall, if we remove what before fupported them.

The ancients were little acquainted with this property. They believed that there were bodies which had, naturally, a tendency to rife, fuch as fmoke and vapours; and fuch bodies they termed light, to diftinguifh them from thofe which have a tendency to fall. But it has been difcovered, by experiment, that it is the air which raifes thefe fubftances aloft; for in a fpace void of air, it is well known; by means of the air-pump, that fmoke and vapours defcend as well as ftone, and that thefe fubftances are, of their own nature, heavy, like others. When, therefore, they rife into the air, the fame law acts upon them which acts upon a log of wood plunged into the water. Notwithftanding it's gravity, it fprings up, as foon as you leave it to itfelf, and fwims, becaufe it is not fo heavy as water; and, in virtue of a general rule, all bodies rife in a fluid of more gravity than themfelves.

If you throw a piece of iron, of copper, of filver, and even of lead, into a veffel full of quickfilver, they fwim on the furface, and if you force them down, they re-afcend when left to themfelves. Gold alone

alone ſinks, becauſe it is heavier than quickſilver. And, ſince there are bodies which riſe in water, and in other fluids, notwithſtanding their gravity, for this reaſon merely, that they are not ſo heavy as water, or thoſe other fluids; it is not at all ſurprizing, that certain bodies, leſs weighty than air, ſuch as ſmoke and vapours; ſhould riſe in it.

I have already remarked, that air itſelf poſſeſſes gravity, and that by means of this gravity, it ſupports the mercury in the barometer. When, therefore, it is affirmed, that all bodies are heavy, it is to be underſtood, that all bodies, without a ſingle exception, would fall downward in a vacuum. I might venture to add, that they would fall with an equal degree of rapidity; for a feather and a piece of gold deſcend with equal velocity in an exhauſted receiver.

It might be objected to this general property of body, that a ſhell, diſcharged from a mortar, does not at once fall to the ground, like a ſtone, which I let drop from my hand, but mounts into the air. It cannot, however, be inferred, that the ſhell has no gravity; for it is evident, that the ſtrength of the powder hurls the bomb aloft, and but for this, it would, without doubt, immediately fall to the ground. And we ſee, in fact, that it does not continue always to aſcend, but as ſoon as the force, which carries it upward, is exhauſted, down it comes with a rapidity, that cruſhes every thing it meets, a ſufficient proof of it's gravity.

When, therefore, it is affirmed, that all bodies are heavy, no one means to deny that they may be

N 2 ſtopped,

ſtopped, or that they may be thrown aloft; but this is effected by an external power, and it remains indubitably certain, that all bodies whatever, as ſoon as left to themſelves, at reſt, or without motion, will aſſuredly fall when no longer ſupported. There is a cellar under my apartment, but the floor ſupports me, and preſerves me from falling into it. Were the floor ſuddenly to crumble away, and the arch of the cellar to tumble in at the ſame time, I muſt infallibly be precipitated into it, becauſe my body is heavy, like all other bodies with which we are acquainted. I ſay, *with which we are acquainted*, for there may, perhaps, be bodies deſtitute of weight; ſuch as, poſſibly, light itſelf, the elementary fire, the electric fluid, or that of the magnet.*

Except theſe bodies, the gravity of which is not

* I muſt once more take the pious *Euler* out of the hands of the *quondam* Marquis, and let him ſpeak for himſelf. The inſtance which the Author adduces, of bodies that, poſſibly, are deſtitute of gravity, is one taken from divine Revelation, that of the angels. "Such," ſays he, "as the bodies of angels, which have "formerly appeared to men. A body, like this, would not fall "downward, though the floor were ſuddenly to be removed from "under it, but would move as firmly through the air, as on the "earth." It is amuſing to obſerve, with what ſolicitude the Pariſian Annotator keeps clear of every thing that ſavours of religion. He ſeems apprehenſive, that a ſingle drop of water from Scripture, would contaminate the whole maſs of philoſophy. His terror is, with a little variation, that of Macbeth.

Will all great Neptune's ocean waſh this blood
Clean from my hand? No; this my hand will rather
The multitudinous ſeas incarnadine,
Making the green one red. SHAKESPEAR.

yet

yet confirmed by experiment, gravity may be confidered as a general property of all the bodies which we know, in virtue of which, they all have a tendency to fall downward, and actually do fo, when nothing oppofes their defcent.

23d Aug. 1760.

4.5.2 Letter 51. Gravity of the Moon

LETTER LI.

Gravity of the Moon.

I HAVE ſaid that a terreſtrial body, placed at the diſtance of the moon, would be reduced to the 3600th part of it's weight, or, in other words, would be forced toward the centre of the earth with a power 3600 times leſs than it has at the ſurface of the globe. This power, however, would be ſufficient to make it deſcend to the earth, if it were no longer ſupported. It is true we are incapable of proving this by any experiment, as no means exiſt of raiſing ourſelves to ſuch a height. There is, however, a body at that height, the moon: ſhe muſt, therefore, be ſubject to this effect of gravity, and yet we ſee ſhe does not fall to the earth.

To this I anſwer, that if the moon were at reſt, ſhe would certainly fall, but the rapid motion which carries her along prevents her falling. There are experiments which prove the ſolidity of this anſwer. A ſtone dropped from the hand, without having any motion impreſſed upon it, falls immediately, in the direction of a ſtraight vertical line; but if you throw this ſtone, impreſſing on it a motion which forces it out of that direction, it does not fall immediately downward, but moves in a curve line before it reaches the earth, and this will appear more ſenſibly in proportion to the velocity impreſſed upon it.

A cannon ball, diſcharged in a horizontal direc-

O 4 tion,

tion, does not come to the earth till it has got to a confiderable diftance; and were it fired from the top of a high mountain, it might, perhaps, fly feveral miles before it reached the ground. If the direction of the cannon is farther elevated, and the quantity and ftrength of the powder increafed, the ball will be carried much farther. This might be carried fo far, that the ball fhould not light till it had reached the antipodes: nay, farther ftill, till it fhould not fall at all, but return to the place where it was fhot off, and thus perform a new tour round the globe. It would thus be a little moon, making it's revolutions round the earth like the real moon.

You will now pleafe to reflect on the height of the moon, and the prodigious velocity with which fhe moves, and you will no longer be furprifed that fhe fhould not fall to the earth, though forced by gravity toward it's centre. There is another reflection which will place this in a clearer light. We have only to confider the path defcribed by a ftone thrown, or a cannon ball fhot off, in an oblique direction. It is always a curve, fuch as reprefented in the annexed figure *(plate* III. *fig*. 3).

Let A be the fummit of a mountain from which the cannon ball is fired off, which, after having moved in the direction A E F B, falls to the ground at B; and the path which it defcribes is a curve line. I remark, then, that if the ball were not heavy, that is, if it were not forced toward the earth by the power of gravity, it would not fall, though left to itfelf, as gravity is the only caufe of it's defcent;

much

much lefs, being fired off at A, as reprefented in the figure, would it ever fall to the ground. Hence we fee, it is gravity that brings it down to the ground, after having defcribed the curve A E F B; gravity, therefore, directs it's path in the curve A E F B; and if it were deftitute of gravity, the ball would not defcribe a curve, but proceed forward in the direction of the ftraight line A C, the direction in which it was fired off.

This being laid down, let us attend to the moon, which affuredly does not move in a ftraight line; her path muft of neceffity be a curve, as fhe always preferves nearly the fame diftance from us, and that curve almoft a circle, fuch as you would defcribe round the earth, with a radius equal to the moon's diftance.

It is very reafonable to demand, Why the moon does not move in a ftraight line? But the anfwer is obvious; for as gravity occafions the curve direction of the path purfued by a ftone thrown, or a cannon ball fired off, there is good ground for maintaining, that gravity acts likewife upon the moon, forcing her toward the earth; and that this gravity occafions alfo the curve direction of her orbit. The moon, then, has a certain weight, fhe is, of confequence, forced toward the earth; but this weight is 3600 times lefs than it would be at the furface of the earth. This is not merely a probable conjecture, but a truth demonftrated. For this gravity being fuppofed, we are enable to determine, on the moft eftablifhed mathematical

4.5.3 Letter 52. Discovery of universal Gravitation by Newton

thematical principles, the path which the moon muſt purſue; and this is found perfectly to agree with that in which ſhe actually does move; and this is a complete demonſtration of the truth of the aſſertion.

1ſt September, 1760.

LETTER LII.

Diſcovery of univerſal Gravitation by Newton.

GRAVITY, then, or weight, is a property of all terreſtrial bodies, and it extends, likewiſe, to the moon. It is in virtue of gravity that the moon preſſes toward the earth; and gravity regulates her motion juſt as it directs that of a ſtone thrown, or of a cannon ball fired off.

To *Newton* we are indebted for this important diſcovery. This great Engliſh philoſopher and geometrician, happening one day to be lying under an apple-tree, an apple fell upon his head, and ſuggeſted to him a multitude of reflections. He readily conceived that gravity was the cauſe of the apple's falling, by overcoming the force which attached it to the branch. Any perſon whatever might have made the ſame reflection; but the Engliſh philoſopher purſued it much farther. Would this force have always acted upon the apple, had the tree been a great deal higher? He could entertain no doubt of it.

But had the height been equal to that of the moon? Here

Here he found himſelf at a loſs to determine whether the apple would fall or not. In caſe it ſhould fall, which appeared to him, however, highly probable, ſince it is impoſſible to conceive a bound to the height of the tree, at which it would ceaſe to fall, it muſt ſtill have a certain degree of gravity forcing it toward the earth; therefore, if the moon were at the ſame place, ſhe muſt be preſſed toward the earth by a power ſimilar to that which would act upon the apple. Neverthel eſs as the moon did not fall on his head, he conjectured that motion might be the cauſe of this, juſt as a bomb frequently flies over us, without falling vertically.

This compariſon of the motion of the moon to that of a bomb, determined him attentively to examine this queſtion; and, aided by the moſt ſublime geometry, he diſcovered, that the moon in her motion was ſubject to the ſame laws which regulate that of a bomb, and that if it were poſſible to hurl a bomb to the height of the moon, and with the ſame velocity, the bomb would have the ſame motion as the moon, with this difference only, that the gravity of the bomb at ſuch a diſtance from the earth, would be much leſs than at it's ſurface.

You will ſee, from this detail, that the firſt reaſonings of the philoſopher on this ſubject were very ſimple, and ſcarcely differed from thoſe of the clown; but he ſoon puſhed them far beyond the level of the clown. It is, then, a very remarkable property of the earth, that not only all bodies near it, but thoſe alſo which are remote, even as far as to the diſtance of of the moon, have a tendency toward the centre of the earth, in virtue of a power which is called gravity, and which diminiſhes in proportion as bodies remove from the earth.

The Engliſh philoſopher did not ſtop here As he knew that the other planets are perfectly ſimilar to the earth, he concluded, that bodies adjacent to each planet poſſeſs gravity, and that the direction of this gravity is toward the centre of ſuch planet. This gravity might be greater or leſs there than on the earth; in other words, that a body of a certain weight with us, tranſported to the ſurface of any planet, might there weigh more or leſs.

Finally, this power of gravity of each planet extends, likewiſe, to great diſtances around them; and as we ſee that Jupiter has four ſatellites, and Saturn five, which move round them juſt as the moon does round the earth, it could not be doubted, that the motion of the ſatellites of Jupiter was regulated by their gravity toward the centre of that planet; and that of the ſatellites of Saturn by their gravitation toward the centre of Saturn. Thus, in the ſame manner as the moon moves round the earth, and their reſpective ſatellites move round Jupiter and Saturn, all the planets themſelves move round the ſun. Hence *Newton* drew this illuſtrious and important concluſion: That the ſun is endowed with a ſimilar property of attracting all bodies toward it's centre, by a power which may be called *ſolar gravity*.

This power extends to a prodigious diſtance around him, and far beyond all the planets, for it is this

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power which modifies all their motions. The fame great philofopher difcovered the means of determining the motion of bodies from the knowledge of the power by which they are attracted to a centre; and as he had difcovered the powers which act upon the planets, he was enabled to give an accurate defcription of their motion. In truth, before he arofe, the world was in a ftate of profound ignorance refpecting the motion of the heavenly bodies; and to him alone we are indebted for all the light which we now enjoy in the fcience of aftronomy.

It is aftonifhing to think how much of their progrefs all the fciences owe to an original idea fo very fimple. Had not *Newton* accidentally been lying in an orchard, and had not that apple by chance fallen on his head, we might, perhaps, ftill have been in the fame ftate of ignorance refpecting the motions of the heavenly bodies, and a multitude of other phenomena depending upon them.* This fubject, undoubtedly, is altogether worthy of your attention, and fhall therefore be refumed in a future letter.

3d September, 1760.

* *Newton* was afked one day, How he had difcovered the fyftem of the univerfe? *By continually thinking upon it*, replied he. This anecdote has a greater air of probability than the ftory of the apple.—*F. E.*

4.5.4 Letter 53. Continuation. Of the mutual attraction of the heavenly bodies

LETTER LIII.

Continuation. Of the mutual Attraction of the heavenly Bodies.

THE Newtonian ſyſtem, you will eaſily believe, made at firſt a great noiſe, and with good reaſon, as no one had hitherto hit upon a diſcovery ſo very fortunate, and which diffuſed, at once, ſuch clear light over every branch of ſcience. It has been expreſſed by ſeveral names, of which it is proper you ſhould be informed, becauſe it is frequently the ſubject of converſation.

It has been denominated, the ſyſtem of univerſal gravitation; for *Newton* maintained, that not only the earth, but all the heavenly bodies, in general, are endowed with this property, of attracting thoſe which ſurround them, with a power ſimilar to that of weight, or gravity: hence is derived the term *Gravitation.* This power is, however, totally inviſible; for we ſee nothing acting upon bodies, and preſſing them toward the earth, and ſtill leſs toward the heavenly bodies.

The loadſtone, by which iron and ſteel are attracted, without our being able to diſcern the cauſe, preſents a phenomenon ſomewhat ſimilar. Though it be now certain, that this is produced by a ſubſtance extremely ſubtile, which penetrates through the pores of the loadſtone and of the iron, it may, however, be affirmed, that the loadſtone attracts iron, and that iron

iron is attracted by it, provided this manner of ſpeaking does not exclude the true cauſe. It may likewiſe be aſſirmed, then, that the earth attracts all bodies that are near it, nay thoſe which are at very great diſtances; and we may conſider the weight, or gravity, of bodies, as the effect of the attraction of the earth, which acts even upon the moon.*

Again, the ſun, and all the planets, are endowed with a ſimilar power of attraction, which extends to all bodies. In conformity to this manner of ſpeaking, we ſay, that the ſun attracts the planets, and that Jupiter and Saturn attract their reſpective ſatellites; hence *Newton*'s ſyſtem has likewiſe been denominated, the ſyſtem of *Attraction*. As there can be no doubt that bodies very near the moon muſt likewiſe be preſſed to it by a power ſimilar to gravity, it may likewiſe be affirmed, that the moon, too, attracts adjoining bodies.

It was natural to ſuppoſe, that this attraction of the moon ſhould extend as far as the earth, though it muſt be, undoubtedly, very feeble, as we have ſeen

* So far is the exiſtence of a magnetic fluid from being undeniable, that it is highly improbable, if not abſurd. The various phenomena of magnetiſm may clearly be derived from two laws, or general facts; than which a greater ſimplicity can hardly be expected. If we recur to the agency of a fluid, we muſt gratuitouſly beſtow on it a number of properties; and, after all, we ſhall find it extremely difficult, I might ſay, impoſſible, to preſerve conſiſtency in our complicated hypotheſis; nor ſhall we ever be able, from our aſſumptive principles, to account for the facts obſerved. Such, at leaſt, has been the fate of the ſpeculations hitherto offered on the ſubject of magnetiſm.—*E. E.*

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that of the earth upon the moon to be; now, the fame philofopher has placed this, alfo, beyond the reach of doubt, by demonftrating that the flux and reflux of the waters of the fea, of which I fhall take occafion to fpeak afterwards, are caufed by the attraction of the moon. It can no longer be doubted, therefore, that Jupiter and Saturn are reciprocally attracted by their refpective fatellites; and that the fun itfelf is fubject to the attraction of the planets, though this attractive power be exceedingly fmall.

This is the origin of the fyftem of univerfal attraction, in which it is maintained, and with good reafon, that not only does the fun attract the planets, but is reciprocally attracted by each of them; nay, that all the planets exert their attractive power upon each other. The earth, then, is attracted, not only by the fun, but alfo by all the other planets, though their power be almoft imperceptible, compared to that of the fun.

You will eafily comprehend, that the motion of a planet, which is attracted not only by the fun, but by the other planets, in however fmall a degree, muft be fomewhat different from what it would have been, were it attracted by the fun only; and that, confequently, the attractions of the other planets muft caufe fome fmall derangement of that motion. Now thefe derangements are, likewife, confirmed by experience; and this has carried the fyftem of univerfal attraction to the higheft poffible degree of certainty, fo that no one now prefumes to difpute it's truth.

I muft,

I muſt likewiſe remark, that comets, too, are ſubject to this law; that they are principally attracted by the ſun, whoſe action regulates their motion; but that they, likewiſe, feel the attractive power of all the planets, eſpecially when they are not very diſtant from them. It is a general rule, as we ſhall ſee afterwards, that the attraction of all the heavenly bodies diminiſhes in proportion to the diſtance, and increaſes in proportion to the nearneſs. Now, comets, likewiſe, are endowed with a power, by which other bodies are attracted toward them, and ſo much the more ſenſibly, as they approach nearer. When, therefore, a comet paſſes ſomewhat more cloſely to a planet, it may derange the motion of that planet by it's attractive power; and it's own will likewiſe be diſturbed by that of the planet. Theſe conſequences are verified by real obſervation.

Examples might be adduced to prove, that the motion of a comet has been deranged by the attraction of the planets, near which it happened to paſs,*

* The comet of 1682, which ſhould have re-appeared in 1757, underwent, from the attractive powers of Jupiter and Saturn, near which it paſſed, a conſiderable derangement, which retarded it's appearance nearly two years. Mr. *Clairaut* calculated, theoretically, the perturbations which it's motion muſt have ſuffered, and predicted the return of that comet, with a degree of exactneſs, which conſtitutes a convincing proof in favour of the ſyſtem of gravitation. There was, however, an error of two months. But Mr. *de la Place* has ſince demonſtrated, that it would have been much leſs, had we then been able to calculate the perturbations of Jupiter and Saturn, with as much exactneſs as it now can be done.—*F. E.*

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and that the motion of the earth, and of the other planets, has already undergone ſome derangement, from the attraction of comets.

The fixed ſtars being bodies ſimilar to the ſun, are likewiſe endowed, no doubt, with an attractive power, but their enormous diſtance prevents our feeling any ſenſible effect from it.

5th Sept. 1760.

4.6 Tides

During the early eighteenth century, a Cartesian–Newtonian controversy existed in western continental Europe, mainly at the Paris Academy. The goal was to find which had superior methods to obtain exact answers. In the prefaces and body of the *Principia mathematica,* Newton recommended studies on the shape of Earth, the tides, the orbit of comets, lunar motion, and the orbits of Jupiter and Saturn to determine whether Cartesian science with its theory of vortices or Newtonian dynamics with its inverse square law of gravitational attraction was more accurate. In 1740 the tides were the topic for the annual prize of the Paris Academy. The Cartesians had already lost the debate over the shape of Earth. In 1740 Euler, Daniel Bernoulli, and two others shared the prize. While Bernoulli employed a Newtonian explanation, Euler rejected Galileo's account that the motion of Earth caused the tides and Descartes' view that the Moon alone did this. Euler still believed that a small adjustment was needed in Newton's gravitation law. He began with Kepler's forces. He showed that the forces of the Sun and Moon influence the tides and developed the calculus integrals to compute the motion of the tides. He had developed the calculus beyond Newton's early stage. With regard to the Cartesian–Newtonian controversy, in letter 64 Euler began by rejecting Descartes' tidal theory and giving Newton's inverse square law as the starting point for explaining the tides. In the Cartesian–Newtonian controversy at the Paris Academy, the Cartesians had by 1740 mostly lost.

4.6.1 Letter 63. Different opinions of philosophers respecting the flux and reflux of the sea

LETTER LXIII.

Different Opinions of Philoſophers reſpecting the Flux and Reflux of the Sea.

WHEN the water of the ſea riſes at any place, we are not to imagine that it ſwells from any internal cauſe, as milk does when put in a veſſel upon the fire. The elevation of the ſea is produced by a real increaſe of water flowing hither from ſome other place. It is a real current which is very perceptible at ſea, conveying the waters toward the place where the flux is.

In order to have a clearer comprehenſion of this, you muſt conſider that in the vaſt extent of the ocean there are always places where the water is low, while it is high at others; and that it is conveyed from the former to the latter. When the water riſes at any place, there is always a current, conveying it from other places, where it is of courſe at that time low. It is an error, therefore, to imagine, with ſome authors, that during the flux of the ſea the total maſs of water becomes greater, and that it diminiſhes during the reflux. The entire maſs or bulk of water remains ever the ſame; but it is ſubject to a perpetual oſcillation, by which the water is alternately tranſported from certain regions to others; and when the water is high at any place, it is of courſe low ſomewhere elſe, ſo that the increaſe at places where it is

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high is precifely equal to the decreafe at thofe where it is low.

Such are the phenomena of the flux and reflux of the fea, the caufe of which ancient philofophers endeavoured to difcover, but in vain. *Kepler*, in other refpects a great aftronomer, and the ornament of Germany, believed that the earth, as well as all the heavenly bodies, was a real living animal, and confidered the flux and reflux of the fea as the effect of it's refpiration. According to this philofopher, men and beafts were juft like infects feeding on the back of the huge animal. You will hardly expect I fhould go into the refutation of an opinion fo ridiculous.

Defcartes, that great French philofopher, endeavoured to introduce a more rational philofophy; and remarked, that the flux and reflux of the fea was principally regulated by the moon's motion; which was indeed a very important difcovery, though the ancients had already fufpected a connection between thefe two phenomena. For if high water or the top of the flux happen to-day at noon, it will be low water at 11 minutes after fix in the evening: it will rife till 22 minutes after midnight; and the next low water will be 33 minutes after fix in the morning of the day after; and the enfuing high water, or flux, will be three quarters of an hour after noon: fo that from one day to another the fame tides are later by three quarters of an hour.

And as the fame thing precifely takes place in the moon's motion, which rifes always three quarters of an an hour later than the preceding day, it was presumable that the tides followed the courſe of the moon. If at any given place, for example, on the day of new moon, high water happen to be at three of the clock, afternoon, you could reſt aſſured, that ever after, on the firſt day of the moon, the flux would invariably be at the height at three o'clock afternoon, and that every following day it would fall later by three quarters of an hour.

Again, not only the time when every flux and reflux happen exactly follows the moon, but the ſtrength of the tides, which is variable, appears ſtill to depend on the poſition of the moon. They are every where ſtronger after the new and full moon, that is, at theſe periods the elevation of the water is greater than at other times; and after the firſt and laſt quarters, the elevation of the water, during the flux, is ſmaller. This wonderful harmony between the tides, and the motion of the moon, was, undoubtedly, ſufficient ground to conclude, that the chief cauſe of the flux and reflux of the ſea was to be ſought for in the action of the moon.

Deſcartes accordingly believed, that the moon, in paſſing over us, preſſed the atmoſphere, or the air which ſurrounds the earth, and that the air preſſing on the water, in it's turn, forced it to ſubſide. Had this been the caſe, the water muſt have been depreſſed at the places over which the moon was, and that the ſame effect ſhould be produced 12 hours after, in the enſuing tide; which, however, does not happen. Beſides the moon is too diſtant from the earth,

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and the atmofphere too low to be impreffed by the moon; and admitting that the moon, or any other great body, were to pafs along the atmofphere, it would be very far from undergoing any preffure from it, and ftill lefs would the fea feel this pretended preffure.

This attempt of *Defcartes* to explain the flux and reflux of the fea, has therefore failed; but the connection of this phenomenon with the moon's motion, which this philofopher has fo clearly unfolded, enabled his fucceffors to employ the application of their refearches with more fuccefs. This fhall be the fubject of fome following letters.

30*th September*, 1760.

4.6.2 Letter 64. Explanation of the flux and reflux, from the attractive power of the Moon

LETTER LXIV.

Explanation of the Flux and Reflux, from the attractive Power of the Moon.

DESCARTES's method of explaining the flux and reflux of the fea, by the preffure of the moon upon our atmofphere, not having fucceeded, it was reafonable to look for the caufe of it in the attraction which the moon exercifes upon the earth, and confequently alfo upon the fea.

The attractive power of the heavenly bodies having been already fufficiently eftablifhed, by fo many other phenomena, as I have fhewn, it could not be doubted that the flux and reflux of the fea muft be an effect of it. As soon as it is demonstrated that the moon, as well as the other heavenly bodies, is endowed with the property of attracting all bodies, in the direct ratio of their mass, and in the inverse ratio of the square of their distance, it is easily comprehended that it's action must extend to the sea; and the more so, as you must frequently have observed, that the smallest force is capable of agitating a fluid. All that remains, therefore, is to enquire, whether the attractive power of the moon, such as we suppose it, is capable of producing in the sea the agitation known to us by the name of flux and reflux.

Let the annexed figure (*plate* III. *fig.* 5.) represent the earth and the moon. A is the place where we see the moon over the earth; B that which is directly opposite, or the antipodes of A; and C is the centre of the earth. As the point A is nearer the moon than the point B, a body at A is more powerfully attracted toward the moon than a similar body at B. And if we suppose a third similar body to be placed at the centre of the earth C, it is evident that the body A will be more powerfully attracted toward the moon than the body C, and this last than the body B, because the body A is nearer to the moon, and the body B more remote than the body C. But similar bodies placed at E and F, are almost as much attracted by the moon as that which is at the centre of the earth C, as they are all three nearly equi-distant from the moon.

Hence we see that bodies placed on the surface of the earth are not all equally attracted toward the moon. This inequality of attraction depends on the inequality of their diſtance from the centre of the moon L, ſo that a body is ſo much the more powerfully attracted by the moon, as it's diſtance is leſs; and the contrary takes place according as the diſtance is greater.

To theſe differences in the action of the moon on bodies differently ſituated, we muſt here chiefly pay attention; for if all bodies were equally attracted toward the moon, they would equally obey this power, and no derangement could take place in their mutual ſituation.

You can eaſily form the idea of ſeveral carriages drawn along by powers perfectly equal; they will proceed on the road, always preſerving the ſame order, and the ſame diſtances; but as ſoon as ſome of them advance more briſkly, and others more ſlowly, the order will be deranged. The ſame thing takes place in the caſe of the different bodies which are attracted by the moon; if they all felt, in the ſame degree, the action of that luminary, they would preſerve the ſame relative ſituation, and we ſhould perceive no change in them: but as ſoon as the force with which they are attracted toward the moon varies as to each of them, their order and their relative ſituation neceſſarily change, unleſs they are attached to each other by bands which that power is unable to burſt aſunder.

But this is not the caſe with the ſea, as all the particles of a fluid are eaſily ſeparated from each other, and

and every one may obey the impreſſions which it receives. It is evident, then, that when the powers which act on the different parts of the ſea are not equal to one another, an agitation, or derangement, muſt be the conſequence.

We have juſt ſeen that the different parts of the ſea are attracted unequally by the moon, according as they are unequally diſtant from her centre; the ſea muſt, therefore, be agitated by the force of the moon, which, continually changing her ſituation, with reſpect to the earth, and performing a revolution round it in about twenty-four hours and three quarters, makes the ſea undergo the ſame changes, and preſents the ſame phenomena in the ſame period of twenty-four hours and three quarters; the flux and reflux muſt, therefore, be retarded from one day to another three quarters of an hour, which is confirmed by conſtant experience.

It now remains that we ſhew, How the alternate elevation and depreſſion of the ſea, which ſucceed each other after an interval of ſix hours and eleven minutes, reſult from the inequality of the powers of the moon. This I propoſe to examine in my next letter.

4th October, 1760.

4.7 Principle of least action

Pierre Moureau de Maupertuis, the president of the Berlin Academy, claimed priority for the principle of least action based on articles from 1741 to 1746, and its universality. In an article in *Acta eruditorum* in 1751, Johann Samuel König held that Leibniz had stated it in a missing letter from 1707. Euler, who actually deserved credit from his *Methodus inveniendi,* tried to stay out of the argument, but after a search of Leibniz's documents failed to produce the letter, announced the judgment by a vote of the Berlin Academy: the letter was a fraud. Euler later provided the mathematics for what is now called the Euler–Maupertuis principle and proved its universality. Voltaire was visiting Frederick II, who supported Maupertuis, the president of the Berlin Academy. Voltaire wrote the pamphlet *Dr Akakia*. This made the Maupertuis–König affair both scientific and literary. *Dr Akakia*, which pilloried Maupertuis, became very popular. Later Condorcet criticized Euler for what he considered the Academy's and Euler's harsh decision.

4.7.1 Letter 78. The same subject. Principle of the least possible action

LETTER LXXVIII.

The ſame Subject. Principle of the leaſt poſſible Action.

YOU have now made very conſiderable progreſs in the knowledge of nature, from the explanation of the real origin of the powers capable of changing the ſtate of bodies; and you are, at preſent, in a condition eaſily to comprehend, why all

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thofe of this world are fubject to an inceffant change of ftate, from reft to motion, or from motion to reft.

Firft, we are certain, that the world is filled with matter. Here below, it is evident, that the fpace which feparates the grofs bodies fenfible to feeling, is occupied by the air, and that, when we make a vacuum in any fpace, the ether inftantly fucceeds, and it, likewife, fills the fpace in which the heavenly bodies move. All fpace being thus full, it is impoffible that a body in motion fhould continue it a fingle inftant, without meeting others, through which it muft pafs, if they were not impenetrable. And, as this impenetrability of bodies exerts always, and univerfally, a force which prevents all penetration, it muft be continually changing the ftate of bodies; it is not at all furprizing, then, that we fhould obferve perpetual changes in the ftate of bodies, though every one has a tendency to preferve itfelf in the fame ftate.

If they could penetrate each other freely, nothing would prevent any one from remaining perfeveringly in it's ftate; but being impenetrable, there muft thence, neceffarily, refult force fufficient to prevent all penetration; and no more refults than what is precifely needful.

While they can continue in the fame ftate, without any injury to impenetrability, they then exert no force, and bodies remain in their ftate; it is only to prevent penetration, that impenetrability becomes active, and fupplies a force fufficient to oppofe it.

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When, therefore, a ſmall force ſuffices to prevent penetration, impenetrability exerts that, and no more; but when a great force is neceſſary for this purpoſe, impenetrability is ever in a condition to ſupply it.

Thus, though impenetrability ſupplies theſe powers, it is impoſſible to ſay, that it is endowed with a determinate force; it is rather in a condition to ſupply all kinds of force, great or ſmall, according to circumſtances; it is even an inexhauſtible ſource of them. As long as bodies are endowed with impenetrability, this is a ſource which cannot be dried up; this force abſolutely muſt be exerted, or bodies muſt mutually penetrate, which is contrary to nature.

It ought, likewiſe, to be remarked, that this force is never the effect of the impenetrability of a ſingle body; it reſults always from that of all bodies at once, for if one of the bodies was penetrable, the penetration would take place, without any need of a power to effect a change in their ſtate. When, therefore, two bodies come into contact, and when they cannot continue in their ſtate without penetrating each other, the impenetrability of both acts equally; and it is by their joint operation, that the force neceſſary to prevent the penetration is ſupplied: we then ſay, that they act upon each other, and that the force, reſulting from their impenetrability, produces this effect. This force acts upon both of them; for as they have a tendency toward mutual penetration, it repels both the one and the other, and thus prevents their penetration.

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It is certain, then, that bodies may act upon each other; and we ſpeak ſo frequently of this action, as when two billiard balls claſh, it is ſaid, the one acts upon the other, that you muſt be well acquainted with this mode of expreſſion. But it muſt be carefully remarked, that, in general, bodies do not act upon each other, but in ſo far as their ſtate becomes contrary to impenetrability; from whence reſults a force capable of changing it, preciſely ſo much as is neceſſary to prevent any penetration; ſo that a ſmall force would not have been ſufficient to produce this effect.

It is very true, that a greater force would, likewiſe, prevent the penetration; but when the change produced in the ſtate of bodies is ſufficient to prevent mutual penetration, the impenetrability acts no farther, and there reſults from it the leaſt force that is capable of preventing the penetration. Since, then, the force is the ſmalleſt, the effect which it produces, that is, the change of ſtate which it operates, in order to prevent penetration, will be proportional; and, conſequently, when two or more bodies come into contact, ſo that no one could continue in it's ſtate without penetrating the others, a mutual action muſt take place, which is always the ſmalleſt that was capable of preventing penetration.

You will find here, therefore, beyond all expectation, the foundation of the ſyſtem of the late Mr. *de Maupertuis*, ſo much cried up by ſome, and ſo violently attacked by others. His principle is, that of the

the leaſt poſſible action; by which he means, that, in all the changes which happen in nature, the cauſe which produces them, is the leaſt that can be.

From the manner in which I have endeavoured to unfold this principle to you, it is evident, that it is perfectly founded in the very nature of body, and that thoſe who deny it, are much in the wrong, though ſtill leſs than thoſe who would turn it into ridicule. You will already, perhaps, have remarked, that certain perſons, no great friends to Mr. *de Maupertuis*, take every opportunity of laughing at the principle of *the leaſt poſſible action*, as well as at the hole continued down to the centre of the earth; but, fortunately, truth ſuffers nothing by their pleaſantry.

22*d Nov.* 1760.

Chapter 5

Selected letters from Volume 2 (letters 80–154) and Volume 3 (letters 155–234)[1]

5.1 Liberty, happiness, and truth

The *Letters to a German Princess* were written at the height of the eighteenth-century Enlightenment. Its main headquarters was in Paris with a group known as the *philosophes*, most notably Montesquieu, Diderot, d'Alembert, and Voltaire, and the salons of a few women. There were other important locations: England, Scotland, German lands, Italian cities, imperial Russia, Sweden, and the new United States of America. The *philosophes*, known as the party of humanity, applied critical reason to fixed beliefs, sought freedom of thought and action, and attacked religious fanaticism, especially repression and cruelty. The most noteworthy publication was Denis Diderot's 28 volume *Encyclopédie* (1751–72). It was not simply an accumulation of knowledge: it gave prominence to the sciences and attempted to change the 'general way of thinking'.

Among the crucial issues that Euler addressed were freedom or liberty, happiness, and methods to find truth. Liberty, he wrote, is essential to all spiritual beings, and not even God can divest beings of it. He rejected the Wolffians, who put the body on the same footing with the mind and the soul. Actions depend on motives. The motives for spirits are always voluntary. To life and liberty, the Enlightenment law from William Blackstone and philosophy added happiness. Euler held that the disposition to attain supreme happiness was through the love of God and the pursuit of virtue. In the previous century scholars and clerics had disputed whether faith in the Scriptures or reason was superior to reach truths. To resolve the many Enlightenment disputes, Euler divided all knowledge as was done in his time into three classes: historical (from

[1] In this chapter, we are using excerpts of the 1853 publication of the Letters by Harper and Brothers. The numbering of the letters is different from that used in modern publications. (https://books.google.com/books?id=hFTkAAAAMAAJ&printsec=frontcover&dq=Letters+of+Euler+to+a+German+Princess,+on+Different+Subjects+in+Physics+and+Philosophy,+Volume+2&hl=en&sa=X&ved=0ahUKEwjDwuW3qurcAhXxpVkKHbFRBb4Q6AEIUjAH#v=onepage&q&f=false).

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the senses, empirical), reason (from demonstrations based on principles), and faith (from persons worthy of credit and the Scriptures). The first two classes can be proven, but the third class is most subject to disagreement.

5.1.1 Letter 85. Of the liberty of spirits; and a reply to objection against liberty

LETTER LXXXV.

Of the Liberty of Spirits; and a Reply to Objections against Liberty.

THE greateſt difficulties on the ſubject of liberty, even thoſe which appear inſurmountable, ariſe from want of diſtinguiſhing, with ſufficient attention, between the nature of ſpirit and that of body. The Wolfian philoſophers even go ſo far as to put ſpirits, and the elements of body, on the ſame footing, and give to both the one and the other the name of *monads*, the nature of which, according to them, conſiſts in the power of changing their ſtate; from whence reſult all the changes in bodies, and all the repreſentations and actions of ſpirits.

Since, then, in this ſyſtem, the actual ſtate of bodies and of ſpirits derives it's determination from that which immediately preceded; and as the actions of ſpirits are derived, like thoſe of bodies, from their preceding ſtate, it is evident, that liberty is no more an attribute of ſpirit than it is of body. As to body, it is impoſſible to conceive the leaſt ſhadow of liberty in it; for liberty always ſuppoſes the power of committing, of admitting, or of ſuſpending an action,

the ſyſtem of pre-eſtabliſhed harmony, but common to all. It is certain that in ſuch and ſuch circumſtances I will perform ſuch an action, and yet I ſhall have the power of acting otherwiſe. This action will *infallibly* take place but not *neceſſarily*. This is the real difficulty.—*F. E.*

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and this is directly oppoſite to all that paſſes in body. Would it not, then, be ridiculous to expect that a watch ſhould point to any other hour than what it actually does, and to think of puniſhing it on that account? Would it not be abſurd to fly into a paſſion at a puppet, becauſe, after ſeveral other geſtures, it had turned it's back to us?

All the changes which take place in bodies, and which are all reducible to their ſtate of reſt, or of motion, are the neceſſary conſequence of the powers which act upon them; and their action once admitted, no changes in bodies can take place, but preciſely ſuch as do take place: what reſpects body, therefore, is an object of neither praiſe nor blame. However ingeniouſly a piece of mechaniſm may be conſtructed, the commendation which we beſtow upon it reverts to the artiſt; the machine itſelf has no intereſt in what paſſes; the artiſt, too, is alone reſponſible for the defects of a clumſy and aukward machine; the machine itſelf is perfectly innocent. While, therefore, the enquiry is reſtricted to bodies, they are clearly in no reſpect reſponſible; no reward, no puniſhment can poſſibly attach to them; all the changes and motions produced in them, are the neceſſary conſequences of their ſtructure.

But ſpirits are of a very different nature, and their actions depend on principles directly oppoſite. Liberty, entirely excluded from the nature of body, is the eſſential portion of ſpirit, to ſuch a degree, that without liberty, a ſpirit could not exiſt; and this it is which renders it reſponſible for it's actions. This property

property is as eſſential to ſpirit as extenſion or impenetrability is to body; and as it would be impoſſible for the divine Omnipotence itſelf to diveſt body of theſe qualities, it would be equally impoſſible for it to diveſt ſpirits of liberty. A ſpirit without liberty, would no longer be a ſpirit, as a body without extenſion would no longer be a body.

It has in all ages been a ſubject of eager enquiry among philoſophers, How God could have permitted ſin to enter into the world? Had they reflected that the ſouls of men are beings neceſſarily free, from their very nature, the controverſy would have been eaſily ſettled.

The objections commonly made to human liberty are theſe: A ſpirit, it is ſaid, or a man, is never determined to an action, but from motives; and after having carefully weighed the reaſons on both ſides, he finally decides in favour of that which he deems the preferable. Hence they conclude that motives determine the actions of men, juſt as the motion of a ball on the billiard table is determined by the ſtroke impreſſed upon it, and that the actions of men are no more free than the motion of the ball. But it muſt be conſidered that the motives which engage a man to undertake any enterprize, refer very differently to the ſoul, from what the ſtroke does to the ball. The ſtroke produces it's effect neceſſarily; but a motive, however powerful, prevents not the action from being voluntary. I had very powerful motives to undertake a journey to Magdeburg: a regard to my promiſe; the proſpect of enjoying the felicity of

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paying my refpects to your Highnefs; but I am perfectly fenfible, at the fame time, that I was not forced to it: and that it was entirely in my own power to take that journey, or to have remained at Berlin. But a body, impelled by any power, neceffarily obeys, and it cannot be affirmed that it was at liberty to obey, or not, as it pleafed.

The motive which determines a fpirit to regulate it's refolves, is of a nature wholly different from a *caufe* or *force* acting upon body. Here, the effect is produced neceffarily; and there, the effect remains always voluntary, and the foul has power over it. On this is founded the *imputability* of the actions of a fpirit, which makes it refponfible for them, and which is the true foundation of right and wrong. As foon as we have fettled this infinite difference between fpirit and body, the queftion refpecting liberty prefents very little difficulty.

16th December, 1760.

5.1.2 Letter 91. The liberty of intelligent beings in harmony with the doctrines of the christian religion

LETTER XCI.

The Liberty of intelligent Beings in Harmony with the Doctrines of the Chriftian Religion.

LIBERTY is a quality fo effential to every fpiritual being, that God himfelf cannot diveft them of it, juft as He cannot diveft a body of it's extenfion, tenſion, or of it's *inertia*, without entirely deſtroying, or annihilating it: to diveſt a ſpirit of liberty, therefore, would be the ſame thing as to annihilate it. This muſt be underſtood of the ſpirit, or ſoul itſelf, and not of the actions of the body, which the ſoul directs, in conformity to it's will. If you would prevent me from writing, you have but to bind my hands; to write is, undoubtedly, an exerciſe of liberty; but then, though you may ſay, that you have deprived me of the liberty of writing, you have only deprived my body of the faculty of obeying the dictates of my ſoul. Bind me ever ſo hard, you cannot extinguiſh in my ſpirit an inclination to write; all you can do is to prevent the execution of it.

We muſt always carefully diſtinguiſh between inclination, or the act of willing, and execution, which is performed by the miniſtration of the body. The act of willing cannot be reſtrained by any exterior power, not even by that of God, for liberty is independent of all exterior force. But there are means of acting on ſpirits, by motives which have a tendency, not to conſtrain, but to perſuade. Let a man be firmly determined to engage in any enterprize, and let us ſuppoſe the execution of it prevented; without making any change in his intention, or will, it might be poſſible to ſuggeſt motives, which ſhould engage him to abandon his purpoſe, without employing any manner of conſtraint: however powerful theſe motives may be, he is always maſter of his own will; it never can be ſaid, that he was forced, or conſtrained, to it, at leaſt the expreſſion

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would be improper ; for the proper term is *perſuade*, which is ſo ſuitable to the nature, and the liberty, of intelligent beings, that it cannot be applied to any other. It would be very ridiculous, for example, in playing at billiards, to ſay, that I perſuaded the ball to run into the hazard.

This ſentiment, reſpecting the liberty of ſpirits, appears, however, to ſome perſons, contrary to the goodneſs, or the power, of the Supreme Being. Liberty, from it's very nature, can ſubmit to no degree of conſtraint, even on the part of God. But without exerciſing any conſtraint over ſpirits, God has an infinite variety of means of preſenting them with perſuaſive motives ; and, I believe, that all poſſible caſes are adapted by Providence to our condition, in ſuch a manner, that the moſt abandoned wretches might derive from them the moſt powerful motives to converſion, if they would but liſten to them : and that a miracle would not produce a better effect on theſe vicious ſpirits ; they might be affected by it, for a ſeaſon, but would not become better. It is thus that God co-operates in our converſion, by furniſhing us with motives the moſt efficacious, and by the circumſtances and opportunities which his providence ſupplies.

If, for example, a man, who hears an awakening ſermon, is affected by it, repents, and is converted ; the act of his ſoul is evidently his own work ; but the occaſion of the ſermon, which he was ſo happy as to hear, preciſely at the time, when he was diſpoſed to profit by it, was nothing leſs than his work ; the divine

divine Providence over-ruled that circumſtance, ſo ſalutary to him. In fact, without the opportunity, over which the man had no power, he would have perſiſted in a ſinful courſe.

Hence, you will eaſily comprehend the meaning of ſuch expreſſions as theſe: "Man can do nothing of "himſelf; all depends on divine grace; it is God "that worketh to will and to do." The favourable circumſtances which Providence ſupplies to men, are ſufficient to elucidate theſe expreſſions, without having recourſe to a ſecret force, which acts by conſtraint on human liberty; as theſe circumſtances are directed of God, in conformity to the moſt conſummate wiſdom, in the view of conducting every intelligent being to happineſs and ſalvation, unleſs he wilfully rejects the means by which he might have attained true felicity.

6th January, 1761.

5.1.3 Letter 97. Refutation of the idealists

LETTER XCVII.

Refutation of the Idealiſts.

I WISH it were in my power to furniſh you with the arms neceſſary to combat the Idealiſts and the Egotiſts, by demonſtrating, that there is a real connection between our ſenſations and the objects themſelves, which they repreſent; but the more I think of it, the more I feel my own incapacity.

It would be ridiculous to think of engaging with the Egotiſts: for a man who imagines he alone exiſts, and who does not believe in my exiſtence, would act in contradiction to his own ſyſtem, if he paid any attention to my reaſoning, which, according to him, would be that of an imaginary being. It is, likewiſe, a hard taſk to confute the Idealiſts, nay, it is impoſſible to convince, of the exiſtence of bodies, a man obſtinately determined to deny it. Though no ſuch philoſophers exiſted, it would be highly intereſting to be able to convince ourſelves, that as often as our ſoul experiences ſenſations, it may be with certainty concluded, that bodies likewiſe exiſt; and that, when my ſoul is affected by the ſenſation of the moon, I may thence boldly infer the exiſtence of the moon.

But the union which the Creator has eſtabliſhed between the ſoul and the brain, is a myſtery ſo unfathomable, that all our knowledge of it amounts only to this: Certain impreſſions made in the brain, where the ſeat of the ſoul is, excite in it certain ideas, or ſenſations; but the *how*, of this influence, is abſolutely unknown to us. We ought to ſatisfy ourſelves with knowing, that this influence ſubſiſts, which experience ſufficiently confirms; and it is in vain to inveſtigate *how* this is produced. Now, the ſame experience which proves it, informs us, likewiſe, that every ſenſation always diſpoſes the ſoul to believe that there exiſts, out of it, ſome object which excited ſuch ſenſation; and that ſenſation diſcovers to us ſeveral properties of the object.

It is, then, a moſt undoubted fact, that the ſoul always concludes, from any ſenſation whatever, the exiſtence of a real object, out of us. This is ſo natural to us, from our earlieſt infancy, and ſo univerſally the caſe with all men, and even with animals, that it cannot, with any propriety, be called a prejudice. The dog that barks when he ſees me, is certainly convinced that I exiſt; for my preſence excites in him the idea of my perſon. The dog, then, is not an idealiſt. Even the meaneſt inſects are aſſured that bodies exiſt, out of them, and they could not have this conviction, but by the ſenſations excited in their ſouls.

I believe, therefore, that ſenſations include much more than thoſe philoſophers are diſpoſed to admit. They are not only ſimple perceptions of certain impreſſions made in the brain; they ſupply the ſoul not with ideas only, but they effectively repreſent

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to it objects externally exifting, though we cannot comprehend how this is done.

In fact, what refemblance can there be between the luminous idea of the moon, and the flight impreffion which it's rays may produce in the brain, by means of nerves?

The idea, even in as far as the foul perceives it, has nothing material; it is an act of the foul, which is a fpirit: it is not neceffary, therefore, to look for a real relation between the impreffions of the brain, and the ideas of the foul; it is enough for us to know, that certain impreffions made in the brain, excite certain ideas in the foul, and that thefe ideas are reprefentations of objects externally exifting, of whofe exiftence they give us the affurance.

Thus, when my brain excites in my foul the fenfation of a tree, or of a houfe, I pronounce, without hefitation, that a tree, or a houfe, really exifts, out of me, of which I know the place, the fize, and other properties. Accordingly, we find neither man nor beaft, who calls this truth in queftion. If a clown fhould take it into his head to conceive fuch a doubt; and fhould fay, for example, he does not believe that his bailiff exifts, though he ftands in his prefence, he would be taken for a madman, and with good reafon; but when a philofopher advances fuch fentiments, he expects we fhould admire his knowledge and fagacity, which infinitely furpafs the apprehenfions of the vulgar.

It appears to me, accordingly, abundantly certain,

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that ſuch extravagant ſentiments would never have been maintained, but from pride, and an affectation of ſingularity: and you will readily agree, that the common people have, in this reſpect, much more good ſenſe than thoſe learned gentlemen, who derive no other advantage from their reſearches, but that of bewildering themſelves in a labyrinth of chimeras, unintelligible to the reſt of mankind.*

Let it be eſtabliſhed, then, as a certain rule, that every ſenſation not only excites in the ſoul an idea, but ſhews it, if I may ſo expreſs myſelf, an external object, of whoſe exiſtence it gives full aſſurance, without practiſing a deception. A very formidable objection, however, is ſtarted againſt this, ariſing from dreams, and the reveries of ſick perſons, in which the ſoul experiences a great variety of ſenſations of objects which no where exiſt. The only reflection I ſhall ſuggeſt on this ſubject is, that it muſt be very natural for us to judge that the objects, the ſenſations of which the ſoul experiences, really exiſt, as we judge after this manner even in ſleep, though then we deceive ourſelves; but it does not thence follow, that we likewiſe deceive ourſelves when we are awake. In order to ſolve this objection, it is

* Mr. *Euler* ſeems here to be confounding two different queſtions, that of the exiſtence of exterior objects, and that of a kind of real reſemblance between theſe objects and the idea which we have of them. *Barclay* has, however, carefully diſtinguiſhed them, and has clearly pointed out the difference. All we can at preſent do, is to refer the reader to the article *Exiſtence*, in the *Encyclopedia*, the only work in which theſe queſtions have been treated with an exact analyſis.—*F. E.*

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neceſſary to know better the difference of the ſtate of the man who is aſleep, and of him who wakes; and none, perhaps, know this leſs than the learned, which muſt ſurely be a matter of ſome ſurprize to you.

27th January, 1761.

5.1.4 Letter 114. Of true happiness. Conversion of sinners

LETTER CXIV.

Of true Happineſs. Converſion of Sinners. Reply to Objections on the Subject.

THE holy life of the apoſtles, and of the other primitive Chriſtians, appears to me an irreſiſtible proof of the truth of the Chriſtian Religion. If true happineſs conſiſts in union with the Supreme Being, which it is impoſſible for a moment to doubt, the enjoyment of this happineſs neceſſarily requires, on our part, a certain diſpoſition, founded on ſupreme love to God, and the moſt perfect charity toward our neighbour, ſo that all thoſe who are deſtitute of this diſpoſition, deſtroy their own pretenſions to celeſtial felicity; and wicked men are, from their very nature, neceſſarily excluded from it, it being impoſſible for God himſelf to render them happy. For the Divine Omnipotence extends only to things which are in their nature poſſible, and liberty is ſo eſſential to ſpirits, that no degree of conſtraint can take place with reſpect to them.

It is only by motives, therefore, that ſpirits can be determined to that which is good: now what motives could be propoſed to the apoſtles and other diſciples of Jeſus Chriſt, to embrace a virtuous life, more powerful than the inſtructions of their divine Maſter, his miracles, his ſufferings, his death and reſurrection, of which they were witneſſes. All theſe ſtriking events, united to a doctrine the moſt ſublime,

blime, muſt have excited, in their hearts, the moſt fervent love and the moſt profound veneration for God, whom they could not but conſider and adore as at once their heavenly Father, and the abſolute Lord of the whole univerſe. Theſe lively impreſſions muſt neceſſarily have ſtifled in their breaſts every vicious propenſity, and have confirmed them, more and more, in the practice of virtue.

This ſalutary effect on the minds of the apoſtles, has nothing in it, of itſelf, miraculous, or which encroaches, in the ſmalleſt degree, on their liberty, though the events be ſupernatural. The great requiſite was, ſimply, a heart docile and uncorrupted by vice and paſſion. The miſſion, then, of Jeſus Chriſt into the world, produced, in the minds of the apoſtles, this diſpoſition, ſo neceſſary to the attainment and the enjoyment of ſupreme happineſs; and that miſſion ſtill ſupplies the ſame motives to purſue the ſame end. We have only to read attentively, and without prejudice, the hiſtory of it, and ſeriouſly to meditate on all the events.

I confine myſelf to the ſalutary effects of our Saviour's miſſion, without preſuming to dive into the myſteries of the work of our redemption, which infinitely tranſcend the powers of human underſtanding. I only remark, that theſe effects, of the truth of which we are convinced by experience, could not be produced by illuſion, or human impoſture; they are too ſalutary not to be divine. They are likewiſe perfectly in harmony with the inconteſtable principles which

which we have laid down, that ſpirits can be governed only by motives.

Theologians have maintained, and ſome ſtill maintain, that converſion is the immediate operation of God, without any co-operation on the part of man. They imagine that an act of the Divine will is ſufficient to transform, in an inſtant, the greateſt miſcreant into a virtuous man. Theſe good gentlemen may mean extremely well, and conſider themſelves as thus exalting the divine Omnipotence; but this ſentiment ſeems to me inconſiſtent with the juſtice and goodneſs of God, even though it were not ſubverſive of human liberty. How, it will with reaſon be ſaid, if a ſimple exertion of the divine Omnipotence is ſufficient for the inſtantaneous converſion of every ſinner, can it be poſſible that the decree ſhould not actually paſs, rather than leave ſo many thouſands to periſh, or employ the work of redemption, by which a part only of mankind is ſaved? I acknowledge that this objection appears to me much more formidable than all thoſe which infidelity raiſes againſt our holy religion, and which are founded entirely in ignorance of the true deſtination of man; but, bleſſed be God, it can have no place in the ſyſtem which I have taken the liberty to propoſe.

Some divines will perhaps accuſe me of hereſy, as if I were maintaining that the power of man is ſufficient for his converſion; but this reproach affects me not, as I am conſcious of intending to place the goodneſs of God in it's cleareſt light. In the work

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of converſion, man makes perfect uſe of his liberty, which is unſuſceptible of conſtraint, but man is always determined by motives. Now, theſe motives are ſuggeſted by the circumſtances and conjunctures of his condition. They depend entirely on divine Providence, which regulates all events, conformably to the laws of ſovereign wiſdom. It is God, therefore, who places men every inſtant in circumſtances the moſt favourable, and from which they may derive motives the moſt powerful, to produce their converſion; ſo that men are always indebted to God for the means which promote their ſalvation.

I have already remarked, that however wicked the actions of men may be, they have no power over their conſequences, and that God, when he created the world, arranged the courſe of all events, ſo that every man ſhould be every inſtant placed in circumſtances to him the moſt ſalutary. Happy the man who has wiſdom to turn them to good account!

This conviction muſt operate in us the happieſt effects: unbounded love to God, with a firm reliance on his providence, and the pureſt charity toward our neighbour. This idea of the Supreme Being, as exalted as it is conſolatory, ought to repleniſh our hearts with virtue the moſt ſublime, and effectually prepare us for the enjoyment of life eternal.

28th March, 1761.

5.1.5 Letter 115. The true foundation of human knowledge. Sources of truth, and classes of information derived from it

LETTER CXV.

The true Foundation of human Knowledge. Sources of Truth, and Claſſes of Information derived from it.

HAVING taken the liberty to lay before you my opinion reſpecting the moſt important article of human knowledge, I flatter myſelf it will be ſufficient to diſſipate the doubts which naturally ariſe out of the ſubject, from want of exact ideas of the liberty of ſpirits.

I ſhall now have the honour of ſubmitting to your conſideration the true foundation of all our knowledge, and the means we have of being aſſured of the truth and certainty of what we know. We are very far from being always certain of the truth of all our ſentiments; for we are but too frequently dazzled by appearances, ſometimes exceedingly ſlight, and whoſe falſehood we afterwards diſcover. As we are, therefore, continually in danger of deceiving ourſelves, a reaſonable man is bound to uſe every effort to avoid error, though he may not always be ſo happy as to ſucceed.

The thing to be here chiefly conſidered is the ſolidity of the proofs on which we found our perſuaſion of any truth whatever, and it is abſolutely neceſſary that we ſhould be in a condition to judge if they are ſufficient to convince us or not. For this effect I remark, firſt, that all truths within our reach are

are referable to three claſſes, eſſentially diſtinguiſhed from each other.

The firſt contains the truths of the ſenſes; the ſecond, thoſe of the underſtanding; and the third, thoſe of belief. Each of theſe claſſes requires peculiar proofs of the truths included in it, and in theſe three claſſes all human knowledge is comprehended.

Proofs of the firſt claſs are reducible to the ſenſes, and are thus expreſſed:

This is true, for I ſaw it, or am convinced of it by the evidence of my ſenſes.

It is thus I know that the magnet attracts iron, becauſe I ſee it, and experience furniſhes me with inconteſtable proofs of the fact. Truths of this claſs are called *ſenſible*, becauſe they are founded on the ſenſes, or on experience.

Proofs of the ſecond claſs are founded in ratiocination; thus:

This is true, for I am able to demonſtrate it on principles of juſt reaſoning, or by fair ſyllogiſms.

To this claſs, principally, logic is to be referred, which preſcribes rules for reaſoning conſequentially. It is thus, we know, that the three angles of a rectilineal triangle are together equal to two right angles. In this caſe I do not ſay I ſee it, or that my ſenſes convince me of it; but I am aſſured of it's truth by a proceſs of reaſoning. Truths of this claſs are called *intellectual*, and here we muſt rank all the truths of geometry, and of the other ſciences, in as much as they are ſupported by demonſtration. You muſt be ſenſible, that ſuch truths are wholly different from thoſe of the firſt claſs, in ſupport of which we adduce no other proofs but the ſenſes, or experience, which aſſure us that the fact is ſo, though we may not know the cauſe of it. In the example of the magnet, we do not know how the attraction of iron is a nececeſſary effect of the nature of the magnet, and of iron; but we are not the leſs convinced of the truth of the fact. Truths of the firſt claſs are as certain as thoſe of the ſecond, though the proofs which we have of them are entirely different.

I proceed to the third claſs of truths, that of faith, which we believe, becauſe perſons worthy of credit relate them; or when we ſay:

This is true, for ſeveral creditable perſons have aſſured us of it.

This claſs, accordingly, includes all *hiſtorical truths.* You believe, no doubt, that there was formerly a king of Macedon, called Alexander the Great, who made himſelf maſter of the kingdom of Perſia, though you never ſaw him, and are unable to demonſtrate, geometrically, that ſuch a perſon ever exiſted. But we believe it on the authority of the authors, who have written his hiſtory, and we entertain no doubt of their fidelity. But may it not be poſſible that theſe authors have concerted to deceive us? We have every reaſon to reject ſuch an inſinuation, and we are as much convinced of the truth of theſe facts, at leaſt of a great part of them, as of truths of the firſt and ſecond claſſes.

The proofs of theſe three claſſes of truths are extremely different; but if they are ſolid, each in it's kind,

kind, they muſt equally produce conviction. You cannot poſſibly doubt that Ruſſians and Auſtrians have been at Berlin, though you did not ſee them: this, then, is to you a truth of the third claſs, as you believe it on the report of others; but to me it is one of the firſt claſs, becauſe I ſaw them, and converſed with them, and as many others were aſſured of their preſence by means of other ſenſes. You have, nevertheleſs, as complete conviction of the fact as we have.

31ſt March, 1761.

END OF THE FIRST VOLUME.

T. Gillet, Printer, Saliſbury-ſquare.

5.2 Systems of monads and pre-established harmony

At the core of Gottfried Leibniz's system of thought are the pre-established harmony and the monadic doctrine. Since Leibniz's monads are windowless, they cannot interact with their surroundings. The pre-established harmony deals with the causal relations between mind and body. This contributed to a debate over causation in the seventeenth century. According to the pre-established harmony, every non-miraculous state of a substance is programmed at creation. Leibniz likened this to two synchronized clocks between mind and body running together for all eternity, but with total independence of the other. Although God could build a perfect machine, Euler found innate ideas as the source of knowledge absurd. Observational confirmation for the influence of the body upon the soul and mind did not exist. Euler appealed to psychotheology to try to refute the pre-established harmony. He called the thought that the soul acts on even the smallest particle of matter pretense. He saw an infinite difference between the soul and body, and the regulations between the soul and body are not a cause but are voluntary. Another objection was that all actions since Creation must be in conformity to it. Thus, the pre-established harmony was 'utterly destructive to human liberty', letter 94.

5.2.1 Letter 76. System of monads of Wolff

LETTER LXXVI.

Syſtem of the Monads of Wolff.

BEFORE I attempt to make you ſenſible of the truth of the principle, that all bodies, of themſelves, always preſerve the ſame ſtate of reſt, or motion, I muſt remark, that if we conſult experience only on the ſubject, without thoroughly inveſtigating it by the powers of reaſoning, we would be diſpoſed to draw the directly oppoſite concluſion, and to maintain, That bodies always have a propenſity to be continually changing their ſtate; as we ſee nothing in the whole univerſe, but a perpetual change in the ſtate of bodies. But we have juſt ſhewn what are the cauſes which produce theſe changes, and we are aſſured, that they are not to be found in the bodies whoſe ſtate is changed, but out of them.

The principle, then, which we have eſtabliſhed, is ſo far from being contradicted by experience, that it is, on the contrary, confirmed by it. You will eaſily judge from this, how ſeveral great philoſophers, miſled by an experience not accurately underſtood, have fallen into the error of maintaining, That all bodies are endowed with powers, diſpoſing them continually to change their ſtate.

It is thus that *Wolff* has reaſoned. He ſays: 1. Experience ſhews us all bodies perpetually changing their ſtate; 2. Whatever is capable of changing the ſtate of bodies, is called force; 3. All bodies, therefore,

fore, are endowed with a force capable of changing their ſtate; 4. Every body, therefore, is making a continual effort to change; 5. Now, this force belongs to body, only ſo far as it contains matter; 6. It is, therefore, a property of matter to be continually changing it's own ſtate; 7. Matter is a compound of a multitude of parts, denominated the elements of matter; therefore, 8. As the compound can have nothing but what is founded in the nature of it's elements, every elementary part muſt be endowed with the power of changing it's own ſtate.

Theſe elements are ſimple beings; for if they were compoſed of parts, they would be no longer elements, but their parts would be ſo. Now, a ſimple being is likewiſe denominated *monad*; every monad, therefore, has the power of continually changing it's ſtate. Such is the foundation of the ſyſtem of monads, which you may have heard mentioned, though it does not now make ſuch a noiſe as it formerly did. I have marked by figures the ſeveral propoſitions on which it is eſtabliſhed, for the purpoſe of making a more diſtinct reference, in the reflections I mean to make upon them.

I have nothing to ſay reſpecting the firſt and ſecond; but the third is very equivocal, and altogether falſe, in the ſenſe in which it is taken. Without meaning to ſay, that the forces which change the ſtate of bodies, proceed from ſome ſpirit, I readily agree, that the force, by which the ſtate of every body is changed, ſubſiſts in body, but, it being always underſtood, that it ſubſiſts in another body, and never

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in that which undergoes the change of ſtate; which has rather the contrary quality, that of perſevering in the ſame ſtate. In ſo far, then, as theſe forces ſubſiſt in bodies, it ought to be ſaid, that theſe bodies, as long as they have certain connections with each other, may be capable of ſupplying forces by which the ſtate of another body is changed. It follows, that the fourth propoſition muſt be abſolutely falſe; and the reſult, from all that went before, rather is, that every body is endowed with the power of remaining in the ſame ſtate, which is directly the oppoſite of the concluſion which theſe philoſophers have drawn..

And I muſt here remark, that it is rather abſurd to give the name of *force* to that quality of bodies by which they remain in their ſtate; for if we are to underſtand by the term *force* every thing that is capable of changing the ſtate of bodies, the quality by which they perſevere in their ſtate, is rather the oppoſite of a force. It is, therefore, by an abuſe of language, that certain authors give the name of force to the *inertia*, which is that quality, and which they denominate the *inert force*.

But, not to wrangle about terms, though this abuſe may lead to very groſs errors, I return to the ſyſtem of monads: and as propoſition 4, is falſe, thoſe that follow, which are ſucceſſively founded upon it, muſt, of neceſſity, be ſo too. It is falſe, then, likewiſe, that the elements of matter, or monads, if ſuch there be, are poſſeſſed of the power of changing their ſtate. The truth is rather to be founded in the oppoſite quality,

quality, that of perfevering in the fame ftate; and thereby the whole fyftem of monads is completely fubverted.

Thefe philofophers attempted to reduce the elements of matter to the clafs of *beings*, which comprehends fpirits and fouls, endowed, beyond the power of contradiction, with the faculty of changing their ftate; for, while I am writing, my foul continually reprefents other objects to itfelf, and thefe changes depend entirely on my will: I am thoroughly convinced of it, and not the lefs fo, that I am mafter of my own thoughts; whereas the changes which take place in bodies, are the effect of an extraneous force.

Add to this, the infinite difference between the ftate of body, capable only of one velocity and of one direction, and the thoughts of fpirit, and you will be entirely convinced of the falfehood of the fentiments of the materialifts, who pretend that fpirit is only a modification of matter. Thefe gentlemen have no knowledge of the real nature of bodies.

15th November, 1760.

5.2.2 Letter 125. Of monads

LETTER X.

Of Monads.

WHEN we talk in company on philosophical subjects, the conversation usually turns on such articles as have excited violent disputes among philosophers.

The divisibility of body is one of them, respecting which the sentiments of the learned are greatly divided. Some maintain that this divisibility goes on to infinity, without the possibility of ever arriving at particles so small as to be susceptible of no further division. But others insist that this division extends only to a certain point, and that you may come at length to particles so minute that, having no magnitude, they are no longer divisible. These ultimate particles, which enter into the composition of bodies, they denominate *simple beings* and *monads*.

There was a time when the dispute respecting monads employed such general attention, and was conducted with so much warmth, that it forced its way into company of every description, that of the guard-room not excepted. There was scarcely a lady at court who did not take a decided part in favour of monads or against them. In a word, all conversation was engrossed by monads—no other subject could find admission.

The Royal Academy of Berlin took up the controversy, and being accustomed annually to propose a question for discussion, and to bestow a gold medal, of the value of fifty ducats, on the person who, in the judgment of the Academy, has given the most ingenious solution, the question respecting monads was selected for the year 1748. A great variety of essays on the subject were accordingly produced. The president, *Mr. de Maupertuis*, named a committee to examine them, under the direction of the late *Count Dohna*, great chamberlain to the queen; who, being an impartial judge, examined with all imaginable attention the arguments adduced both for and against the existence of monads. Upon the whole, it was found that those which went to the establishment of their existence were so feeble and so chimerical, that they tended to the subversion of all the principles of human knowledge. The question was therefore determined in favour of the opposite opinion, and the prize adjudged to *Mr. Justi*, whose piece was deemed the most complete refutation of the monadists.

You may easily imagine how violently this decision of the Academy must have irritated the partisans of monads, at the head of whom stood the celebrated *Mr. Wolff*. His followers, who were then much more numerous and more formidable than at present, exclaimed in high terms against the partiality and injustice of the Academy; and their chief had wellnigh proceeded to launch the thunder of a philosophical anathema against it. I do not now recollect to whom we are indebted for the care of averting this disaster.

As this controversy has made a great deal of noise, you will not be displeased, undoubtedly, if I dwell a little upon it. The whole is reduced to this simple question, Is body divisible to infinity? or, in other words, Has the divisibility of bodies any bound, or has it not? I have already remarked as to this, that extension, geometrically considered, is on all hands allowed to be divisible in infinitum; because however small a magnitude may be, it is possible to conceive the half of it, and again the half of that half, and so on to infinity.

This notion of extension is very abstract, as are those of all *genera*, such as that of man, of horse, of tree, &c., as far as they are not applied to an individual and determinate being. Again, it is the most certain principle of all our knowledge, that whatever can be truly affirmed of the genus must be true of all the individuals comprehended under it. If therefore all bodies are extended, all the properties belonging to extension must belong to each body in particular. Now all bodies are extended, and extension is divisible to infinity; therefore every body must be so likewise. This is a syllogism of the best form; and as the first proposition is indubitable, all that remains is to be assured that the second is true, that is, whether it be true or not that bodies are extended.

The partisans of monads, in maintaining their opinion, are obliged to affirm that bodies are not extended, but have only an appearance of extension. They imagine that by this they have subverted the argument adduced in support of the divisibility in infinitum. But if body is not extended, I should be glad to know from whence we derived the idea of extension; for if body is not extended, nothing in the world is, as spirits are still less so. Our idea of extension, therefore, would be altogether imaginary and chimerical.

Geometry would accordingly be a speculation entirely useless and illusory, and never could admit of any application to things really existing. In effect, if no one thing is extended, to what purpose investigate the properties of extension? But as geometry

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is beyond contradiction one of the most useful of the sciences, its object cannot possibly be a mere chimera.

There is a necessity then of admitting, that the object of geometry is at least the same apparent extension which those philosophers allow to body; but this very object is divisible to infinity: therefore existing beings endowed with this apparent extension must necessarily be extended.

Finally, let those philosophers turn themselves which way soever they will in support of their monads, or those ultimate and minute particles divested of all magnitude, of which, according to them, all bodies are composed, they still plunge into difficulties, out of which they cannot extricate themselves. They are right in saying that it is a proof of dulness to be incapable of relishing their sublime doctrine; it may however be remarked, that here the greatest stupidity is the most successful.

5th May, 1761.

5.2.3 Letter 83. Examination of the system of pre-established harmony. An objection to it

LETTER LXXXIII.

Examination of the Syſtem of pre-eſtabliſhed Harmony. An Objection to it.

THERE was a time, when the ſyſtem of pre-eſtabliſhed harmony had acquired ſuch a high reputation over all Germany, that to dare to call it

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in queſtion was to incur the imputation of ignorance, or bigotry. The ſupporters of this ſyſtem boaſted, that, by means of it, the omnipotence and omniſcience of the Supreme Being were ſet in their cleareſt light, and that it was impoſſible for any one, who believed in theſe exalted perfections of God, to entertain a doubt of the truth of this ſublime ſyſtem.

In fact, ſay they, we ſee, that poor, pitiful mortals, are capable of conſtructing machines ſo ingeniouſly, as to fill the vulgar ſpectator with aſtoniſhment: how much ſtronger reaſon, then, have we to admit, that God having known, from all eternity, all that my ſoul would wiſh and deſire, at every inſtant, ſhould have been able to conſtruct ſuch a machine, which, at every inſtant, ſhould produce motions conformable to the determinations of my ſoul? Now, this machine is preciſely my body, which is united to my ſoul, only by this harmony; ſo that if the organization of my body were deranged to ſuch a degree, as to be no longer in harmony with my ſoul, this body would no more belong to me, than the body of a rhinoceros in the heart of Africa: and if, in the caſe of a derangement of my body, God ſhould adjuſt that of a rhinoceros, ſo that it's motions were in ſuch harmony with the determinations of my ſoul, as to raiſe it's paw at the moment I willed it; this body would then be mine, and would belong to my ſoul, as my preſent body now belongs to it, without having undergone itſelf, on that account, any change whatever.

Mr. *Leibnitz* himſelf has compared the ſoul and the body

body to two clocks, which continually indicate the ſame hour. A clown who ſhould ſee this beautiful harmony of theſe two clocks, would undoubtedly conclude, that they acted upon each other, but he would be under a miſtake, for the one performs it's motions independently of the other. The ſoul and the body are likewiſe two machines totally independent, the one being ſpiritual, the other material; but their operations are always in a harmony ſo complete, that we are induced to believe them to belong to each other, and that the one has a real influence upon the other, which is, however, a mere illuſion.

In order to form a judgment of this ſyſtem, I remark, firſt, That it cannot be denied to be poſſible for God to create a machine which ſhould be always in harmony with the operations of my ſoul; but it appears to me that my body belongs to me by other rights than ſuch a harmony, however beautiful it may be: and, I believe, you will not be diſpoſed haſtily to adopt a ſyſtem which is founded on this principle alone, that no ſpirit can act upon a body; and that, reciprocally, a body cannot act upon, or ſupply ideas to, a ſpirit. This principle is, beſides, deſtitute of all proof, the chimeras of it's partiſans, reſpecting ſimple beings, having been completely refuted. And if God, who is a ſpirit, has the power of acting upon bodies, it is not abſolutely impoſſible that a ſpirit, ſuch as the human ſoul, ſhould be able likewiſe to act upon a body. Accordingly, we do not pretend to ſay, that our ſoul acts upon all bodies, but only upon a ſmall particle of matter, with

reſpect

respect to which it has received the power of God himself, though to exercise it in a manner which we are utterly unable to comprehend.

Farther, the system of pre-established harmony labours under other great difficulties. According to it the soul derives all it's knowledge from it's own proper fund, without any contribution on the part of the body and the senses. Thus, when I read in the Gazette that the Pope is dead, and I come to the knowledge of the Pope's death, the Gazette and my reading have nothing to do with the communication of this knowledge, as these circumstances respect only my body and my senses, which have no manner of connection with my soul. But, conformably to this system, my soul derives, at the same time, from it's own proper fund, the ideas which it has of this same Pope. It concludes, he must absolutely be dead, and this knowledge comes to it with the reading of the Gazette, so that I imagine the reading of the Gazette furnished me with this knowledge, though I really derived it from the proper fund of my soul.

But this idea is perfectly absurd. How was it possible for me so boldly to assert, that the Pope must necessarily have died at the moment mentioned in the Gazette, and that, only from the idea which I had of the Pope's condition and health, though, perhaps, I knew nothing about him, while I am infinitely better acquainted with my own situation, without knowing, however, what shall befall me tomorrow.

In like manner when you do me the honour to read

read theſe letters, and derive the knowledge of ſome truth from them, it is your ſoul which extracts that truth from it's own proper fund, without my contributing at all to it by my letters. The reading of them ſerves only to maintain the harmony which the Creator meant to eſtabliſh between the ſoul and the body. It is only a formality, altogether ſuperfluous, with reſpect to the knowledge itſelf. I ſhall, neverthelеſs, continue to tender you my inſtructions.

9th December, 1760.

LETTER LXXXIV.

Another Objection.

THERE is another objection to be made to the ſyſtem of pre-eſtabliſhed harmony; namely, that it is utterly deſtructive of human liberty. In fact, if the bodies of men are machines, ſimilar to a watch, all their actions are a neceſſary conſequence of their conſtruction. Thus, when a thief ſteals my purſe, the motion made by his hands is an effect as neceſſary of the machine of his body, as the motion of the hand of my clock, now pointing to nine. You will readily comprehend what muſt be the concluſion. As it would be unjuſt, nay, ridiculous, to think of being angry at the clock, and of chaſtiſing it, becauſe it pointed to nine, it would be equally ſo, with reſpect to the thief, whom it would be abſurd to puniſh for having ſtolen my purſe.

5.3 Calculation of longitude and latitude

Letters 155–68 from vol III are devoted to the astronomical solution to the practical problem of navigation in an open sea. Euler made many contributions to the Moon theory: explanation of the lunar motion irregularity and calculation of lunar tables. He started a prize competition in St. Petersburg Imperial Academy of Sciences to explore whether the motion of the Moon agrees with the Newtonian theory of gravitation. He invited two outstanding French scientists, A C Clairaut (1713–65) and J L d'Alembert (1717–83), to take part in the competition in 1753. Clairaut won the prize for his lunar theory development[2].

Understanding of lunar motion was of prime commercial interest in the eighteenth century. To emphasize its importance, the British Parliament promised in its 1714 Longitude Act a prize of 20 000 Sterling for the first scholar who could determine the geographical longitude of a ship at open sea with the accuracy of half-a-degree, which corresponds to about 50 km close to the Equator. In an open ocean the only reference points are the stars in the sky, as well as the Moon and the Sun.

At that time, it was clear that a practical solution to this problem required accurate lunar tables. The tables would tell exactly where the Moon is in the sky relative to the fixed stars at a given moment of universal time (Greenwich Mean Time). 'Reading the time by the Moon' was a simple matter of comparing it to the local time and calculating the longitude based on the difference.

Euler was awarded 300 pounds Sterling in 1763 from the Board of Longitude for the tables, which were based on his lunar theory. Afterward, the French government gave him an award. The tables were actually calculated and sent to Greenwich by German astronomer Tobias Mayer (1723–62). Mayer's widow was awarded 3000 pounds from the British Parliament fund. Euler's method of longitude determination in open sea was used by navigators even after John Harrison's invention (1765) of a marine chronometer. Later, the effective production of naval chronometers began worldwide. John Harrison (1693–1776), an English clockmaker, invented the first naval chronometer. He received half of the major prize of 20 000 Sterling for developing a naval chronometer which gave longitude better than the lunar method, and which was successfully tested the year before. The marine chronometer was widely adopted and used. However, Mayer's lunar tables based on Euler's method and his equations of lunar motion remained in British nautical almanacks and aided sea travels and navigators for more than a century[3].

From 1764 on, Euler had prepared his 775-page work *Theoria Motuum Lunae, Nova Methodo Pertractata* (The theory of lunar motion, treated by means of a new method), published in 1772, which is generally taken to constitute his second lunar theory. His first theory (see above) was in the late 1740s as *Theoria Motus Lunae…* (Theory of the Moon…) Euler produced his own lunar tables, which were better than the semi-empirical ones of Mayer and Clairaut and clearer and easier to use.

Thus, while Newton had created the geometrical form of celestial mechanics, Euler founded its analytical form through his research in astronomy on his first theory of 1740 and especially in his second *Theoria Motuum Lunae…* of 1772. Euler's lunar theories were the most advanced of the time, and even today this part of the manuscript commentary remains a popular reference for modern astronomers.

[2] Bodenmann S 2010 The 18th century battle over lunar motion *Physics Today* (Jan), pp 27–32.
[3] Howse D 1980 *Greenwich Time and the Discovery of the Longitude* (Oxford: Oxford University Press).

5.3.1 Letter 160. Method of determining the latitude, or the elevation of the pole

LETTER XLV.

Method of determining the Latitude, or the Elevation of the Pole.

IT being a matter of such importance to know the latitude and longitude of every place, in order to ascertain exactly the spot of the globe where you are, you must be sensible that it is equally important to discover the means of certainly arriving at such knowledge.

Nothing can be more interesting to a man who has been long at sea, or after a tedious journey through unknown regions, than to be informed at what precise spot he is arrived; whether or not he is near some known country, and what course he ought to pursue in order to reach it. The only means of relieving such a person from his anxiety would undoubtedly be to give him the latitude and longitude of the place where he is; but what must he do to attain this most important information? Let us suppose him on the ocean, or in a vast desert, where there is no one whom he can consult. After having ascertained, by the help of a terrestrial globe, or of maps, the latitude and longitude of the place where he is, he will with ease from them determine his present position, and be furnished with the necessary information respecting his future progress.

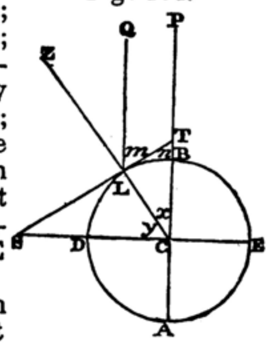
I proceed therefore to inform you that it is by astronomy chiefly we are enabled to determine the latitude and longitude of the place where we are; and that I may not tire you by a tedious detail of all the methods which astronomers have employed for this important purpose, I shall satisfy myself with presenting a general idea of them, trusting that this will be sufficient to convey to you the knowledge of the principles on which every method is founded.

I begin with the latitude, which is involved in scarcely any difficulty; whereas the determination of the longitude seems hitherto to have defied all human research, especially at sea, where the utmost precision is requisite. For the discovery of this last, accordingly, very considerable prizes have been proposed, as an encouragement to the learned to direct their talents and their industry towards a discovery so interesting, both from its own importance and from the honour and emolument which are to be the fruit of it.

I return to the latitude, and the means of ascertaining it, referring to some future opportunity a more ample discussion of the longitude, and of the different methods of discovering it, especially at sea

Let the points B and A, *Fig.* 104, be the poles of the earth; B A its axis, and B its centre; let the semicircle B D A represent a meridian, intersected by the equator at the point D; and B D, A D, will be each the quadrant of a circle, or an arch of 90 degrees; the straight line D C will therefore be a radius of the equator, and D E its diameter.

Fig. 104.



Let there now be assumed in this meridian B D A the point L, the given place of which the latitude is required; or, in other words, the number of degrees contained in the arch L D, which measures the distance of the point L from the equator; or again, drawing the radius C L, as the arch L D measures the angle D C L, which I shall call y, this angle y will express the latitude of the place L, which we want to find.

Now, it being impossible to place ourselves at the centre of the earth, from which we could take the measure of that angle, we must have recourse to the heavens. There the prolongation of the axis of the earth A B terminates in the north pole of the heavens P, which we are to consider as at an immense distance from the earth. Let the radius C L likewise be carried forward till it terminate in the heavens at the point Z, which is called the zenith of the place; then, drawing through the point L the straight line S T, perpendicular to the radius C L, you will recollect that this line S T is a tangent of the circle, and that consequently it will be horizontal to the place

L; our horizon always touching the surface of the earth at the place where we are.

Let us now look from L towards the pole of the heavens P, which being infinitely distant, the straight line L Q directed to it will be parallel to the line A B P, that is, to the axis of the earth: this pole of the heavens will appear, therefore, between the zenith and the horizon L T; and the angle T L Q, indicated by the letter *m*, will show how much the straight line L Q, in the direction of the pole, is elevated above the horizon; hence this angle *m* is denominated the *elevation of the pole*.

You have undoubtedly heard frequent mention made of the elevation of the pole, or, as some call it, the *height of the pole;* which is nothing else but the angle formed by the straight line L Q in the direction of the pole and the horizon of the place where we are. You have a perfect comprehension of the possibility of measuring this angle *m*, by means of an astronomical instrument, without my going into any further detail.

Having measured this angle *m*, or the height of the pole, it will give you precisely the latitude of the place L, that is, the angle *y*. To make this appear, it is only necessary to demonstrate that the two angles *m* and *y* are equal.

Now the line L Q being parallel to C P, the angles *m* and *n* are alternate, and consequently equal. And the line L T being perpendicular to the radius C L, the angle C L T of the triangle C L T must be a right angle, and the other two angles of that triangle, *n* and *x*, must be together equal to a right angle. But the arch B D being the quadrant of a circle, the angle B C D must likewise be a right angle; the two angles *x* and *y*, therefore, are together equal to the two angles *n* and *x*. Take away the angle *x* from both, and there will remain the angle *y* equal to the angle *n*; but the angle *n* has been proved equal to the angle m, therefore the angle y is likewise equal to the angle m.

It has already been remarked that the angle y expresses the latitude of the place L, and the angle m the elevation or height of the pole at the same place L; the latitude of any place, therefore, is always equal to the height of the pole at that same place. The means which astronomy supplies for observing the height of the pole indicate therefore the latitude required.

Astronomical observations made at Berlin have accordingly informed us that there the height of the pole is 52° 32′, and hence we conclude that the latitude of that city is likewise 52° 32′.

This is one very remarkable instance to demonstrate how the heavens may assist us in the attainment of the knowledge of objects which relate only to the earth.

5th September, 1761.

5.3.2 Letter 167. The motion of Moon a fifth method

LETTER LII.

The Motion of the Moon, a fifth Method.

The heavens furnish us with one resource more for discovering the longitude without the assistance of telescopes, in which astronomers seem to place the greatest confidence. It is the moon, not only when eclipsed but at all times, provided she be visible; an unspeakable advantage considering that eclipses are so rare, and that the immersions and emersions of the satellites of Jupiter are of such difficult observation; there being a considerable time every year during which the planet Jupiter is not visible to us, whereas the moon is almost constantly in view.

You must undoubtedly have already remarked, that the moon rises every day almost three-quarters of an hour later than the preceding, not being attached to one fixed place relatively to the stars, which always preserve the same situation with respect to each other, though they have the appearance of being carried round by the heavens, to accomplish every day their revolution about the earth. I speak here according to appearances; for it is the earth which revolves every day round its axis, while the heavens and the fixed stars remain at rest; while the sun and planets are continually changing their place relatively to these. The moon has likewise a motion abundantly rapid from one day to another, with relation to the fixed stars.

If you were to see the moon to-day near a certain fixed star, it will appear to-morrow at the same hour at a considerable distance from it towards the east; and the distance sometimes exceeds even 15 degrees. The velocity of her motion is not always the same, yet we are able to determine it very exactly for every day; by which means we can calculate before-hand her true place in the heavens for every hour of the day, and for any known meridian, say that of Berlin, or Paris.

Suppose, then, that after a long voyage I find myself at sea, in a place altogether unknown, what use can I make of the moon, in order to discover the longitude of the place where I am? There is no difficulty with respect to the latitude, even at sea, where there are means abundantly certain for ascertaining the height of the pole, to which the latitude is always equal. My whole attention, then, will be directed to the moon; I will compare her with the fixed stars which are nearest, and thence calculate her true place relatively to them. You know there are celestial globes on which all the fixed stars are arranged, and that celestial charts are likewise constructed similar to geographical maps, on which are represented the fixed stars which appear in a certain quarter of the heavens. On taking, then, a celestial chart on which the fixed stars to which the moon is near are marked, it will be an easy matter to determine the true place where the moon at that time is; and my watch, which I have taken care to regulate there, from an observation of the moment of noon, will indicate to me the time of my lunar observation. Then, from my knowledge of the moon's motion, I calculate for Berlin, at what hour she must appear in the same place where I have seen her. If the time observed exactly correspond with the time of Berlin, it will be a demonstration that the place where I am is precisely under the meridian of Berlin, and that consequently the longitude is the same. But if the time of my observation is not that of Berlin, the difference will give that which is between the meridians; and reckoning 15 degrees for every hour of time, I compute how much the longitude of the place I am at is greater or less than that of Berlin: the place where time is more advanced has always the greater longitude.

This is an abstract of the manner of determining longitude by simple observations of the moon. I remark, that the happiest moments for successfully performing this operation, and for accurately determining the moon's place, are, when a fixed star happens to be concealed behind her body; this is called *occultation*, and there are two instances favourable to observation, that when the moon in her motion completely covers the star, and that when the star reappears. Astronomers are particularly attentive to catch these instants of occultation, in order to calculate from them the moon's true place.

I foresee, however, an objection you will probably make respecting the time-piece with which I suppose our navigator provided, after having maintained the impossibility of constructing one that shall be proof against every agitation of a ship at sea But this impossibility respects only such time-pieces as are expected to preserve a regular motion for a long time together, without the necessity of frequent adjustment; for as to the observations in question, a common watch is quite sufficient, provided it go regularly for some hours, after having been carefully adjusted to the noon of the place where we are; supposing a doubt to arise, whether we could calculate from it the succeeding evening or night, at the time we observe the moon, the stars likewise will afford the means of a new and accurate adjustment. For as the situation of the sun with relation to the fixed stars is perfectly known for any time whatever, the simple observation of any one star is sufficient to determine the place where the sun must then be; from which we are enabled to calculate the hour that a well regulated timepiece ought to indicate. Thus, at the very instant of making an observation by the moon, we are enabled likewise to regulate our timepiece

5.3.3 Letter 168. Advantages of this last Method: its degree of precision

LETTER LIII.

Advantages of this last Method; its Degree of Precision.

THIS last method of finding the longitude, founded on lunar observations, seems to merit the preference, as the others are subjected to too many difficulties, or the opportunities of employing them occur too seldom to be useful. And you must be abundantly sensible that success depends entirely on the degree of precision attained in forming the calculation, and that the errors which may be committed would lead to conclusions on which we could place no dependence. It is of importance, therefore, to explain what degree of precision we may reasonably hope to attain in reducing this method to practice, founded on the considerable change which the moon undergoes from one day to another in her position. It may be affirmed, that if the moon's motion were more rapid, it would be more adapted to the discovery of the longitude, and would procure for us a higher degree of precision. But if, on the contrary, it were much slower, so that we could scarcely discern any change of her position from day to day, we could derive very little, if any, assistance from her towards the discovery of the longitude.

Let us suppose, then, that the moon changes her place among the fixed stars a space of 12 degrees in twenty-four hours; she will, in that case, change it one degree in two hours, and half a degree, or thirty minutes in an hour: if we were to commit a mistake in observing the moon's place of thirty minutes, it would be the same thing as if we observed the moon an hour earlier or later, and we should commit a mistake of one hour in the conclusion respecting the difference of the meridians. Now, one hour's difference in the meridians corresponds to 15 degrees in their longitude; consequently, we should be mistaken 15 degrees in the longitude itself of the place we look for; which would undoubtedly be an error so enormous that it were almost as well to know nothing about it; and a simple computation of the distance and the direction, however uncertain, could not possibly lead to a mistake so very gross. But a man must have gone to work in a very slovenly manner to commit a mistake of 30 minutes respecting the moon's place; and the instruments which he employed must have been very bad, a thing not to be supposed.

Nevertheless, however excellent the instruments may be, and whatever degree of attention may have been bestowed, it is impossible to keep clear of all error; and he must have acquitted himself very well indeed who has not committed the mistake of one minute in determining the moon's place. Now, as it changes half a degree, or 30 minutes, in one hour, it will change one minute of distance in two minutes of time. When, therefore, the mistake of the moon's place amounts to no more than one minute, the mistake in the difference of meridians will amount to two minutes of time. And one hour, or 60 minutes, being equivalent to 15 degrees of longitude, there will result from it an error of half a degree in the longitude; and this point of precision might be sufficient for every purpose, were it but attainable.

I have hitherto supposed our knowledge of the moon's motion to be so perfect, that, for a known meridian, we could determine the moon's true place for every moment without an error; but we are still very far short of that point of perfection. Within these twenty years, the error in this calculation was more than six minutes; and it is but lately that the ingenious *Professor Mayer* of Gottingen, pursuing the track I had pointed out to him, has succeeded so far as to reduce this error to less than a minute. It may very easily happen, then, that in the calculation likewise, the error of one minute may be committed, which, added to that of a minute committed in the observation of the moon's place, will double that which results from it respecting the longitude of the place where we are; and, consequently, it may possibly amount to a whole degree: it is proper further to remark, that if the moon in twenty-four hours should change her relative situation more than 12 degrees, the error in the longitude would be less considerable. The means may perhaps be discovered of diminishing still further the errors into which we are liable to fall, in the observation and in the calculation; and then we should be able to ascertain the longitude to a degree, or less. Nay, we ought not to despair of attaining a still higher degree of precision. We have only to make several observations, which can be easily done by remaining several days together at the same place. It is not to be apprehended, in that case, that all the conclusions should be equally defective; some will give the longitude sought too great, others too small, and by striking a medium between all the results, we may rest assured that this longitude will not be one degree removed from the truth.

The English nation, generously disposed to engage genius and ability in this important research, has proposed three prizes for ascertaining the longitude —one of 10,000*l.*, one of 15,000*l.*, and one of 20,000*l.* The first of these is to be bestowed on the person who shall determine the longitude to a degree, or about it, so as to give perfect assurance that the error shall not exceed one degree at most. The second is to be given to him who shall discover a method still more exact, so that the error shall never exceed two-thirds of a degree, or 40 minutes. The highest prize is destined to the man who shall ascertain the longitude so exactly that the error shall never exceed half a degree, or 30 minutes; and a higher degree of precision is hardly to be expected. No one of these prizes has hitherto been allotted: I do not take into the account the gratification bestowed on the artist who pretended to it from his construction of perfect timepieces. *Mr. Mayer* is at this moment cláiming the highest, and I think he is entitled to it.*

3d October, 1761.

5.4 Electricity and magnetism

Euler employs his concept of the properties of ether to explain electric and magnetic phenomena. He highlights the common nature of electricity and magnetism, even though based on the ether theory. The modern reader will find a huge gap that separates Euler's mathematically precise mechanics with the almost 'colloquial' physics of electromagnetism. The century between Euler's *Letters* and Maxwell's work on electromagnetism was a period of colossal progress in terms of perceptions of the scientific community of physics' capacity as an area of knowledge and what it is able to do.

5.4.1 Letter 139. The true principle of nature on which are founded all the phenomena of electricity (from volume 2)

LETTER XXIV.

The true Principle of Nature on which are founded all the Phenomena of Electricity.

The summary I have exhibited of the principal phenomena of electricity has no doubt excited a curiosity to know what occult powers of nature are capable of producing effects so surprising.

The greatest part of natural philosophers acknowledge their ignorance in this respect. They appear to be so dazzled by the endless variety of phenomena which every day present themselves, and by the singularly marvellous circumstances which accompany these phenomena, that they are discouraged from attempting an investigation of the true cause of them. They readily admit the existence of a subtile matter, which is the primary agent in the production of the phenomena, and which they denominate the electric fluid; but they are so embarrassed about determining its nature and properties, that this important branch of physics is rendered only more perplexed by their researches.

There is no room to doubt that we must look for the source of all the phenomena of electricity only in a certain fluid and subtile matter; but we have no need to go to the regions of imagination in quest of it. That subtile matter denominated *ether*, whose reality I have already endeavoured to demonstrate,* is sufficient very naturally to explain all the surprising effects which electricity presents. I hope I shall be able to set this in so clear a light, that you shall be able to account for every electrical phenomenon, however strange an appearance it may assume.

The great requisite is to have a thorough knowledge of the nature of ether. The air which we breathe rises only to a certain height above the surface of the earth; the higher you ascend the more subtile it becomes, and at last it entirely ceases. We must not affirm that beyond the region of the air there is a perfect vacuum which occupies the immense space in which the heavenly bodies revolve. The rays of light, which are diffused in all directions from these heavenly bodies, sufficiently demonstrate that those vast spaces are filled with a subtile matter.

If the rays of light are emanations forcibly projected from luminous bodies, as some philosophers have maintained, it must follow that the whole space of the heavens is filled with these rays—nay, that they move through it with incredible rapidity. You have only to recollect the prodigious velocity with which the rays of the sun are transmitted to us. On this hypothesis, not only would there be no

* See Letter XV. vol. I.

vacuum, but all space would be filled with a subtile matter, and that in a state of constant and most dreadful agitation.

But I think I have clearly proved that rays of light are no more emanations projected from luminous bodies than sound is from sonorous bodies. It is much more certain that rays of light are nothing else but a tremulous motion or agitation of a subtile matter, just as sound consists of a similar agitation excited in the air. And as sound is produced and transmitted by the air, light is produced and transmitted by that matter, incomparably more subtile, denominated ether, which consequently fills the immense space between the heavenly bodies.

Ether, then, is a medium proper for the transmission of rays of light: and this same quality puts us in a condition to extend our knowledge of its nature and properties. We have only to reflect on the properties of air, which render it adapted to the reception and transmission of sound. The principal cause is its elasticity or spring. You know that air has a power of expanding itself in all directions, and that it does expand the instant that obstacles are removed. The air is never at rest but when its elasticity is everywhere the same; whenever it is greater in one place than another the air immediately expands. We likewise discover by experiment that the more the air is compressed, the more its elasticity increases: hence the force of air-guns, in which the air, being very strongly compressed, is capable of discharging the ball with astonishing velocity. The contrary takes place when the air is rarefied: its elasticity becomes less in proportion as it is more rarefied, or diffused over a larger space.

On the elasticity of the air, then, relative to its density, depends the velocity of sound, which makes a progress of 1142 feet in a second. If the elasticity of the air were increased, its density remaining the same, the velocity of sound would increase; and the same thing would take place if the air were more rare or less dense than it is, its elasticity being the same. In general, the more that any medium, similar to air, is elastic, and at the same time less dense, the more rapidly will the agitations excited in it be transmitted. And as light is transmitted so many thousand times more rapidly than sound, it must clearly follow that the ether, that medium whose agitations constitute light, is many thousand times more elastic than air, and, at the same time, many thousand times more rare or more subtile, both of these qualities contributing to accelerate the propagation of light.

Such are the reasons which lead us to conclude that ether is many thousand times more elastic and more subtile than air; its nature being in other respects similar to that of air, in as much as it is likewise a fluid matter, and susceptible of compression and of rarefaction. It is this quality which will conduct us to the explanation of all the phenomena of electricity.

23*d* *June*, 1761.

5.4.2 Letter 176. True magnetic direction; subtile matter which produces the magnetic power

5.4.3 Letter 177. Nature of magnetic matter, and its rapid current. Magnetic canals

LETTER LXI.

True Magnetic Direction; subtile Matter which produces the Magnetic Power.*

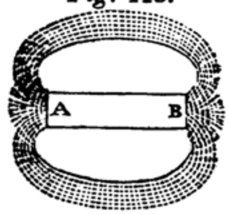
In order to form a just idea of the effect of the earth's magnetic power, we must attend at once to the declination and inclination of the magnetic needle, at every place of the globe. At Berlin, we know the declination is 15° west, and the inclination of the northern extremity 72°. On considering this double effect, the declination and inclination, we shall have the true magnetic direction for Berlin. We draw first, on a horizontal plane, a line which shall make with the meridian an angle of 15° west, and thence descending towards the vertical line, we

* See Note on Letter LVI.

trace a new line, which shall make with it an angle of 72°; and this will give us the magnetic direction for Berlin: from which you will comprehend how the magnetic direction for every other place is to be ascertained, provided the inclination and declination are known.

Every magnet exhibits phenomena altogether similar. You have only to place one on a table covered with filings of steel, and you will see the filings arrange themselves round the loadstone A B, nearly as represented in *Fig.* 113, in which every particle of the filings may be considered as a small magnetic needle, indicating at every point round the loadstone the magnetic direction. This experiment leads us to inquire into the cause of all these phenomena.

Fig. 113.



The arrangement assumed by the steel filings leaves no room to doubt that it is a subtile and invisible matter which runs through the particles of the steel, and disposes them in the direction which we here observe. It is equally clear that this subtile matter pervades the loadstone itself, entering at one of the poles, and going out at the other, so as to form, by its continual motion round the loadstone, a vortex which reconducts the subtile matter from one pole to the other; and this motion is, without doubt, extremely rapid.

The nature of the loadstone consists, then, in a continual vortex, which distinguishes it from all other bodies; and the earth itself, in the quality of a loadstone, must be surrounded with a similar vortex, acting everywhere on magnetic needles, and making continual efforts to dispose them according to its own direction, which is the same I formerly denominated the magnetic direction: this subtile matter is continually issuing at one of the magnetic poles of the earth, and after having performed a circuit round to the other pole, it there enters, and pervades the globe through and through to the opposite pole, where it again escapes.

We are not yet enabled to determine by which of the two magnetic poles of the earth it enters or issues; the phenomena depending on this have such a perfect resemblance, that they are indistinguishable. It is undoubtedly, likewise, this general vortex of the globe which supplies the subtile matter of every particular loadstone to magnetic iron or steel, and which keeps up the particular vortices that surround them.

Previous to a thorough investigation of the nature of this subtile matter, and its motion, it must be remarked, that its action is confined to loadstone, iron, and steel;* all other bodies are absolutely indifferent to it: the relation which it bears to those must therefore be by no means the same which it bears to others. We are warranted to maintain, from manifold experiments, that this subtile matter freely pervades all other bodies, and even in all directions for when a loadstone acts upon a needle, the action is perfectly the same whether another body interposes or not, provided the interposing body is not iron, and its action is the same on the filings of iron. This subtile matter, therefore, must pervade all bodies, iron excepted, as freely as it does air, and even pure ether; for these experiments succeed equally well in a receiver exhausted by the air-pump. This matter is consequently different from ether, and even much more subtile. And, on account of the general vortex of the earth, it may be affirmed that the globe is completely surrounded by it, and

* Professor Hansteen has lately found that every vertical object, of whatever materials it is composed, has a *magnetic south pole above*, and a *magnetic north pole below*. This curious fact he has put beyond a doubt, by measuring the velocity of the oscillations of a magnetic needle on different sides of the extremities of the vertical object.—See the *Edinburgh Philosophical Journal*, vol. iv. p. 299, 300.—*Ed.*

freely pervaded, as all other bodies are, excepting the loadstone and iron; for this reason iron and steel may be denominated magnetic bodies, to distinguish them from others.

But if this magnetic matter passes freely through all non-magnetic bodies, what relation can it have to those which are such? We have just observed, that the magnetic vortex enters at one of the poles of every loadstone, and goes out at the other; whence it may be concluded that it freely pervades loadstones likewise, which would not distinguish them from other bodies. But as the magnetic matter passes through the loadstone only from pole to pole, this is a circumstance very different from what takes place in others. Here, then, we have the distinctive character. Non-magnetic bodies are freely pervaded by the magnetic matter in all directions: loadstones are pervaded by it in one direction only; one of the poles being adapted to its admission, the other to its escape. But iron and steel, when rendered magnetic, fulfil this last condition; when they are not, it may be affirmed that they do not grant a free transmission to the magnetic matter in any direction.

This may appear strange, as iron has open pores, which transmit the ether, though it is not so subtile as the magnetic matter. But we must carefully distinguish a simple passage, from one in which the magnetic matter may pervade the body, with all its rapidity, without encountering any obstacle.

31st October, 1761.

LETTER LXII.

Nature of the Magnetic Matter, and of its rapid Current. Magnetic Canals.

I AM very far from pretending to explain perfectly the phenomena of magnetism; it presents difficulties which I did not find in those of electricity. The cause of it undoubtedly is, that electricity consists in too great or too small a degree of compression of a subtile fluid which occupies the pores of bodies, without supposing that subtile fluid, which is the ether, to be in actual motion: but magnetism cannot be explained unless we suppose a vortex in rapid agitation, which penetrates magnetic bodies.

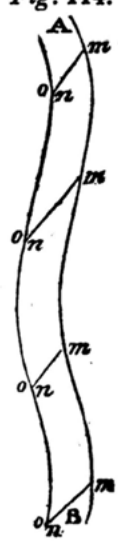
The matter which constitutes these vortices is likewise much more subtile than ether, and freely pervades the pores of loadstones, which are impervious even to ether. Now, this magnetic matter is diffused through and mixed with the ether, as the ether is with gross air; or, just as ether occupies and fills up the pores of air, it may be affirmed that the magnetic matter occupies and fills the pores of ether.

I conceive, then, that the loadstone and iron have pores so small that the ether in a body cannot force its way into them, and that the magnetic matter alone can penetrate them: and which, on being admitted, separates itself from the ether by what may be called a kind of filtration. In the pores of the loadstone alone, therefore, is the magnetic matter to be found in perfect purity: everywhere else it is blended with ether, as this last is with the air.

You can easily imagine a series of fluids, one always more subtile than another, and which are perfectly blended together. Nature furnishes instances of this. Water, we know, contains in its pores particles of air, which are frequently seen discharging themselves in the form of small bubbles: air again, it is equally certain, contains in its pores a fluid incomparably more subtile—namely, ether—and which on many occasions is separated from it, as in electricity. And now we see a still further progression, and that ether contains a matter much more subtile than itself—the magnetic matter—which may perhaps contain, in its turn, others still more subtile, at least this is not impossible.

Having considered the nature of this magnetic matter, let us see how the phenomena are produced. I consider a loadstone, then; and say, first, that besides a great many pores filled with ether, like all other bodies, it contains some still much more narrow, into which the magnetic matter alone can find admission. Secondly, these pores are disposed in such a manner as to have a communication with each other, and constitute tubes or canals, through which the magnetic matter passes from the one extremity to the other. Finally, this matter can be transmitted through these tubes only in one direction, without the possibility of returning in an opposite direction. This most essential circumstance requires a more particular elucidation.

Fig. 114.



First, then, I remark, that the veins and lymphatic vessels in the bodies of animals are tubes of a similar construction, containing valves, represented in *Fig*. 114, by the strokes *m n*, which, by raising themselves, grant a free passage to the blood when it flows from A to B, and to prevent its reflux from B to A. For if the blood attempted to flow from B to A, it would press down the moveable extremity of the valve *m* on the side of the vein *o*, and totally obstruct the passage. Valves are thus employed in aqueducts, to prevent the reflux of the water. I do not consider myself, then, as supposing any thing contrary to nature, when I say that the canals in loadstones, which admit the magnetic matter only, are of the same construction.

Fig 115.

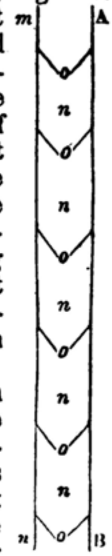


Fig. 115, represents this magnetic canal, according to my idea of it. I conceive it furnished inwardly with bristles directed from A towards B, which present no opposition to the magnetic matter in its passage from A to B, for in this case they open of themselves at *n*, to let the matter pass at *o*; but they would immediately obstruct the channel were it to attempt a retrograde course from B to A. The nature of magnetic canals consists, then, in granting admission to the magnetic matter only at A, to flow towards B, without the possibility of returning in the opposite direction from B towards A.

This construction enables us to explain how the magnetic matter enters into these tubes, and flies through them with the greatest rapidity, even when the whole ether is in a state of perfect rest, which is the most surprising; for how can a motion so rapid be produced? This will appear perfectly clear to you, if you will please to recollect that ether is a matter extremely elastic; accordingly, the magnetic matter, which is scattered about, will be pressed by it on every side. Let us suppose the magnetic canal A B still quite empty, and that a particle of magnetic matter *m* presents itself at the entrance A; and this particle pressed on every side at the opening of the canal, into which the ether cannot force admission, it will there be pressed forward with prodigious force, and enter into the canal with equal rapidity: another particle of magnetic matter will immediately present itself, and be driven forward with the same force; and in like manner all the following particles. There will thence result a continual flux of magnetic matter, which, meeting with no obstruction in this canal, will escape from it at B with the same rapidity that it enters at A.

My idea then is, that every loadstone contains a great multitude of these canals, which I denominate magnetic; and it very naturally follows, that the magnetic matter dispersed in the ether must enter into them at one extremity, and escape at the other, with great impetuosity; that is, we shall have a perpetual current of magnetic matter through the canals of the loadstone: and thus I hope I have surmounted the greatest difficulties which can occur in the theory of magnetism.

3d November, 1761.

Chapter 6

Afterword

This work has examined the *Letters to a German Princess*, a masterpiece by the pre-eminent mathematician, scientist, and educator of the eighteenth century. It provides for the first time in English its context in the history of science and reviews his pedagogical mastery. Written 250 years ago, this book has been translated into about a dozen languages and more than a hundred editions; yet, it remains unknown to most physics-teaching practitioners of today.

During the Enlightenment Era, Euler's physics became the turning point from the medieval static teaching methods and the Cartesian foundations of the fifteenth century to a new physics horizon. There was already *the new physics*, the fundamentals of which we recognize today, created in Europe by the great Newton and Euler. Physics was brought to North America where it was further developed by Franklin and other great minds. By their efforts, the *new physics* succeeded in obtaining *the coordinates origin.*

Our goal was to demonstrate conceptual arguments and more precise measurements with new instruments between the Newtonians and Cartesians, mainly at the Paris Academy and the Newtonians with the Leibnizians and Wolffians. The battle in astronomy involved arguments and better measurements involving the shape of Earth, and irregularities in lunar motion, as well as mechanics with analytical mechanics, fluid dynamics, and priority for the principle of least possible actions. The reader can see from our work that *the new physics* successfully won all these significant disputes, and its triumph paved the way to twentieth century physics.

It is our hope that in addition to historians of science, this *book about a book* will be of interest to physics teachers and informal educators looking for unique ways of making the teaching of conceptual physics meaningful and applicable to learners' experiences. As science education is moving away from concept regurgitation to *sense-making* of the natural phenomena, the *Letters to a German Princess* presents a collection of powerful teaching ideas to give learners from all backgrounds and walks of life access to the understanding of the beauty of the natural world.

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Chapter 7

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Chapter 8

Glossary of principal names

Bernoulli, Daniel (Gröningen, Netherlands 1-29-1700–3-17-1782 Basel, Switz)	Swiss natural philosopher, mathematician, and physician, faculty at the University of Basel, author of *Hydrodynamica* (1738), became Euler's best friend, Member of the Petersburg Academy, the Berlin Academy, the Paris Academy, and FRS.
Brahe, Tycho (Knutstorp Castle, Sweden 12-14-1546–10-24-1601 Prague, Imperial Austria)	Danish astronomer.
Brewster, David (Jedburgh, Scotland 12-11-1781–2-10-1868 Allerly, Melrose, Scotland)	Scottish physicist, inventor, writer, and academic, Scottish Templar, FRS.
Condorcet, Marie Jean Antoine Nicolas de Caritat, Marquis de (Ribemont, France 9-17-1743–3-29-1794 Bourg-la-Reine, France)	French aristocrat, philosopher, mathematician, and official of the Paris Academy of Sciences. He worked with Euler and Benjamin Franklin. In his final years, he wrote *Esquisse d'un tableau historique des progrès de l'esprit humain* (*Sketch for a Historical Picture of the Progress of the Human Mind*) published posthumously in 1795.
D'Alembert, Jean-Baptiste le Rond (Paris, France 11-17-1717–10-29-1783 Paris)	French mathematician, natural philosopher, philosopher, and music theorist, corresponded with Euler on mathematics, science co-editor of the *Encyclopédie,* Member of the Paris Academy, HM of the Berlin Academy, FM of the Petersburg Academy and FRS.
Delisle, Jean Nicholas (Paris 4 April 1688–11 September 1768, Paris)	French astronomer and cartographer. He went to St. Petersburg to the new astronomical

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	institute and science academy in 1725. He remained there for most of the next twenty-two years. He trained Leonhard Euler and the first generation of Russian astronomers. He participated in the expedition to Siberia (Kamchatka), but clouds blocked him. In 1747 he returned to Paris and was appointed geographic astronomer to the navy department. In 1761 he organized the study on a global scale of the transit of Venus.
Desagulier, John Theophilus (New Rochelle, France 12 March 1683–29 February 1744, London)	While born in France, he became a British natural philosopher, Anglican cleric, engineer, and Freemason. He studied at Oxford, where he attended the lectures and demonstrations of John Keill. In 1714 Isaac Newton invited Desagulier to be his experimental assistant at the weekly meetings of the Royal Society, which elected him a fellow that year. Desagulier popularized Newtonian science and demonstrated their practical applications. He was ordained an Anglican priest in 1717 in London. He gave public lectures on mechanics, hydrostatics, pneumatics, optics, and astronomy. He received the Royal Society's Copley Medal in 1734 for his work with Stephen Gray on electricity.
Diderot, Denis (Langres, France 5-10-1713–31-7-1784 Paris)	French philosopher, art critic, and writer, chief editor of the multi-volume *Encyclopédie,* prominent Enlightenment figure.
Fontenelle, Bernard Le Bovier de (Rouen, France 2-11-1657–1-9-1757 Paris, France)	French writer, philosopher, and man of letters, member of the French and Paris Academies, wrote *Entretiens sur la pluralité de mondes* (1686), defender of the new Cartesian natural philosophy.
Formey, Jean-Henri Samuel (Berlin, 31-5-1711–7-3-1791 Berlin)	German Calvinist minister and writer, minister at the church that Euler attended, perpetual secretary of the Berlin Academy, popularized Wolffian philosophy, critic of Rousseau.
Franklin, Benjamin (Boston 17 January 1706–17 April 1790 Philadelphia)	American writer, diplomat, natural philosopher, polymath, and satirist. He was a leading author and printer in Philadelphia and a signer of the United States' Declaration of Independence. Franklin was the first postmaster general of the United States and ambassador to France. He conducted important

	research on electricity, including his famous kite experiment, and wrote *Experiments and Observations on Electricity* (1751). Franklin founded the American Philosophical Society in Philadelphia and was its first president. He was also a founder of the University of Pennsylvania. He was a foreign member of the Royal Society of London (FRS) 1756–, and the Petersburg Academy, 1789–.
Frederick II, the Great (Berlin, Prussia 1-24-1712–8-17-1786, Sanssouci, Potsdam, Prussia, reigned 1740–86)	Prussian monarch, advocated the reorganization and expansion of the Berlin Academy and its Observatory, 1744. He stressed the primacy of the state. Expanded Prussian territory, supported religious toleration.
Frederike Charlotte Leopoldine Ludovica Louise (Schwedt, Prussia 8-18-1745–1-23-1808 Altona, today part of Hamburg, Germany)	German princess of 'royal blood', studied Euler's *Letters to a German Princess*, Abbess of Herford Abbey, 1764–1802.
Fuss, Nicolas (Basel, Switz. 29-1-1755–4-1-1826 St. Petersburg, Russia)	Swiss mathematician, secretary recruited from Basel and collaborator to Euler, leader of Euler circle.
Gassendi, Pierre (Champtercier, France 1-22-1592–10-24-1655 Paris)	French philosopher, priest, scientific observer, defender of new empiricism and physics, and skeptic. He attempted to make Epicurean atomism acceptable to Christianity.
Hunter, Henry (Culross, Scotland 25-8-1741–27-10-1802 Bristol, England)	Scottish minister, translator of Euler's *Letters to a German Princess.*
Lacroix, Sylvestre François (Paris 28-4-1765–24-5-1843 Paris)	French mathematician, member of the Paris Academy, 1789, and the Institut National of the Sciences, 1799–, author of a textbook on differential and integral calculus, 1797–98.
Lagrange, Joseph Louis, born Giuseppe Luigi (Turin, Italy 25 January 1736–10 April 1813 Paris)	Italian-born mathematician and astronomer. Lagrange taught at the Royal Military Academy in Turin and founded the Turin Academy of Sciences in 1758. He moved to the Berlin Academy in 1766 and the Paris Academy in 1786. He was made a professor of mathematics at the new École polytechnique in 1794. Lagrange made significant contributions to both classical and celestial mechanics, generalizing the results of Euler and Maupertuis. He wrote *Mécanique analytique* in 1788. He worked on developing satisfactory foundations for calculus. He was considered 'the successor to Euler'.

Leibniz, Gottfried Wilhelm (Leipzig, Germany 1 July 1646–14 November 1716 Hanover, Germany)	German philosopher, logician, mathematician, natural philosopher, metaphysician, and historian who made substantial contributions to each. He was the next great figure in rationalism after Descartes. He received a law degree in 1667 from Altdorf. Leibniz served Baron von Boyneburg from 1667, as the librarian and councillor to the Duke of Hanover from 1678, historian to the House of Brunswick from 1685, and librarian at Wolfenbuettel from 1691. He proposed the Berlin Society of the Sciences in 1700, and Peter the Great consulted him on creating the Petersburg Academy. Independent of Newton, Leibniz invented an early stage of calculus. He formulated a new dynamism with the conservation of *vis viva*, roughly kinetic energy, as central. The ultimate substance in his universe were monads, geometric points of energy. The monads were windowless, so a pre-established harmony was needed for connections between the body and mind. Leibniz's rational method was based on contradiction and the principle of sufficient reason.
Leopold (III) Friedrich Franz, (Furst) Prince of Anhalt-Dessau (Dessau, Germany 10-8-1740–9-8-1817 Lisium in Oranien-Wörlitz, Germany)	Known as 'Father Franz'.
Lexell, Anders Johan (Turku, Finland 12-24 (11-30 O.S.)1740–[11-30, O.S.]12-11-1784 St. Petersburg, Russia)	Finnish-Swedish mathematician, natural philosopher, and astronomer, Euler's successor at the Petersburg Academy.
Lomonosov, Mikhail Vasilyevich (near Kholinogory close to the White Sea, Russia 19 November [Nov. 8 o.s.] 1711–15 April [April 4 o.s.] 1765 St. Petersburg)	Russian polymath, scientist, chemist, physicist, mineralogist, historian, linguistic reformer, poet, and historian. He studied under Wolff at Marburg and closely followed his experiments. Lomonosov opposed the phlogiston theory in chemistry and laid the foundations of physical chemistry. He was named professor of chemistry in 1745 at the Petersburg Academy. He organized its chemical laboratory and collected plants, minerals, and ores from all over Russia. Lomonosov proposed the founding of Moscow State University in 1754.
Louise Henriette Wilhelmine (Stolzenberg, today Rozanki, Poland 9-24-1750–12-21-1811 Dessau, Germany)	German princess, spouse of Leopold III of Anhalt-Dessau, artistically gifted.

Maupertuis, Pierre Louis Moraeau de (Saint Malo, France 28 September 1698–27 July 1759 Basel, Swiss Confederation)	French mathematician, philosopher, and man of letters. Director of the Paris Academy, 1742–44, President of the Berlin Academy, 1746–57. He led the Lapland Expedition to measure an arc of meridian to help determine the shape of Earth that supported the Newtonian view, 1736–7, and set forth the principle of least action, 1741–6, now known as the Euler–Maupertuis principle. He claimed priority but this was opposed based on a lost letter by Leibniz that provoked a bitter scientific-literary controversy centered at the Berlin Academy. Frederick II supported Maupertuis, and so did Euler, who deserved priority.
Musschenbroek, Pieter van (Leiden, Netherlands 14 March 1692–19 September 1761 Leiden)	Dutch physicist and mathematician. He was a professor of mathematics,the sciences, astronomy, and medicine at the universities of Duiberg, 1719–23, Utrecht, 1723–40, and Leiden, 1740–61. He invented the Leiden jar, the first capacitor that stored electrical charge, and investigated the buckling of struts. He wrote *Elementa physica* (1726) and *Institutiones physicae* (1734). He was elected a foreign member of the Fellows of the Royal Society (FRS) and of the Paris Academy, 1734. He was also a foreign member of the Berlin Academy and an honorary professor at the Petersburg Academy.
Newton, Isaac (Woolsthorpe Manor, England 1-4-1643–3-31-1727 Kensington, London, England)	English physicist, mathematician, astronomer, alchemist, and theologian. Author of *Principia mathematica* (1687) and *Opticks* (1704), inventor of the method of fluxions, an early stage of calculus, Fellow and President of the Royal Society (FRS) in London.
Rohault, Jacques (Amiens, France ca. 1618–12-27-1672 Paris)	French philosopher, natural philosopher, and mathematician, Cartesian, supported mechanical philosophy, wrote *Traite de physique* (1671).
Rumovskij, Stepan (Vladimir, Russia 10-29-1734–7-6-1812 St. Petersburg, Russia)	Russian astronomer and student of Euler, had difficulties with older colleague Lexell.
Thümmig, Ludwig Philipp (Helmbrechts 12 May 1697–15 April 1728 Kassel)	German philosopher and early Wolffian. Adjunct at the University of Halle, where he mainly taught mathematics and natural philosophy, ordinary professor of philosophy at Halle, 1717–23, ordinary professor of philosophy and mathematics at *Collegium*

Carolinum in Kassel, 1724–8. He simplified Wolffian physics in his *Physicae Institutiones Philosophia Wolffianae* and made it more understandable.

Arouet, François-Marie, known by his *nom de plume,* Voltaire (Paris 21 November 1694–30 May1778 Paris)	One of the greatest French writers. He was a leader and a historian of the French Enlightenment. Voltaire opposed tyranny, bigotry, and cruelty by governments and churches. He was famous for his wit and satire. He supported and popularized the science of Newton. He interacted with Frederick the Great and criticized Maupertuis. Two of his books are *Lettres philosophiques* (on the English) (1734), and *Siècle de Louis XIV* (1751).