(A, I) - bounded prism

$I = (d)$

$A/I \to R$ - form smooth

Goal: complex of A-modules

$\Delta_{R/A}$ is related to

$$\Omega^{*}_{R/(A/I)}$$

<u>Def</u>: Pr. site of R relative to A

obj: prisms over (A, I)

$(A, I) \to (B, IB)$

mfg : $(A, I) \to (B, IB)$

with

$$\begin{tikzcd} & R \arrow[r] & B/IB \\ & A/I \arrow[u] \arrow[ur] & \end{tikzcd}$$

sheaves = presheaves

$$\Big(R \to B/IB \leftarrow B \Big)_{\boxed{(R/A)}/A} : X$$

$$\begin{tikzcd} A \arrow[rr] \arrow[d] & & B \arrow[d] \\ A/I \arrow[r] & R \arrow[r] & B/IB \end{tikzcd}$$

$X \xmapsto{\vartheta/A} B \qquad X \xmapsto{\overline{\vartheta}_A} B/IB$

$$\overline{\mathcal{O}}_{/A} = \mathcal{O}_{/A} \big/ I\, \mathcal{O}_{/A}$$

<u>Ex:</u> $A/I = R$

$(R \simeq A/I \leftarrow A)$

<u>Ex:</u> $R = A/I\langle X\rangle$ - p-adic compl. of $A/I[X]$

$A[X]$ $\quad \delta(X) = 0$

$B := (p, I)$-compl. of $A[X]$

$B/IB \cong R$ $\quad (R \cong B/IB \leftarrow B)$

<u>Remark</u> (A, I)-perfect

$$\stackrel{A/I}{\searrow} R \longrightarrow S \circlearrowleft \text{perfectoid}$$

$$(A_{inf}(S), \ker(\theta_S))$$
$$\uparrow$$
$$(A, I)$$

$$(R \longrightarrow S \longleftarrow A_{inf}(S))$$

Functor
$$\left(\begin{array}{c}\text{category of} \\ \text{perf. } R\text{-alg}\end{array}\right) \longrightarrow (R/A)_{\Delta} \qquad \widehat{(R/A)}_{\Delta}$$

$$(R \to B/_{IB} \leftarrow B)$$

with (B, IB)-perfect

essential image $:= (R/A)^{perf}_{/A}$

<u>Ex</u> (A, I)-perf prism

$R = A/_I\langle X\rangle$, $S = A/_I\langle X^{\frac{1}{p^\infty}}\rangle$

$A_{inf}(S) = (p, I)$-completion

$A[X^{1/p^\infty}]$, $\delta(X^{1/p^n}) = 0$

Def. $\Delta_{R/A} := R\Gamma((R/A)_{\Delta}, \mathcal{O}_{\Delta})$

prism. complex $\in D(A)$

- (p, I)-complete
- commutative alg. in $D(A)$
- $\Delta_{R/A} \to \Delta_{R/A}$ - semi-linear

$\Gamma(\mathcal{F}) \otimes \Gamma(\mathcal{F}) \to \Gamma(\mathcal{F} \otimes \mathcal{F})$

$- \otimes \mathcal{F})$

de Rham complex

$\mathbb{R}$-comm. ring

Def. $\mathbb{R}$-dga (E^*, d)

E^* - graded $\mathbb{R}$-alg

$d: E^* \longrightarrow E^{*+1}$

graded commutative -

$$ab = (-1)^{\deg(a)\deg(b)} ba.$$

Strictly gr. comm

$\forall a$ of odd degree $a^2 = 0$

Construction $B \to C$

$(\Omega^*_{C/B}, d_{dR})^! =$

$= (C \to \Omega^1_{C/B} \to \Omega^2_{C/B} \to \ldots)$

Lemma

$-d_{\xi a}$, $E^i = 0$

$C \xrightarrow{\lambda} E^0$ $\forall i < 0$

$\forall x \in C$ $y = d(\lambda(x)) \in E^1$ $y^2 = 0$

Then $C \xrightarrow{1} E^0$ extends to $\Omega^*_{C/B} \to E^*$.

<u>Proof</u>.

$$\begin{tikzcd}
0 \arrow[r] & E^0 \arrow[r] & E^1 \arrow[r] & E^2 \arrow[r] & \cdots \\
& C \arrow[r] \arrow[u, "\eta"] & \Omega^1_{C/B} \arrow[r] \arrow[u, "\eta^1"] & \Omega^2_{C/B} \arrow[r] \arrow[u, "\eta^2"] & \cdots
\end{tikzcd}$$

$d \in I$ $\quad J = (d)$

$\eta : R \to H^0(\Delta_{R/A})$

$$0 \to \mathcal{O}_\Delta/d \xrightarrow{\times d} \mathcal{O}/d^2 \xrightarrow{can} \mathcal{O}/d \to 0$$

$$\begin{matrix} \| \\ d\mathcal{O}_\Delta/d^2 \end{matrix}$$

$$\begin{tikzcd}
C \arrow[r, "d"] \arrow[dr, "der"'] & \Omega_{C/B} \arrow[d, dashed] \\
& M
\end{tikzcd}$$

$$0 \to \mathcal{O}_A/d \to \mathcal{O}_A/d^2 \to \mathcal{O}_A/d \to 0$$

Bockstein map β_d ?

$$H^i(\overline{\Delta}_{R/A}) \to H^{i+1}(\overline{\Delta}_{R/A})$$

$$(H^*(\overline{\Delta}_{R/A}), \beta_d) \qquad \eta : R \to H^0$$

A/I - dga

<u>Lemma</u> $\forall x \in R \quad \beta_d(\eta(x))^2 = 0$

$$\eta_R^* : (\Omega^*_{R/(A/I)}, d_{dR}) \to (H^*(\overline{\Delta}_{R/A}), \beta_d)$$

B' $\quad \widetilde{B'} = B/\ldots$

$B' \to (C, K')$ —

наибольший фактор

без d—пружений

$B_1 := \widehat{\widetilde{B'}} \leftarrow (p, d)$ — derived completion

$\mathrm{colim}\left(B_1 \to \widetilde{B}_1 \to B_2 \to \widetilde{B}_2 \to \ldots\ldots\right)$

$\|$

$\overline{B}$