Равенство Яржинского (Jarzynski), 1997

Г. Н. Бочков, Ю.Е. Кузовлев, 1970–1980-е

$$\vec{x} = (\vec{q}, \vec{p})$$

$$H(\vec{x}, \lambda_t) = H_0(\vec{x}) - \lambda_t Q(\vec{x})$$

$$\rho_0(\vec{x}_0) = e^{-\beta H_0(\vec{x}_0)} / Z_0, \quad Z_0 = \int d\vec{x}_0 \, e^{-\beta H_0(\vec{x}_0)}$$

Работа: $W[\vec{x}_0, \lambda] = H(\vec{x}_\tau, \lambda_\tau) - H(\vec{x}_0, \lambda_0)$

$$\dot{q}_t = \frac{\partial H(x_t, \lambda_t)}{\partial p}$$

$$\dot{p}_t = -\frac{\partial H(x_t, \lambda_t)}{\partial H}$$

$$W[\vec{x}_0, \lambda] = \int_0^\tau dt \, \dot{\lambda}_t \frac{\partial H(\vec{x}_t, \lambda_t)}{\partial \lambda_t} = -\int_0^\tau dt \, \dot{\lambda}_t Q_t$$

Теорема

$$\boxed{\langle e^{-\beta W} \rangle_\lambda = e^{-\beta \Delta F}}$$

$$\Delta F = -\beta^{-1} \ln \frac{Z(\lambda_\tau)}{Z(\lambda_0)} \qquad Z(\lambda_\tau) = \int d\vec{x} \, e^{-\beta H(\vec{x}, \lambda_\tau)}$$

$$\langle e^{-\beta W} \rangle_\lambda = \int e^{-\beta W[\vec{x}_0, \lambda]} \rho(\vec{x}_0) \, d\vec{x}_0 \geq e^{-\beta \int W \rho(x_0) dx_0}$$

Следствие

$$\boxed{\langle W \rangle \geq \Delta F} \Leftarrow e^{-\beta \langle W \rangle} \leq e^{-\beta \Delta F}$$

arXiv: 1012.2268 ;

cond-mat/0702553