(A, I) - bounded prism

$I = (d)$ $\quad A/I \to R$

$(R \to B/IB \leftarrow B) \xrightarrow{\overline{\mathcal{O}}_\Delta} B/IB$

$H^*(\overline{\Delta}_{R/A})$ $\quad \eta : R \to H^0(\overline{\Delta}_{R/A})$

gr. comm R-alg.

$$0 \to \mathcal{O}_\Delta/d \xrightarrow{\times d} \mathcal{O}_\Delta/d^2 \to \mathcal{O}_\Delta/d \to 0$$

β_d: $\quad H^i(\overline{\Delta}_{R/A}) \to H^{i+1}(\overline{\Delta}_{R/A})d$

<u>Lemma</u> $\forall t \in R$

$\beta_d(\eta(t)) \in H^1(\overline{\Delta}_{R/A})$ squares to 0.

<u>Proof</u>. $p = 2$

$$F^0 \rightrightarrows F^1 \ \substack{\longrightarrow\\ \longrightarrow\\ \longrightarrow}\ F^2 \ \substack{\longrightarrow\\ \longrightarrow\\ \longrightarrow\\ \longrightarrow}$$

cosimplitial δ-algebra

$\mathcal{O}_\Delta$ d-torsion free

$F^\bullet / I F^\bullet$ $\quad \partial(-)$ is A-lin

$\partial(-)$ cummutes with φ

$t \in R$ $\quad \eta(t) \in F/IF \quad T \in F^0$

$F^0 \rightrightarrows F^1 \quad u, v \quad \partial(T) = u - v$

$F^0 \Rrightarrow F^2 \quad x, y, z \quad \partial(u) = x - y + z$

$\exists! \, \alpha \in F^1 \quad u - v = d\alpha$

$u - v$ - cocycle $\Rightarrow$

α - cocycle

$\beta_d(t) =$ image of α

in $H^1(F^\bullet / dF^\bullet)$

$\alpha \cup \alpha \overset{?}{=} 0$ in $H^2(F^\bullet / dF^\bullet)$

$\Longleftrightarrow (u - v)(u + v) \overset{?}{=} 0$

in $H^2(d^2F^\bullet / d^3F^\bullet)$

$$\underbrace{(x-y)(y-z)^2}_{\alpha} = 0 \text{ in } H^2\left(d^2F^* / d^3F^*\right)$$

coboundary of

$$d^2\delta(\alpha) \in d^2F$$

$$\boxed{d\alpha = u - v}$$

Note $\delta(a-b) =$

$= \delta(a) - \delta(b) + b(a-b)$

(char $= 2$)

Take $a = u$, $b = v$

$\delta(u-v) = \nu(u-v) + \varepsilon$

where $\varepsilon \in F^1$ - coboundary.

$\partial(\delta(u-v)) = \partial(\nu(u-v)) =$

$= (X-Y)(Y-Z) \in d^2 F^2$

$\partial(d^2\delta(\alpha))$

Apply $\delta(-)$ to

$u - v = d\alpha$

$\delta(u-v) = d^2\delta(\alpha) + \varphi(\alpha)\delta(\alpha)$