# BULK ENTANGLEMENT AND HOLOGRAPHY

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(S.R.Das, A. Kaushal, G. Mandal and S.P. Trivedi : 2004.00613)

# Entanglement in the Bulk

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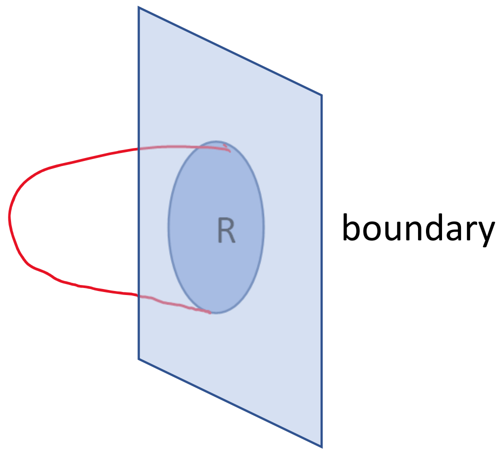


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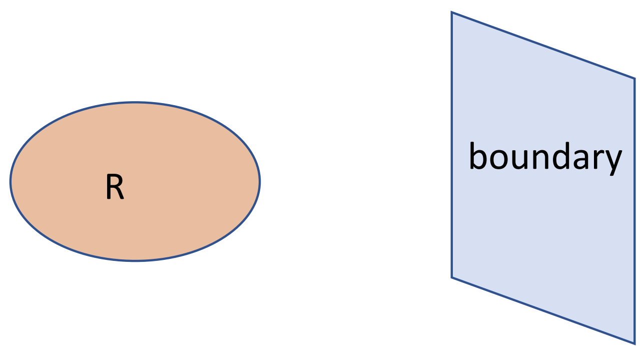


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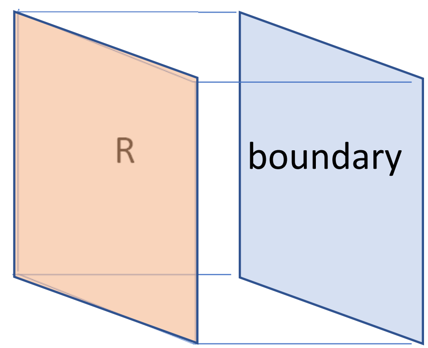


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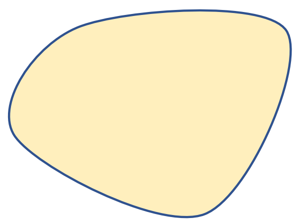
- This cannot be simply related to a geometric entropy of the dual theory – e.g. the dual theory may not have any space at all !

- We will argue that this has a meaning in terms of von Neumann entropy associated with a subalgebra in the dual quantum field theory coming from a subregion of the target space.

- Since the bulk is a theory of gravity, it is not a priori clear what is the meaning of entanglement between two regions – except in the regime where this can be thought of as entanglement of perturbative modes which include matter and gravitons.
- Our motivation is in fact to provide a more precise meaning to this notion which goes beyond the above.
- For gravitational theories with a holographic dual, entanglement in target space provides this notion. In an appropriate limit, this would reduce to the notion of entanglement of perturbative modes.

# EE and UV

- In usual field theory, the leading contribution to the entanglement entropy of a region is given by



$$S_{EE} \sim \frac{A}{\epsilon^{d-1}}$$

- Where $\epsilon$ is a UV cutoff
- In a UV complete theory of gravity, like string theory, one would expect that the result should be finite.
- We may ask : what provides the cutoff ? Is it the string length $l_s$ , or the Newton constant $G$ ?

- We will *conjecture* that to the leading order this entropy saturates the Bekenstein bound

$$S = \frac{A}{4G}$$

and provide some sanity checks for this in theories of Dp branes.

- A similar conjecture was made by *Bianchi and Myers* some time ago – for rather different reasons.

# A clarification

- Sometimes the phrase "bulk entanglement entropy" refers to entanglement of bulk modes across an extremal (or quantum extremal) surface – for a HRT surface this is the $1/N$ correction to the entropy of a subregion of the dual field theory.

(*Faulkner, Lewkowycz and Maldacena; Jaffreis, Lewkowycz, Maldacena and Suh; Agon & Faulkner; Sugishita; Barella, Dong, Hartnoll & Martin; Benin, Iqbal & Lokhande*)

(*Anous, Karczmarek, Mintun, van Raamsdonk & Way*)

- We are interested in a more general question : the entropy we discuss is associated with any region of the bulk – not necessarily ones which are bounded by extremal or quantum extremal surfaces.

# EE in 2d Noncritical String Theory

- Perhaps the earliest example of emergent space is the duality of 2d noncritical strings and gauged quantum mechanics of a single $N \times N$ Hermitian matrix $M_{ij}$ .
- In the $A_t = 0$ gauging means that all states are singlets. We can further make the matrix diagonal

$$M_{ij} \to \text{diag}[\lambda_1, \lambda_2, \cdots \lambda_N]$$

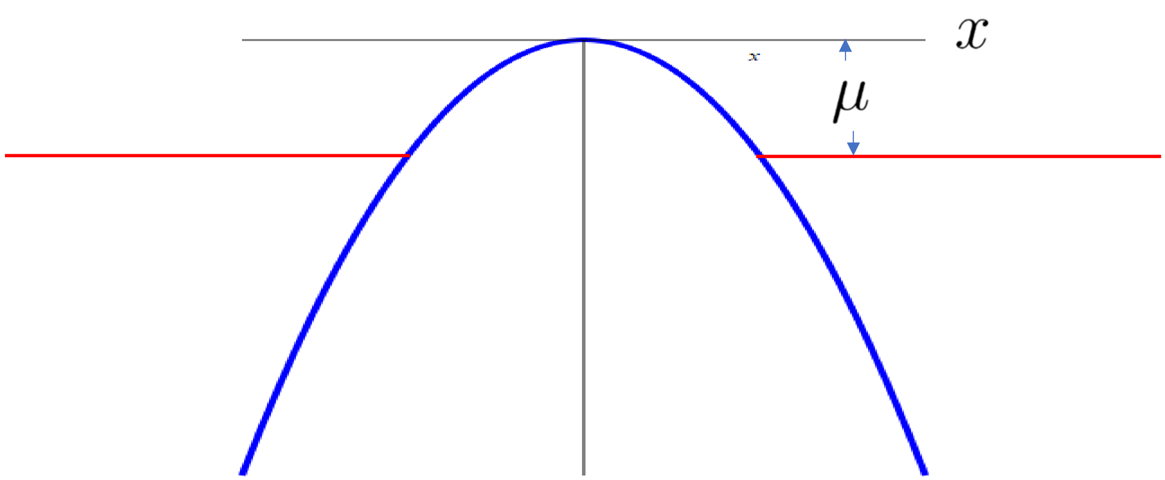
- The eigenvalues $\lambda_i$ become coordinates of fermions. In the double scaling limit one has a Hamiltonian

$$H = \frac{1}{2} \sum_{i=1}^{N} \left[ -g_s \frac{d^2}{d\lambda_i^2} - \frac{1}{g_s} \lambda_i^2 \right]$$

- In second quantization there is a fermion field

$$H = \int dx \left[ \frac{g_s}{2} |\partial_x \psi|^2 - \frac{1}{2g_s} x^2 |\psi|^2 + \frac{1}{2g_s} |\psi|^2 \right] \qquad g_s = \frac{1}{N\mu}$$

- The space of eigenvalues $x$ becomes an emergent space coordinate. This is the bulk space.



- A bosonic formulation is in terms of the density of eigenvalues

$$\rho(x,t) = \frac{1}{N}\sum_{i=1}^{N}\delta(x-\lambda_i(t))$$

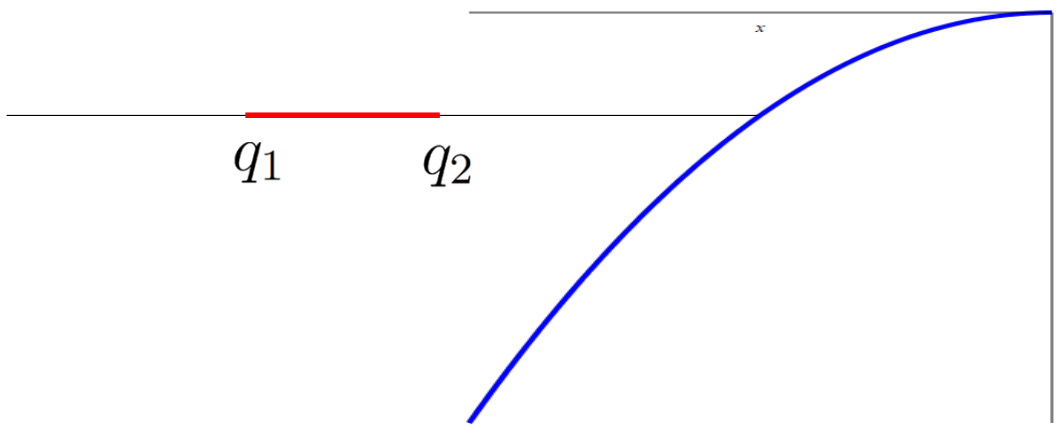
- This is the collective field theory. The fluctuations around the large-N saddle has a space dependent coupling. The classical action is

$$S = \int dqdt\left[\frac{1}{2}\{(\partial_t\eta)^2 - (\partial_q\eta)^2\} + \frac{g_s}{2\sinh^2 q}\{(\partial_t\eta)^2\partial_q\eta + \frac{1}{3}(\partial_q\eta)^3\}\right]$$

$$g_{eff}(q) = \frac{g_s}{\sinh^2 q}$$

- This is the "bulk" description – the field $\eta$ is related to the only dynamical field in two-dimensional string theory – the "massless tachyon"

We can now ask what is the entanglement entropy of a region of this bulk with the complement – as in any field theory




If we treat the collective field theory *perturbatively*, the lowest order result is logarithmically divergent – just like a massless scalar in 1+1 dimension.

- However, the fermionic field theory predicts a finite answer.

  *( S.R. Das (1995)*; *S. Hartnoll and E. Mazenc (2015)*

$$S_{EE} = \frac{1}{3} \log \frac{q_2 - q_1}{\sqrt{g_{eff}(q_1) g_{eff}(q_2)}}$$

- From the point of view of string theory this is a non-perturbative effect.
- The cutoff is provided by the coupling of the theory.
- Seems to indicate that in a general situation the cutoff would be Newton's constant.

**WHAT IS THE MEANING OF THIS QUANTITY IN THE QUANTUM MECHANICS ?**

# Meaning in the Quantum Mechanics

*(S.R.D., G. Mandal and S.P. Trivedi – unpublished )*
*Mazenc and Ranard, 1910.07449*

- Let us start with quantum mechanics of ONE particle in ONE space dimension

$$H = \frac{1}{2}\left[-\frac{d^2}{dx^2} + V(x)\right]$$

Let the wavefunction be $\psi(x)$

We now want to restrict ourselves to some interval $A : a \leq x \leq b$ nd concern ourselves to measurements which can be made in this interval. This defines a sub-algebra of operators which are of the form

$$\hat{O}_A = \int_a^b dx \int_a^b dx' O(x,x')|x >< x'|$$

- We want to define a density matrix in the Hilbert space spanned by $|x >$ where we restrict $a \leq x \leq b$ which evaluates expectation values of such operators.

- The Hilbert space is now a direct sum

$$\mathcal{H} = \mathcal{H}_{1,0} + \mathcal{H}_{0,1} \qquad \mathcal{H}_{1,0} = \{|x>\}, a \leq x \leq b$$

- $\mathcal{H}_{0,1}$ is the complement.
- The density matrix associated with the sector $1,0$ is then given by

$$\tilde{\rho}_{1,0} = \int_a^b dx \int_a^b dx' \psi(x)\psi^\star(x')|x><x'| \qquad <\psi|\hat{O}_{1,0}|\psi> = \mathrm{Tr}(\tilde{\rho}\hat{O}_{1,0})$$

- This has an entanglement entropy

$$S_{1,0} = -\mathrm{Tr}[\tilde{\rho}_{1,0} \log \tilde{\rho}_{1,0}] = -p_A \log p_A$$

- Where $p_A = \int_a^b dx \psi^\star(x)\psi(x)$ is the probability that the particle is in the region.
- Similarly $S_{0,1} = -(1-p_A)\log(1-p_A)$
- The total entropy is then

$$\boxed{S = -p_A \log p_A - (1-p_A)\log(1-p_A)}$$

- This generalizes to N fermions in this setup. Now we have N+1 sectors : the full Hilbert space becomes a direct sum

$$\mathcal{H}_N = \oplus_{p,q;p+q=N} \mathcal{H}_{p,q}$$

- Where $\mathcal{H}_{p,q}$ denotes the sector where $p$ of the coordinates are in the subregion and the rest in the complement.
- For example, for two particles

$$\mathcal{H}_{2,0} = \text{span}\{|x_1, x_2\rangle_a, \quad x_1, x_2 \in A\}$$
$$\mathcal{H}_{1,1} = \text{span}\{|x_1, x_2\rangle_a, \quad x_1 \in A,\, x_2 \in \bar{A}\}$$
$$\mathcal{H}_{0,2} = \text{span}\{|x_1, x_2\rangle_a; \quad x_1, x_2 \in \bar{A}\}$$

where

$$|x_1, x_2\rangle_a \equiv \frac{1}{\sqrt{2!}} \left(|x_1\rangle \otimes |x_2\rangle - |x_2\rangle \otimes |\langle x_1\rangle\right)$$

- This kind of decomposition of the full Hilbert space into a sum over sectors – each of which is a product appears in discussions of entanglement entropy in gauge theories (*Roni and Trivedi*).

- Given a density matrix in the full Hilbert space

$$\rho[\{x_i\},\{x_a\};\{x'_i\},\{x'_a\}] \qquad i = 1\cdots p; a = p+1\cdots N$$

the reduced density matrix for the $p, N-p$ subsector is given by

$$\tilde{\rho}_{p,N-p}[\{x_i\},\{x'_i\}] = \binom{N}{p} \int \prod_{a=p+1}^{N} dx_a \, \rho_a[\{x_i\},\{x_a\};\{x'_i\},\{x_a\}]$$

- The entanglement entropy now becomes

$$S = -\sum_{p,q;p+q=N} \mathrm{Tr}_{\mathcal{H}_A^p} \, \tilde{\rho}_{p,q} \log(\tilde{\rho}_{p,q})$$

- For example, for 2 particles in a state

$$\Psi_a(x_1,x_2) = \frac{1}{\sqrt{2}}[u_1(x_1)u_2(x_2) - u_1(x_2)u_2(x_1)]$$

- This entanglement entropy can be expressed in terms of

$$\int_A dx|u_1(x)|^2 \qquad \int_A dx|u_2(x)|^2 \qquad \int_A dx u_1^\star(x)u_2(x)$$

- This is an example of TARGET SPACE ENTANGLEMENT.
- We proved that this is exactly the same quantity which is computed in the second quantized framework, with the condition that the number of particles is N.
- Target space entanglement entropy has appeared implicitly in discussions of entropy in String Theory using a world-sheet formalism (*Dabholkar*; Witten).
- This is related to discussions of holographic entanglement entropies which involve the internal sphere as well as the boundary (*Graham & Karch*; *Mollabashi, Shiba & Takayanagi,……)*
- This is also related to notions of entwinement (*Erdmenger & Gerbershagen*)

- This quantity is finite because N is finite – a fact which becomes less apparent in the second quantized formalism.
- This then is the origin of finiteness of EE in two-dimensional string theory – this is why the "cutoff" is the bulk coupling which is ~ 1/N.
- From this point of view the finiteness is tracable to the "stringy exclusion principle".
- In terms of the original matrix, these fermion wavefunctions can be related to Schur polynomials made out of multiple traces of the Matrix (*Jevicki*) which realizes the stringy exclusion principle.
- In an exact bosonization at finite N this manifests itself as a discretization of the emergent space with lattice spacing ~ 1/N (*Dhar and Mandal*).

WE NOW APPLY THESE LESSONS TO
HOLOGRAPHIC THEORIES WITH MULTIPLE MATRICES

# D0 BRANE BACKGROUNDS

- Consider the IIA background of N coincident D0 brane

$$
\begin{aligned}
ds^2_{string} &= -H_0(r)^{-1/2}dt^2 + H_0(r)^{1/2}[dx_1^2 + \cdots + dx_9^2] \\
e^{-2\phi} &= H_0(r)^{-3/2}, \qquad r^2 = x_1^2 + \cdots x_9^2. \\
H_0(r) &= \frac{R^7}{r^7}, \qquad R^7 = \frac{(2\pi)^7}{7\Omega_8} l_s^7 (g_s N).
\end{aligned}
$$

- Let us divide the 9-dimensional space into two parts by a surface

$$x_1 = d$$

- We want to give a meaning to the entanglement between the two regions in terms of a holographic description : D0 brane quantum mechanics

- In fact we will consider a heated version

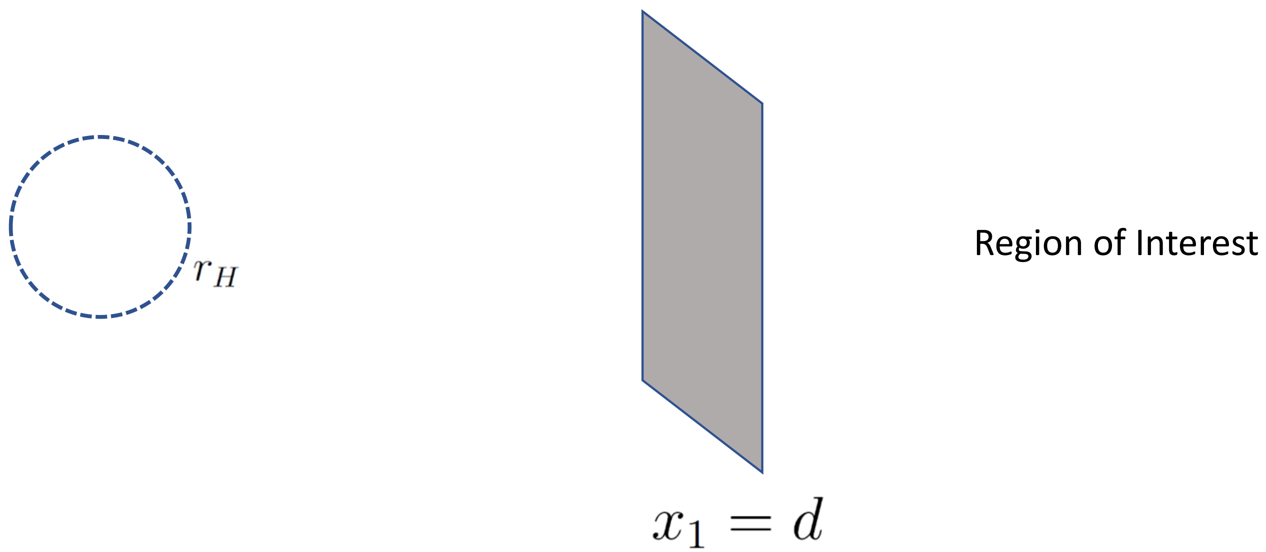
$$ds^2_{string} = -H_0(r)^{-1/2}f(r)dt^2 + H_0(r)^{1/2}[\frac{dr^2}{f(r)} + r^2 d\Omega_8^2]$$

$$f(r) = 1 - \left(\frac{r_H}{r}\right)^7$$

- The temperature given by

$$T = \frac{7}{4\pi R}\left(\frac{r_H}{R}\right)^{5/2}$$

- In this case the $x_1 = d$ surface will be taken to be far from the horizon.



# DO BRANE QUANTUM MECHANICS

- The dual theory is supersymmetric quantum mechanics of 9 matrices

$$S = \frac{N}{2(g_s N) l_s} \mathrm{Tr} \int dt \left[ \sum_{I=1}^{9} (D_t X^I)^2 - \frac{1}{l_s^4} \sum_{I \neq J = 1}^{9} [X^I, X^J]^2 \right] + \text{fermions}$$

- In a gauge $A_t = 0$. the Gauss Law constraint requires wavefunctions to be singlets.
- In terms of suitably rescaled variables the Hamiltonian is

$$H = \frac{(g_s N)^{1/3}}{2 l_s} \mathrm{Tr} \left[ \frac{1}{N} \sum_{I=1}^{9} (\tilde{P}^I)^2 + N \sum_{I \neq J = 1}^{9} [\tilde{X}^I, \tilde{X}^J]^2 \right] + \text{fermions}$$

- The theory has no dimensionless parameter – it is characterized by an energy scale

$$\Lambda = \frac{(g_s N)^{1/3}}{l_s}$$

- This theory has a coulomb branch where all the $< X^I >$ are diagonal. These diagonal elements are the coordinates of the N D0 branes.
- The base space of the gravitational theory becomes the target space of this quantum mechanics.
- The gravity solution we wrote is the dual of the origin of the Coulomb branch $\langle X^I \rangle = 0$ There are also solutions at generic points.
- The wavefunction of course has a spread, leading to

$$< \sqrt{\mathrm{Tr}(X^I)^2} >\sim (g_s N)^{1/3} l_s$$

  This is the size of the bound state.
- In fact in the gravity solution the string frame curvature becomes large when

$$r > r_0 \sim (g_s N)^{1/3} l_s$$

- Furthermore the dilaton becomes large when $r < r_1 \sim (g_s N)^{1/7} l_s$
- We will therefore work in the domain $(g_s N)^{1/7} l_s \ll d \ll (g_s N)^{1/3} l_s$

- In the $A_t = 0$ gauge, the remaining time independent symmetry can be fixed by diagonalizing one of the matrices, $X^1$ . Denote the eigenvalues by $\lambda_i, i = 1, \cdots N$.
- The remaining symmetry is now Weyl transformations.
- In this gauge a generic state may be written as

$$|\psi\rangle = \int [d\mu] \Psi(\lambda_i; X^2_{ij}, \cdots X^9_{ij}) \, |\lambda_i; X^2_{ij}, \cdots X^9_{ij}\rangle + (\text{Weyl Transforms})$$

- A general operator is given by

$$\hat{O} = \int [d\mu] \int [d\mu'] \mathcal{O}(\lambda_i, X^I_{ij}; \lambda'_i, X'^{\,I}) |\lambda_i; X^I\rangle\langle\lambda'_i, X'^{\,I}| + \text{Weyl transforms}$$

- Our proposal is that the quantity of interest is the entropy associated with a restriction in the target space.

- Using the standard relation between the matrices with the coordinates in the supergravity solution, we therefore  the target space subregion to be defined by a restriction on the eigenvalues of $X^1$

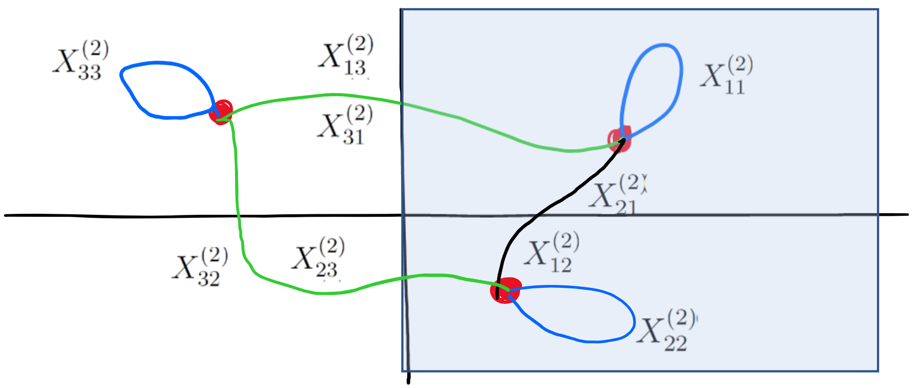
$$\lambda_i > d_0 \qquad\qquad d_0 = \frac{d}{(g_s N)^{1/3} l_s}$$

- This is pretty much like the single matrix problem.
- Now we need to decide what to do with the other matrices.

- To decide on that it is useful to consider a typical snapshot of a configuration of the eigenvalues $\lambda_\alpha$ and the matrix elements $X^I_{\alpha\beta}$. Consider $N = 3$ , and the matrices

$$X^1 = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} \qquad X^2 = \begin{pmatrix} X^{(2)}_{11} & X^{(2)}_{12} & X^{(2)}_{13} \\ X^{(2)}_{21} & X^{(2)}_{22} & X^{(2)}_{23} \\ X^{(2)}_{31} & X^{(2)}_{32} & X^{(2)}_{33} \end{pmatrix}$$

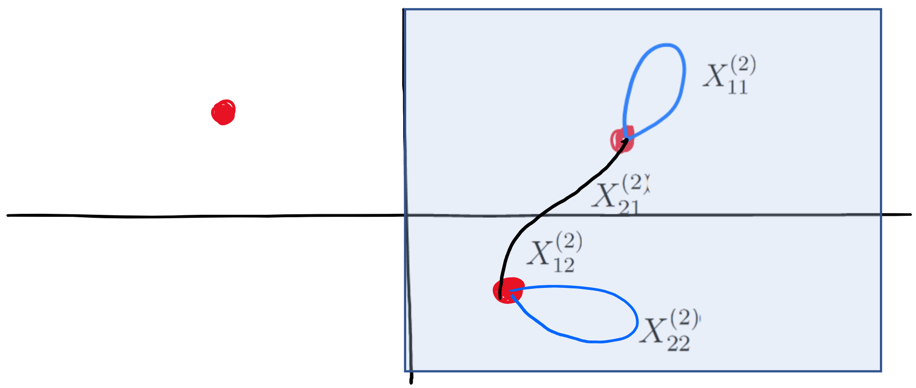
- A typical configuration can be pictorially represented as




- Note this is a picture of a configuration, not a picture of expectation values. The wavefunction evaluated on this configuration provides the probability amplitude.

- One possibility is to keep only the 2 X 2 block and integrate out the rest

$$X^1 = \begin{pmatrix} \lambda_1 & 0 & \\ 0 & \lambda_2 & \\ & & \end{pmatrix} \qquad X^2 = \begin{pmatrix} X^{(2)}_{11} & X^{(2)}_{12} & \\ X^{(2)}_{21} & X^{(2)}_{22} & \\ & & \end{pmatrix}$$



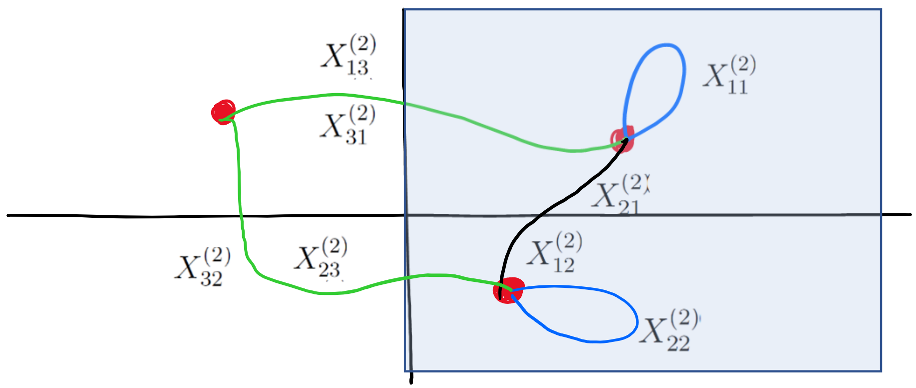

$$\tilde{\rho}_{2,1}\ [\lambda_i, X^{(2)}_{ij}; \lambda'_i, X^{(2)\prime}_{ij}]$$

$$= \int [d\lambda_a dX^{(2)}_{ia} dX^{(2)}_{ai} dX^{(2)}_{ab}]\rho\ [\lambda_i, X^{(2)}_{ij}, \boxed{\lambda_a, X^{(2)}_{ia}, X^{(2)}_{ai}, X^{(2)}_{ab}}; \lambda'_i, X^{(2)\prime}_{ij}, \boxed{\lambda_a X^{(2)}_{ia}, X^{(2)}_{ai}, X^{(2)}_{ab}}]$$

+ Weyl

- A second possibility is to retain the off-block-diagonal matrix elements and integrate out only

$$X^1 = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \end{pmatrix} \qquad X^2 = \begin{pmatrix} X^{(2)}_{11} & X^{(2)}_{12} & X^{(2)}_{13} \\ X^{(2)}_{21} & X^{(2)}_{22} & X^{(2)}_{23} \\ X^{(2)}_{31} & X^{(2)}_{32} & \end{pmatrix}$$




$$\tilde{\rho}_{2,1}\left[\lambda_i, X^{(2)}_{ij}, X^{(2)}_{ia}, X^{(2)}_{ai}; \lambda'_i, X^{(2)\prime}_{ij}, X^{(2)\prime}_{ia}, X^{(2)\prime}_{ai}\right]$$

$$= \int [d\lambda_a dX^{(2)}_{ab}]\rho\left[\lambda_i, X^{(2)}_{ij}, X^{(2)}_{ia}, X^{(2)}_{ai}, \boxed{\lambda_a, X^{(2)}_{ab}}; \lambda'_i, X^{(2)\prime}_{ij}, X^{(2)\prime}_{ia}, X^{(2)\prime}_{ai}, \boxed{\lambda_a, X^{(2)}_{ab}}\right]$$

+ Weyl

- What could the answer look like ?
- Consider the case where the state of the whole system is a thermal state (or more precisely a thermofield double state) with a dimensionless temperature $T_0$
- The answer for the target space entanglement entropy we discussed is therefore some function of $T_0$ and $d_0$
- Since the density matrices in a generic sector encode entanglement of $N^2$ degrees of freedom we expect that the answer should be proportional to $N^2$
- We therefore expect an answer for large N

$$S \sim N^2 \; F(T_0, d_0)$$

- What could the answer look like ?
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- The answer for the target space entanglement entropy we discussed is therefore some function of $T_0$ and $d_0$
- Since the density matrices in a generic sector encodes entanglement of $N^2$ degrees of freedom, we expect that the answer should be proportional to $N^2$
- This becomes evident when we write the expression in terms of the normalized density matrices for each sector,

$$\hat{\rho} = \frac{1}{P_{p,q}} \tilde{\rho}_{p,q} \qquad P_{p,q} = \mathrm{Tr}\tilde{\rho}_{p,q}$$

$$S = -\sum_{p,q} \mathrm{Tr}\tilde{\rho}_{p,q} \log \tilde{\rho}_{p,q} = -\sum_{p,q} [P_{p,q} \log P_{p,q} + P_{p,q} \mathrm{Tr}\hat{\rho}_{p,q} \log \hat{\rho}_{p,q}]$$

- The theory of course has fermionic matrices $\theta_A$
- They should be treated in a manner identical to the bosonic matrices

$$X^I, I = 2, \cdots 9$$

- While the existence of the bound state with N D0 branes has been proved (e.g. Sethi and Stern) explicit expressions for the wavefunction is not known.
- However there has been considerable progress in calculating quantities in D0 brane quantum mechanics and related models numerically.

(*Hanada, Hyakutake, Ishiki & Nishimura (2016); Berkowitz, Rindaldi, Hanada, Ishiki, Shimasaki & Vranas (2016)).*

- This gives us a hope that a numerical calculation of this target space entanglement entropy should be possible in the near future.
- We are currently setting up the problem by utilizing a replica trick in a way which will make such a calculation possible.

# The Conjecture

- Our conjecture is that the target space entanglement entropy we discussed is given by the expression

$$S_{EE}(d) = \frac{A_d}{4G_N}$$

- Where $A_d$ is the Einstein Frame area of the $x_1 = d$ surface.
- For the black D0 brane metric

$$ds^2_{string} = -H_0(r)^{-1/2} f(r) dt^2 + H_0(r)^{1/2} [\frac{dr^2}{f(r)} + r^2 d\Omega_8^2] \qquad r^2 = x_1^2 + x_2^2 + \cdots x_9^2$$

- The result is

$$A_d(T) = \Omega_7 R^{7/2} \int_0^{\rho_0} d\rho\, \rho^7 \frac{1}{(d^2+\rho^2)^{7/4}} [(f(\bar{r})^{-1} - 1) \frac{\rho^2}{d^2+\rho^2} + 1]^{1/2}$$

- Where $\rho$ is the radial coordinate in the $x_1 = d$ plane.

- The integral is IR divergent, which is why we have introduced a cutoff.
- Since the curvature becomes large at $r \sim (g_s N)^{1/3} l_s$ natural to take $\rho_0 \sim (g_s N)^{1/3} l_s$ But this is rather ambiguous.
- The key point is that the difference of this area and the area at zero temperature

$$A_d(T) - A_d(0)$$

is finite – so this quantity is insensitive to this cutoff for large $g_s N \gg 1$

- We can then take the upper limit to infinity and expand the result in powers of $r_H/d$ which is small.
- The leading result for the difference of (conjectured) entropies is

$$S(d,T) - S_{EE}(d,T=0) = C_0 \frac{\Omega_7 R^{7/2} r_H^7}{4 G_N d^{5/2}} \qquad C_0 = \frac{2048}{69615}$$

- To compare with D0 brane QM we need to express this quantity in terms of the dimensionless temperature and location of the entangling surface.

- Recall that

$$R^7 = \frac{(2\pi)^7}{7\Omega_8} l_s^7 (g_s N) \qquad T = \frac{7}{4\pi R}\left(\frac{r_H}{R}\right)^{5/2} \qquad G_N = 8\pi^6 g_s^2 l_s^8$$

- Since the D0 brane QM has just one scale $\Lambda = \frac{(g_s N)^{1/3}}{l_s}$ the appropriate dimensionless temperature is

$$T = T_0 \Lambda$$

- As is standard in AdS/CFT correspondence, the transverse distance is also proportional to the energy scale of the theory

$$d = d_0 (g_s N)^{1/3} l_s$$

- This leads to

$$\boxed{S(d,T) - S_{EE}(d, T=0) = B_0 \ N^2 T_0^{14/5} d_0^{-5/2}}$$

- This is exactly of the form we expected from D0 brane quantum mechanics. In particular this is proportional to $N^2$

- It is important that the "cutoff" appearing in the bulk entanglement entropy is the Newton's constant and not the string length.
- If we used the string length, the answer would have an additional factor of $g_s^2$ .
- This cannot be a result in D0 brane quantum mechanics since this theory does not have any dimensionless parameter.
- The result we displayed is valid in the regime where supergravity is reliable. This requires

$$T_0 \ll 1, N \gg 1$$

- Furthermore, we have taken the entangling surface far from the horizon. This means

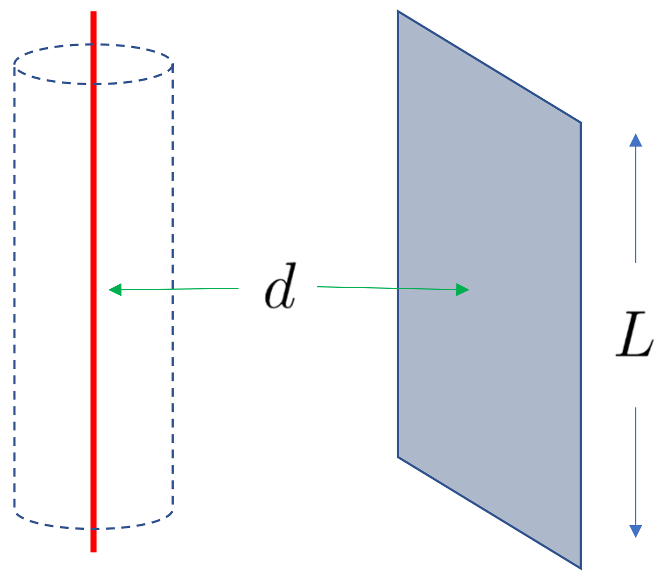
$$d_0 \gg T_0^{2/5}$$

- For smaller $d_0$ the relationship between the transverse coordinates of the background and matrices becomes complicated.
- For higher $T_0$ stringy and loop corrections become important and one expects corrections to the area law conjecture.

- If we compute the area in string frame and use the formula with Newton's constant replaced by string length, once again powers of $g_s^2$ disappear.
- However the result is now $N^0$ rather than $N^2$ which is rather unnatural.

# Dp Branes

- These considerations generalize for Dp branes with $p < 3.$
- We take a surface located at some distance from the center in the transverse direction, and fills the entire Dp brane



- The energy scale of the dual theory is now

$$\Lambda = (g_s N)^{\frac{1}{n-4}} l_s^{-1} \qquad\qquad n \quad = 7 - p$$

- The appropriate dimensionless quantities are now

$$T = T_0 \Lambda \qquad\qquad L = L_0 \Lambda^{-1} \qquad\qquad d = d_0 \Lambda l_s^2$$

- The conjecture for the entanglement entropy now becomes

$$\Delta S_{EE} = B_p \; N^2 \; T_0^{\frac{2n}{n-2}} \; L_0^{7-n} \; d_0^{1-\frac{n}{2}}$$

- The string coupling has disappeared – so this result is conceivable in the dual (p+1) dimensional field theory.
- The entanglement entropy in the field theory is again a target space entanglement – the procedure is pretty similar, except that all the matrix elements are functions of the Dp brane coordinates. We again expect an answer proportional to $N^2$
- We could also consider an entanglement entropy which comes from a restriction in both target space and base space.

# Other entangling surfaces

- We have considered in detail simple entangling surfaces for which the connection to the matrices of the dual theory is simple.
- However we can also consider more interesting surfaces in the bulk, e.g.

$$\sum_{i=1}^{9}(x^i)^2 \leq R^2$$

- The corresponding operator in the dual theory is a Hermitian operator

$$\sum Tr(\hat{X}^i)^2$$

- We can choose to diagonalize this operator.
- This is technically much more involved.

# Epilogue

- We have proposed that entanglement of bulk regions map to target space entanglement – or more generally a combination of target and base space entanglement.
- For simple entangling surfaces this map can be stated precisely – and we find that there are two natural candidates for the reduced density matrix.
- This quantity should be calculable numerically – we are setting up a replica trick method to make this possible.

- We conjectured that the leading answer saturates the area law.
- This means that in a UV complete theory of gravity the cutoff in the entanglement entropy is provided by the Newton constant.
- The target space entanglement entropy is of course defined for all values of the parameters $T_0, d_0, N$ - it may be possible to see how this quantity changes beyond the regime we explored.
- In particular, for finite N and higher $T_0$ bulk locality fails and stringy and loop corrections become important, but the target space entanglement entropy continues to make sense.
- Hopefully this can be calculated explicitly in the near future and our conjecture can be proved or disproved.

**THANK YOU**