Time-periodicities in holographic CFTs

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Scalar field in AdS

$$ds^2 = \frac{1}{\cos^2 x}\left(-dt^2 + dx^2 + \sin^2 x\, d\Omega^2_{d-1}\right)$$

$$0 \leq x < \frac{\pi}{2}$$

$$S = \int d^{d+1}x\sqrt{-g}\left(-\frac{1}{2}\partial\phi\cdot\partial\bar{\phi} - \frac{m^2}{2}|\phi|^2 - \frac{\lambda}{4}|\phi|^4\right)$$

$$0 < \lambda \ll 1$$

Non-interacting limit $\lambda = 0$

$$S = \int d^{d+1}x\sqrt{-g}\left(-\frac{1}{2}\partial\phi\cdot\partial\bar{\phi} - \frac{m^2}{2}|\phi|^2 - \frac{\lambda}{4}|\phi|^4\right)$$

$$ds^2 = \frac{1}{\cos^2 x}\left(-dt^2 + dx^2 + \sin^2 x\, d\Omega_{d-1}^2\right)$$

$$\phi_{\text{linear}}(t,x,\Omega) = \sum_{n=0}^{\infty}\sum_{l,k}\left(A_{nlk}e^{i\omega_{nlk}t} + B_{nlk}e^{-i\omega_{nlk}t}\right)e_{nlk}(x,\Omega)$$

$$\omega_{nlk} = \Delta + 2n + l \qquad\qquad \Delta = \frac{d}{2} + \sqrt{\frac{d^2}{4} + m^2}$$

Weakly nonlinear regime $0 < \lambda \ll 1$

$$S = \int d^{d+1}x\sqrt{-g}\left(-\frac{1}{2}\partial\phi\cdot\partial\bar{\phi} - \frac{m^2}{2}|\phi|^2 - \frac{\lambda}{4}|\phi|^4\right)$$

$$ds^2 = \frac{1}{\cos^2 x}\left(-dt^2 + dx^2 + \sin^2 x\, d\Omega_{d-1}^2\right)$$

$$\phi_{\text{linear}}(t,x,\Omega) = \sum_{n=0}^{\infty}\sum_{l,k}\left(A_{nlk}e^{i\omega_{nlk}t} + B_{nlk}e^{-i\omega_{nlk}t}\right)e_{nlk}(x,\Omega) \qquad \omega_{nlk} = \Delta + 2n + l$$

Naïve perturbation theory: $\phi = \phi_{\text{linear}} + \lambda\phi_{\text{correction}} + \cdots$

Resonances → secular terms → breaks down for $t \sim 1/\lambda$ → resummation required

→ $$\phi(t,x,\Omega) = \sum_{n=0}^{\infty}\sum_{l,k}\left[\alpha_{nlk}(\lambda t)\, e^{i\omega_{nlk}t} + \beta_{nlk}(\lambda t)\, e^{-i\omega_{nlk}t}\right]e_{nlk}(x,\Omega) + \cdots$$

Anharmonic oscillator

$$V(x) = \frac{\omega^2}{2}x^2 + \frac{\lambda}{4}x^4 \qquad\qquad 0 < \lambda \ll 1$$

$$\ddot{x} + \omega^2 x + \lambda x^3 = 0 \qquad\qquad x(t) = x_0(t) + \lambda x_1(t) + \cdots$$

$$x_0(t) = a\cos(\omega t + b)$$

$$\ddot{x}_1 + \omega^2 x_1 = -a^3\cos^3(\omega t + b) = -\frac{a^3}{4}\cos(3\omega t + 3b) - \frac{3a^3}{4}\cos(\omega t + b)$$

$$x(t) = a\cos(\omega t + b) + \lambda\left(\frac{a^3}{32\omega^2}\cos(3\omega t + 3b) - \frac{3a^3}{8\omega}t\sin(\omega t + b)\right) + \cdots$$

Frequency shift

$$\ddot{x} + \omega^2 x + \lambda x^3 = 0 \qquad\qquad 0 < \lambda \ll 1$$

$$\ddot{x}_1 + \omega^2 x_1 = -\frac{a^3}{4}\cos(3\omega t + 3b) - \frac{3a^3}{4}\cos(\omega t + b)$$

$$x(t) = a\cos(\omega t + b) - \frac{3a^3}{8\omega}\lambda t \sin(\omega t + b) + \cdots$$

$$x(t) = a\cos\left(\omega t + b + \frac{3a^2}{8\omega^2}\lambda t\right) + \cdots$$

$$x(t) = a\cos\left(\left(\omega + \frac{3a^2}{8\omega^2}\lambda\right) t + b\right) + \cdots$$

Resonant Hamiltonian

$$H = \frac{1}{2}p^2 + \frac{\omega^2}{2}x^2 + \frac{\lambda}{4}x^4$$

$$x = \frac{1}{\sqrt{2\omega}}(ae^{-i\omega t} + \bar{a}e^{i\omega t}) \qquad\qquad p = i\sqrt{\frac{\omega}{2}}(-ae^{-i\omega t} + \bar{a}e^{i\omega t})$$

$$H = \frac{\lambda}{16\omega^2}\left(e^{-4i\omega t}a^4 + 4e^{-2i\omega t}\bar{a}a^3 + 6\bar{a}^2a^2 + 4e^{2i\omega t}\bar{a}^3a + e^{4i\omega t}\bar{a}^4\right)$$

$$= H_{\text{res}} + \text{oscillating}$$

$$H_{\text{res}} = \frac{3\lambda}{8\omega^2}\bar{a}^2a^2 \qquad\qquad \text{conserves } N = \bar{a}a$$

Maximally rotating waves in AdS_4

$$S = \int d^4x \sqrt{-g} \left(-\frac{1}{2} \partial\phi \cdot \partial\bar{\phi} - \frac{m^2}{2} |\phi|^2 - \frac{\lambda}{4} |\phi|^4 \right)$$

$$ds^2 = \frac{1}{\cos^2 x} \left[-dt^2 + dx^2 + \sin^2 x \, (d\theta^2 + \sin^2\theta d\phi^2) \right] \qquad \omega_{nlm} = \Delta + 2n + l$$

$$\phi(t, x, \Omega) = \sum_{n=0}^{\infty} \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \left[\alpha_{nlm}(\lambda t) \, e^{i\omega_{nlm} t} + \beta_{nlm}(\lambda t) \, e^{-i\omega_{nlm} t} \right] e_{nlm}(x, \theta, \phi)$$

$$i \frac{d\alpha_{n_1 l_1 m_1}}{d(\lambda t)} = \sum_{n_2 l_2 m_2} \sum_{n_3 l_3 m_3} \sum_{\substack{n_4 l_4 m_4 \\ \omega_1 + \omega_2 = \omega_3 + \omega_4}} C_{n_1 l_1 m_1 \cdots n_4 l_4 m_4} \bar{\alpha}_{n_2 l_2 m_2} \alpha_{n_3 l_3 m_3} \alpha_{n_4 l_4 m_4}$$

$$i\dot{\alpha}_n = \sum_{m=0}^{\infty} \sum_{k=0}^{n+m} C_{nmk,n+m-k} \, \bar{\alpha}_m \, \alpha_k \, \alpha_{n+m-k}$$

[BC, Evnin, Luyten]

Maximally rotating waves in AdS_4

$$i\dot{\alpha}_n = \sum_{m=0}^{\infty} \sum_{k=0}^{n+m} C_{nmk,n+m-k}\, \bar{\alpha}_m\, \alpha_k\, \alpha_{n+m-k}$$

Ansatz $\alpha_n = (b + na)p^n$ respected by evolution → solutions with periodic $|\alpha_n|$

[BC, Evnin, Luyten]

Where do they come from?

Holographic implications?

Gross-Pitaevskii from AdS

$$S = \int d^{d+1}x\sqrt{-g}\left(-\frac{1}{2}\partial\phi\cdot\partial\bar{\phi} - \frac{m^2}{2}|\phi|^2 - \frac{\lambda}{4}|\phi|^4\right)$$

$$ds^2 = \frac{1}{\cos^2 x}\left(-dt^2 + dx^2 + \sin^2 x\, d\Omega^2_{d-1}\right)$$

$$\cos^2 x\left(-\partial_t^2\phi + \frac{1}{\tan^{d-1}x}\partial_x(\tan^{d-1}x\partial_x\phi) + \frac{1}{\sin^2 x}\Delta_{S^{d-1}}\phi\right) - m^2\phi = |\phi|^2\phi$$

$$\phi(t,x,\Omega) = \sqrt{2m}\, e^{-imt}\Psi(t, x\sqrt{m}, \Omega) \qquad m\to\infty$$

$$i\frac{\partial\Psi}{\partial t} = \frac{1}{2}(-\nabla^2 + r^2)\Psi + |\Psi|^2\Psi$$

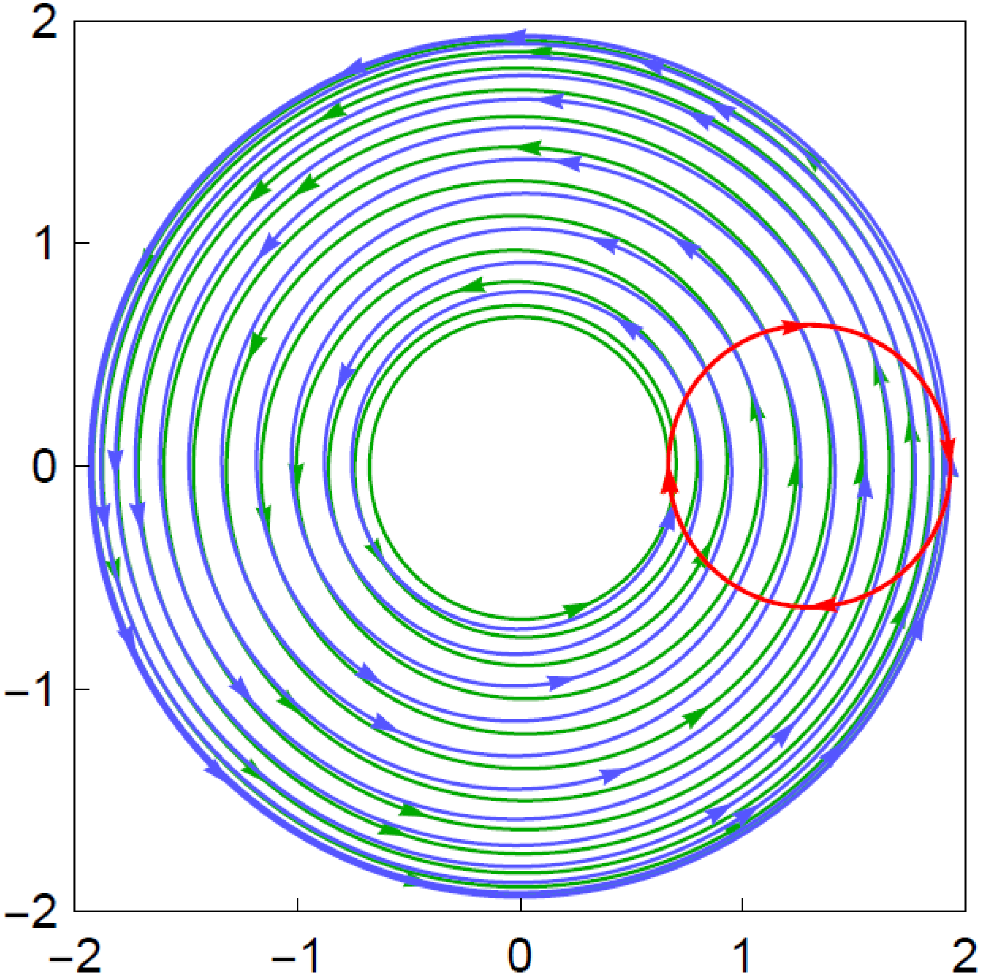
[BC, Evnin, Luyten]

Similar solutions

$$\alpha_n = (b + na)p^n$$

- Cubic Szegö equation [Gérard, Grellier]
- Conformal flow [Bizoń, BC, Evnin, Hunik, Luyten, Maliborski]
- Landau level truncations of GP (e.g. LLL) [Biasi, Bizoń, BC, Evnin]
- Maximally rotating waves in AdS (and on S^3) [BC, Evnin, Luyten]
- Angular momentum truncations of GP [Biasi, Bizoń, BC, Evnin]
- Classes of cubic and quintic resonant systems [Biasi, Bizoń, Evnin]
- Schrodinger-Newton-Hooke system [Bizoń, Evnin, Ficek]

LLL vortex precession



- Fast oscillations with $\omega = 1$
- Slow modulation with $\omega = g/4$

[Biasi, Bizoń, BC, Evnin]

Maximally rotating waves in AdS$_4$

$$i\dot{\alpha}_n = \sum_{m=0}^{\infty} \sum_{k=0}^{n+m} C_{nmk,n+m-k}\, \bar{\alpha}_m\, \alpha_k\, \alpha_{n+m-k}$$

Conserved charges:

$$H_{\text{res}} = \sum_{m_1+m_2=m_3+m_4} C_{m_1 m_2 m_3 m_4} \bar{\alpha}_{m_1} \bar{\alpha}_{m_2} \alpha_{m_3} \alpha_{m_4}$$

$$N = \sum_{m=0}^{\infty} \bar{\alpha}_m \alpha_m \qquad M = \sum_{m=0}^{\infty} m \bar{\alpha}_m \alpha_m$$

$$Z = \sum_{m=0}^{\infty} \sqrt{(m+1)(m+\Delta)}\, \bar{\alpha}_m \alpha_{m+1} \qquad \bar{Z}$$

[Biasi, Bizoń, Evnin]

[BC, Evnin, Luyten]

Breathing modes

$$Z = \sum_{m=0}^{\infty} \sqrt{(m+1)(m+\Delta)}\, \bar{\alpha}_m \alpha_{m+1}$$

Conserved charges like Z appear in all these systems. [Biasi, Bizoń, Evnin]

Breathing mode of original Hamiltonian → conserved charge of resonant Hamiltonian

[BC, De Clerck, Evnin, Khetrapal] [Evnin]

Quantum energy shifts

- Quantum resonant systems, study level spacings.

  [Evnin, Piensuk]

- QM perturbation theory: Hamiltonian → resonant Hamiltonian

  [Evnin]

- Conserved charges → rich algebraic structure.

  [BC, De Clerck, Evnin, Khetrapal]

- Level spacing statistics → no integrability.

  [BC, De Clerck, Evnin, Khetrapal]

- Ladders in quantum level splittings explain LLL periodic energy returns

  [De Clerck, Evnin]

$$E_0 + \frac{g}{4} E_1 + O(g^2)$$

Holography

- Generalized Free Fields, Fock space
- Energy shifts in AdS → energy shifts of CFT on sphere → anomalous dimensions
- $O(\lambda)$ anomalous dimensions: large class of operators with arbitrarily many particles
- Explicit quantum states reproduce classical solutions with periodic energy transfer
- Energy shifts more complicated than in LLL case (two incommensurate ladders).
  Reflects state-dependent frequency of classical energy returns.
- Expect generalization to maximally rotating sectors of gravitational systems

[BC, De Clerck, Evnin]

Conclusions

- $\lambda\phi^4$ in AdS: maximally rotating solutions with slow periodic modulation (period $\sim 1/\lambda$) of fast oscillations
- Studied using averaging/resonant Hamiltonian
- QM counterpart: $O(\lambda)$ energy shifts
- Rich algebraic structure originating from AdS isometries
- Patterns in $O(\lambda)$ anomalous dimensions of classes of operators with arbitrarily many particles
- Explicit coherent-like quantum states that reproduce classical solutions