Black hole dynamics in the SYK model holographic to 2-dim gravity

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Motivation

- BH 'information loss' is associated with BH evaporation

- BH formation is a version of 'information loss':

$$|\tilde{\Psi}\rangle \longrightarrow \text{BH} \quad \tilde{\rho}_\beta$$

mixed state density matrix

- Dual process in AdS/CFT is

$$|\Psi\rangle \longrightarrow \text{thermal state} \quad \rho_\beta$$

quantum quench

- Precise formulation + calculation in SYK.

SYK Model (Sachdev-Ye-Kitaev)

SYK is a 'soluble' model dual to 2-dim gravity

(Kitaev, Maldacena, Stanford, Yang... these are the first works + many others)

- low energy effective action:

  $$S[f] = \frac{N}{J}\alpha_s \int dt \, \{f(t), t\}$$

  Schwarzian

- exhibits chaos : $\lambda_L = 2\pi/\beta$

- micro-states and low energy pure states are easy to construct

- dual to AdS_2 space time

SYK - contd → SYK & KM

QM of N real fermions
with all-to-all random interactions

$\psi_a,\ a=1,..,N \qquad \{\psi_a, \psi_b\} = \delta_{ab}$

$$H_0 = \sum_{1\le a<b<c<d\le N} j_{abcd}\, \psi_a \psi_b \psi_c \psi_d$$

$$\langle j_{abcd} \rangle = 0\ , \langle j^2_{abcd} \rangle = 3!\, \frac{J^2}{N^3}$$

$$H = H_0 + \epsilon(t) H_M$$ quench parameter

$$H_M = -iJ \sum_{k=1}^{N/2} s_k\, \psi_{2k-1} \psi_{2k}$$

(Kourkoulou + Maldacena, 1707.02325)

$\{s_k\} = \{s_1, s_2, \dots, s_{N/2}\}\ ,\ s_k = \pm 1$

Specify a micro-state

Results

The SYK model provides the simplest holographic model that enables a computation of the process of BH formation in the bulk.

- We compute the evolution of pure states through two quantum quenches
- First quench of sufficient strength extracts energy and the spacetime horizon dissapears.
- Second quench pumps in energy; leads to BH formation with $T_{BH} \propto (\epsilon_c - \epsilon)^{1/2}$, Choptuik form

Micro states of SYK

$$\{\psi_a, \psi_b\} = \delta_{ab} \quad , \psi_a \equiv \frac{\gamma_a}{\sqrt{2}}$$

γ_a are $SO(N)$ γ-matrices

$$\gamma_{2k-1} = \underbrace{\mathbb{1} \times \mathbb{1} \times \cdots \mathbb{1}}_{k-1} \times \underbrace{\sigma_1}_{k} \times \underbrace{\sigma_3 \times \cdots \times \sigma_3}_{\ell-k}$$

$$\gamma_{2k} = \mathbb{1} \times \mathbb{1} \times \cdots \mathbb{1} \times \underbrace{\sigma_2}_{k} \times \sigma_3 \times \cdots \times \sigma_3$$

$k = 1, \ldots, N/2$

$$-i\gamma_{2k-1}\gamma_k = \mathbb{1} \times \mathbb{1} \times \cdots \mathbb{1} \times \underbrace{\sigma_3}_{k} \times \sigma_3 \times \cdots \times \sigma_3$$

$\hat{S}_k$ has eigenvalues $s_k = \pm 1$

Hence Hilbert space is spanned by $2^{N/2}$ orthonormal states.

$$|B_s\rangle \equiv |s_1\rangle \otimes |s_2\rangle \otimes \cdots \otimes |s_{N/2}\rangle$$

$s_k = \pm 1$

Pure states at low-energy

$$|B_s(\ell)\rangle \equiv e^{-\ell H_0}|B_s\rangle, \quad \ell = \frac{\beta}{2} \gg 1$$

Over complete low energy states.

The '*Flip group*' $F_G \subset SO(N)$, flips the sign of the even fermions:

$$F_k : \left(\psi_{2k} \to -\psi_{2k}, \; \psi_{2k-1} \longrightarrow \psi_{2k-1}\right)$$

So you have operators which are even and odd under F_G.

In particular $\hat{S}_k \to -\hat{S}_k$ and hence the entire Hilbert space is an orbit of the Flip group: e.g. $|-\rangle|-\rangle|+\rangle = F_1|+\rangle F_2|+\rangle|+\rangle$

Thermal properties of pure states $|B_s(\ell)\rangle$

$$\sum_{\{\Delta_k\}} \langle B_s(\ell)|\hat{O}|B_s(\ell)\rangle \equiv \mathrm{tr}\, e^{-2\ell H_0}\hat{O}$$

Definition

Lemma:

If $\hat{O}$ is flip invariant (FI)

$$\mathrm{Tr}\, e^{-2\ell H_0}\hat{O} = 2^{N/2}\langle B_s(\ell)|\hat{O}|B_s(\ell)\rangle$$

for any micro-state $|B_s\rangle$.

FI operators cannot distinguish between the micro-states $\{|B_s\rangle\}$ and seem thermalized, e.g.

- $Z_\beta = 2^{N/2}\langle B_s(\ell)|B_s(\ell)\rangle = \mathrm{tr}\, e^{-2\ell H_0}$

- $G_\beta(\tau-\tau') = \langle B_s(\ell)|\psi_a(\tau)\psi_a(\tau')|B_s(\ell)\rangle$

$$= \frac{c}{\left[\frac{J\beta}{\pi}\sin\frac{\pi}{\beta}(\tau-\tau')\right]^{1/2}}$$

Time evolution of $|B_s(\ell)\rangle$

$$H = H_0 + \epsilon(t) H_M$$ quench parameter

$$H_M = -iJ \sum_{k=1}^{N/2} s_k \psi_{2k-1} \psi_{2k}$$

The set $\{s_k\}$ in H_M is the same as in $|B_s(\ell)\rangle$.

SYK Hamiltonian H_0 is flip invariant

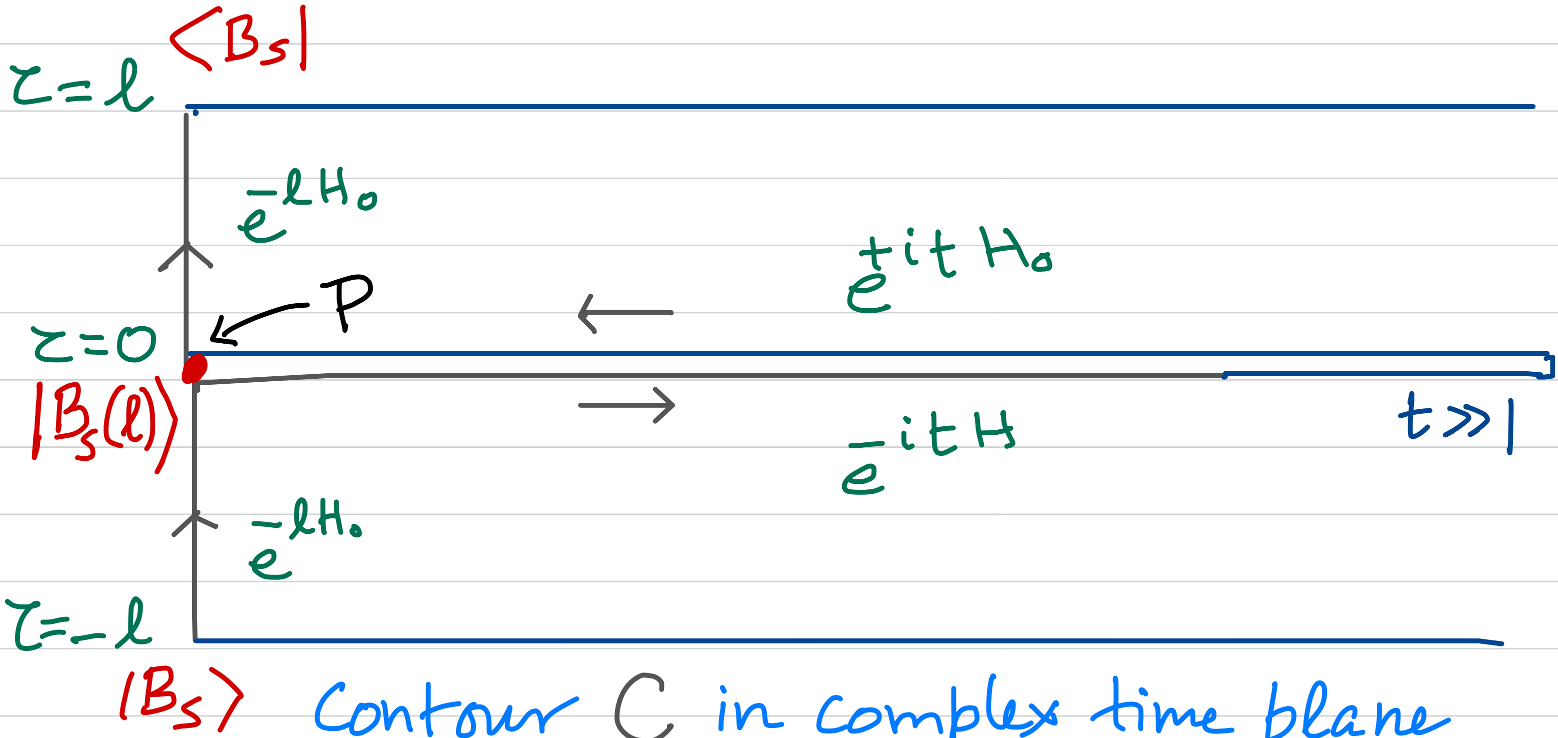
H_M is not; H_M is a relevant operator in the infrared.

To study its effect consider 2 states:

$$|\psi(t)\rangle = e^{-iHt} |B_s(\ell)\rangle$$
$$|\psi_0(t)\rangle = e^{-iH_0 t} |B_s(\ell)\rangle$$

The main object of study is the overlap of these two states:

$$A_s(t,\ell) \equiv \langle \Psi_0(t) | \Psi(t) \rangle$$
$$= \langle B_s(\ell)| e^{+iH_0 t} e^{-iHt} |B_s(\ell)\rangle$$



Contour C in complex time plane

Result at large N limit and low energy

$$A_s(t,\ell) = \int [df]\; e^{iS[f]}$$

$$S[f] = -\frac{N\alpha_s}{J}\int_C dt \Big[\underbrace{\{f,t\}}_{\text{SYK}} - \frac{J^2}{2}\epsilon(t)(f')^{2\Delta}\Big]$$

Bulk Dual Gravity

In 2-dim there are 2 natural possibilities:

1) Polyakov gravity

$$S_{Pol} = \frac{1}{16\pi G_N} \int_{\Gamma} d^2x \sqrt{-g} \left[R \frac{1}{\Box} R - 16\pi \Lambda \right]$$

$$+ \frac{1}{4\pi G_N} \int_{\partial\Gamma} dt \sqrt{-\gamma} \left(K \frac{1}{\Box} R + K \frac{1}{\Box} K \right)$$

Arises from quantizing the co-adjoint orbit of Diff $\mathbb{1}$ / $SL(2,\mathbb{R})$, $\mathbb{1} = \left\{ \begin{array}{l} S^1 \\ \mathbb{R}^1 \end{array} \right.$

(Mandal, Nayak, SRW: 1702.04266)

The co-adjoint orbit construction naturally leads to a theory of gravity in 1+1 dim. (Alekseev, Shatashvili)

2) Jackiw-Teitelboim gravity

$$S_{JT} = \frac{\Phi_0}{16\pi G_N}\left(\int_\Gamma d^2x\sqrt{-g}\,(R+2\Lambda) + \int_{\partial\Gamma} dt\sqrt{-\gamma}\,K\right)$$

$$-\frac{1}{16\pi G_N}\left(\int_\Gamma d^2x\sqrt{-g}\;\Phi(R+2\Lambda) + 2\int_{\partial\Gamma} dt\sqrt{-\gamma}\,(K-1)\right)$$

Φ is the dilaton field

Arises from a s-wave reduction of a near-extremal charged BH

4dim $\longrightarrow$ $S^2 \times (NAdS_2)$

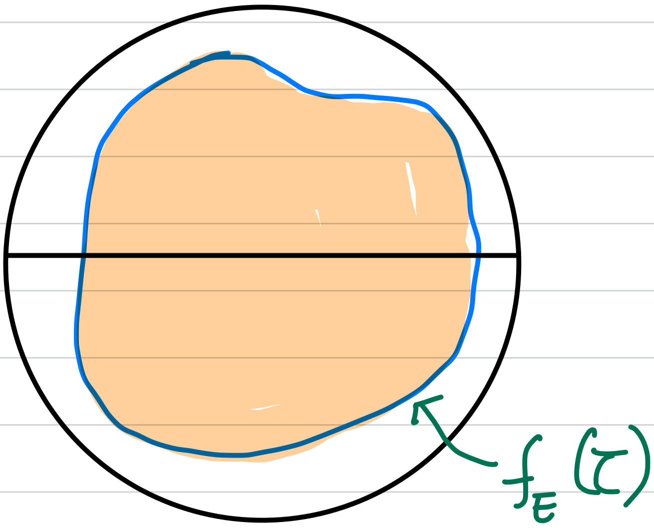
In **both** theories there are no propagating degrees of freedom.

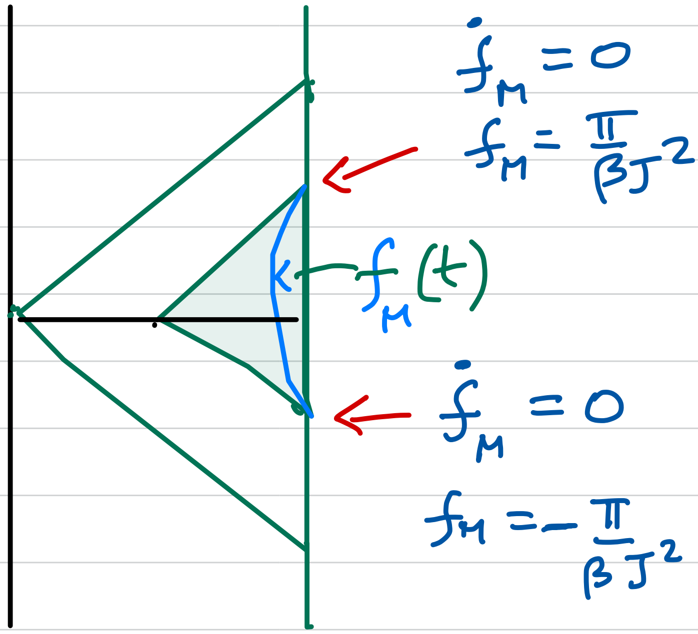
The only physical degrees of freedom are 'large diffeomorphisms':

$$(\tau, z) \to (\tau^f(\tau,z), z^f(\tau,z))$$

$$\simeq (f(\tau), z f'(\tau)) \quad \text{near } z \simeq 0$$

Geometry





Euclidean AdS_2   **Lorentzian AdS_2**

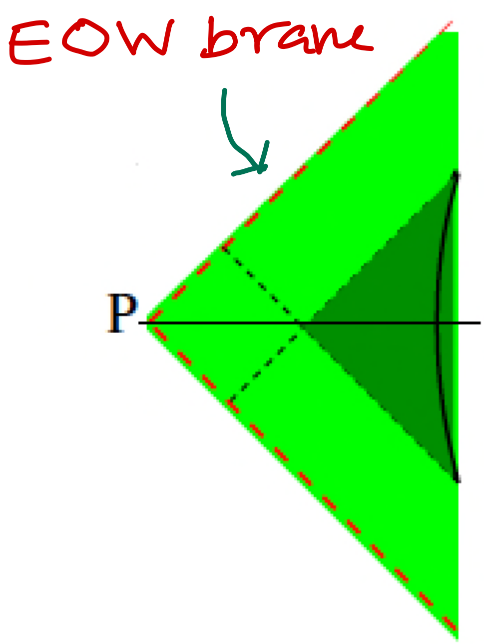
$f_E(\tau) \in \text{diff}\, S^1/SL_2$ $f_M(t) \in \text{diff}\, \mathbb{R}^1/SL_2$

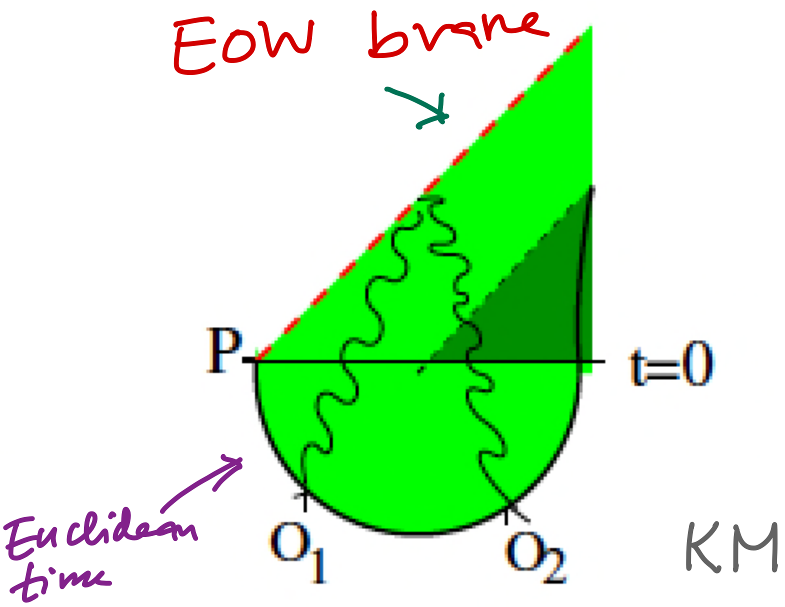
> In Poincare coordinates, near $z \simeq 0$, where $f(t)$ is a large diff.
>
> $$(t^f, z^f) \simeq \left(f(t),\ z f'(t) \right)$$
>
> If $f'(t) = 0$, $z^f = 0$ and an observer cannot receive signals from the entire Poincare patch + there is a horizon.

Solutions of eom :

$$f_E(\tau) = \frac{\pi}{\beta J^2} \tan \frac{\pi}{\beta} \tau \qquad f_M(t) = \frac{\pi}{\beta J^2} \tanh \frac{\pi}{\beta} t$$

Geometry in the presence of $|B_s(\ell)\rangle$

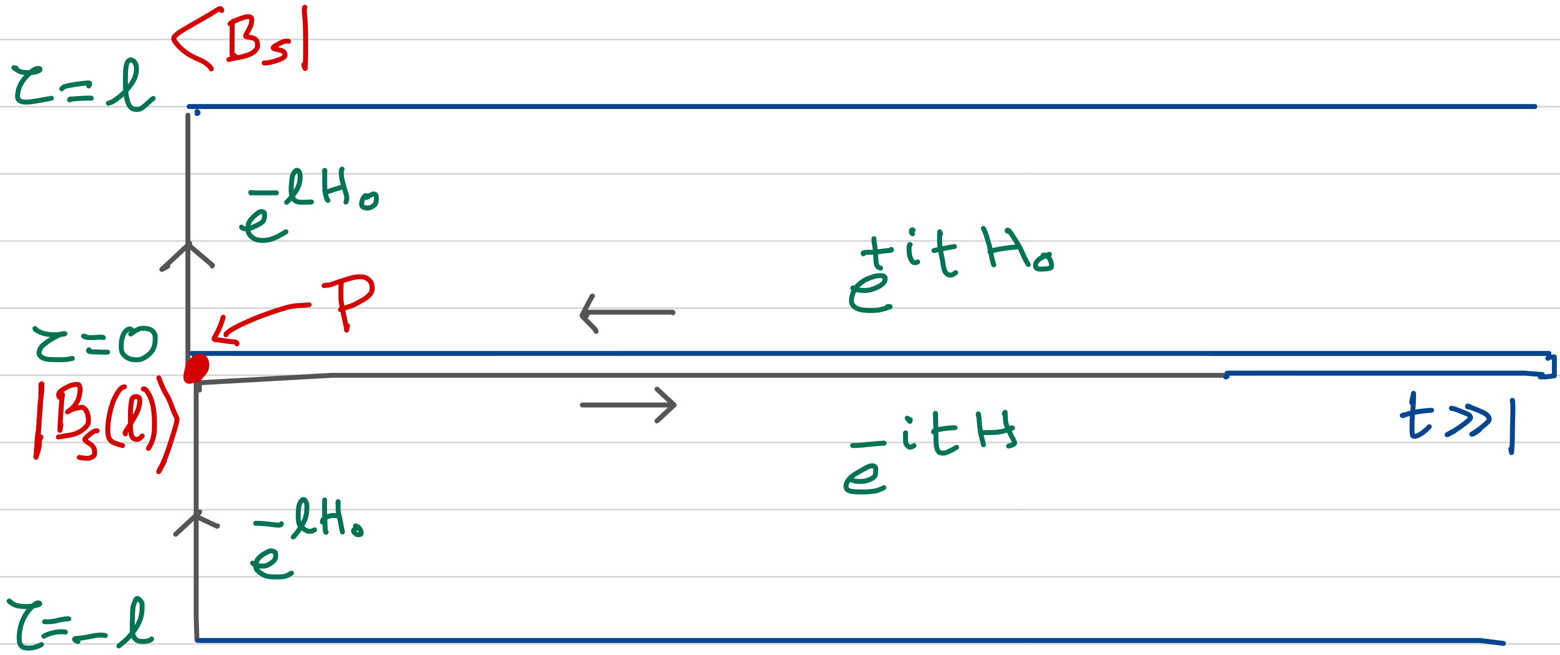




The presence of $|B_s(\ell)\rangle$ at P leads to a geometry with an EOW brane of large mass behind the horizon, that is qualitatively similar to the geometry we discussed before.

Back to the SYK model:

$$A_s(t,\ell) \equiv \langle \Psi_0(t) | \Psi(t) \rangle$$
$$= \langle B_s(\ell) | e^{+iH_0 t} e^{-iHt} | B_s(\ell) \rangle$$



Contour C in complex time plane

Result at large N limit and low energy

$$A_s(t,\ell) = \int [df] \, e^{iS[f]}$$

$$S[f] = -\frac{N\alpha_s}{J} \int_C dt \left[\{f, t\} - \frac{J^2}{2} \epsilon(t) (f')^{2\Delta} \right]$$

SYK

(Detailed derivation in 1812.03979)

We evaluate the path integral along C semi-classically as $N \to \infty$.

Since the initial state at $t=0$ describes a BH, in the

- Euclidean section $[-l, 0]$

$$f_E(\tau) = \frac{\pi}{\beta J^2} \tan \frac{\pi}{\beta}\tau$$

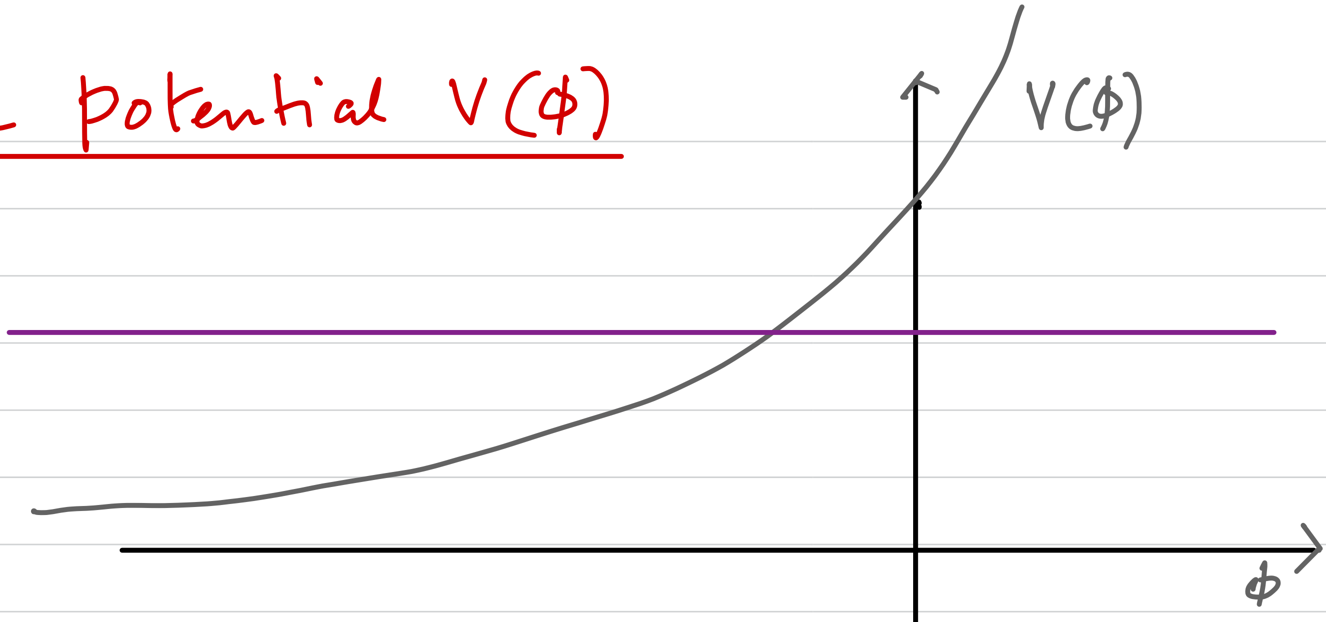
- change variables $\dot{f}(t) = e^{\phi(t)}$ along all of C, use $f_E(\tau)$ to arrive at

$$A_s(t,l) = \int \mathcal{D}\phi(t)\, e^{iS[\phi]} \quad , \; t \geq 0$$

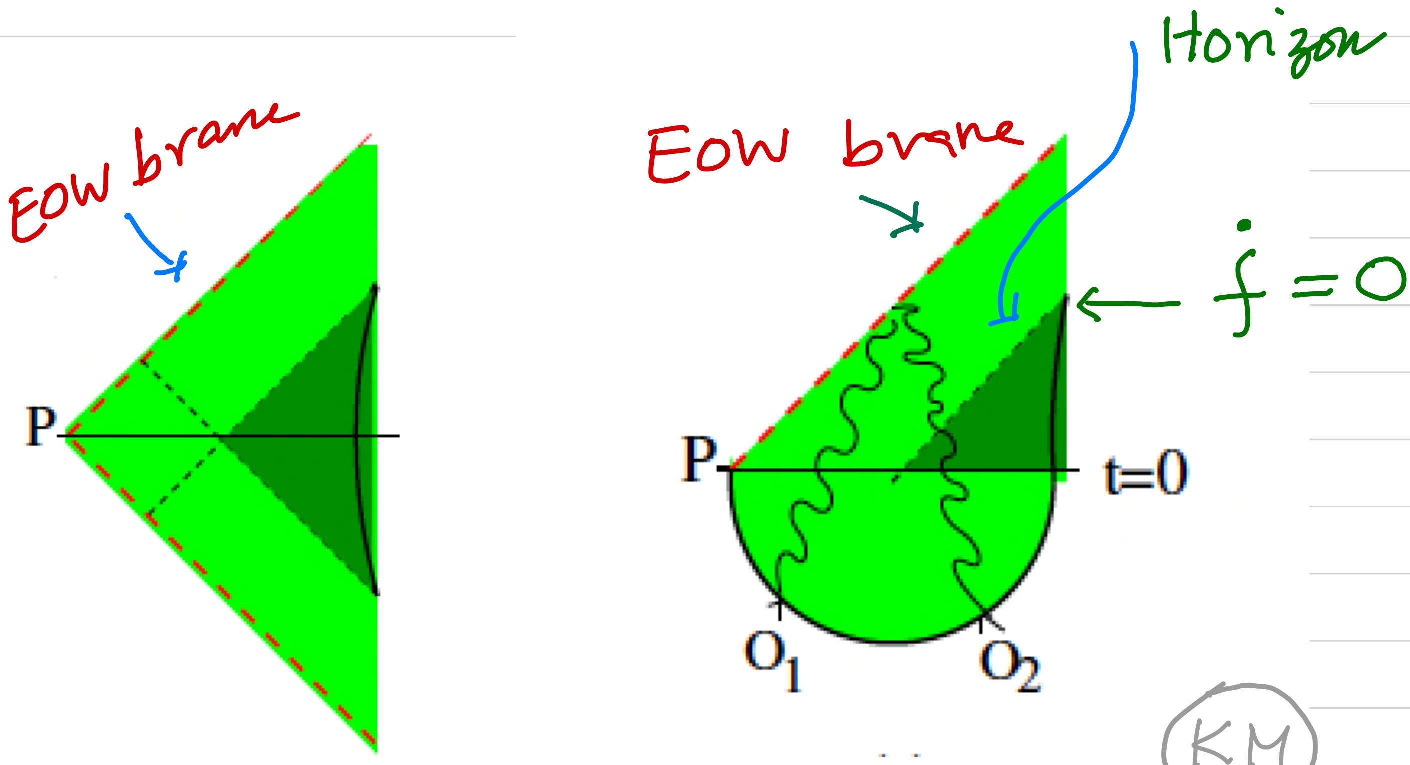
$$S[\phi] = \frac{N\alpha}{J} \int dt \left[\frac{\dot{\phi}^2}{2} - V(\phi) \right]$$

$$V[\phi] = \frac{J^2}{2} \left(e^{\phi} - \epsilon(t)\, e^{\phi/2} \right)$$

The potential $V(\phi)$

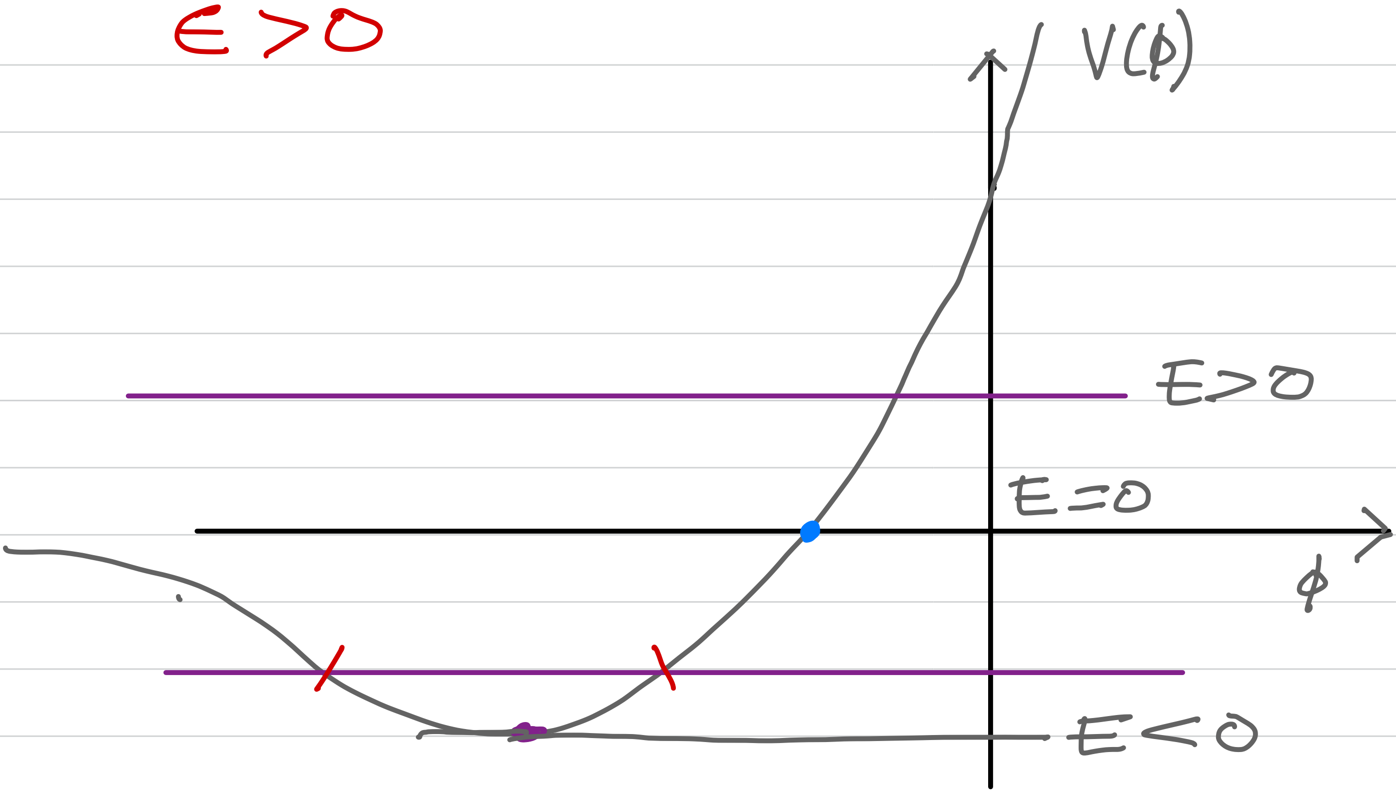


$\epsilon \leq 0$, unbounded motion, $\dot{f} = e^{\phi} \to 0$ as $\phi \to -\infty$



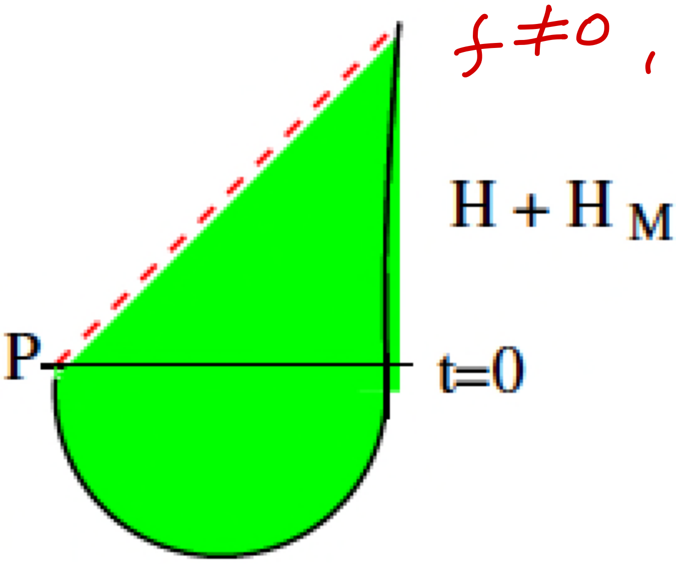
KM

$\epsilon > 0$



Depending on the energy, there is a bounded solution, including a static solution at the bottom of the potential. $e^{\phi} = f \neq 0$

$f \neq 0$, no horizon



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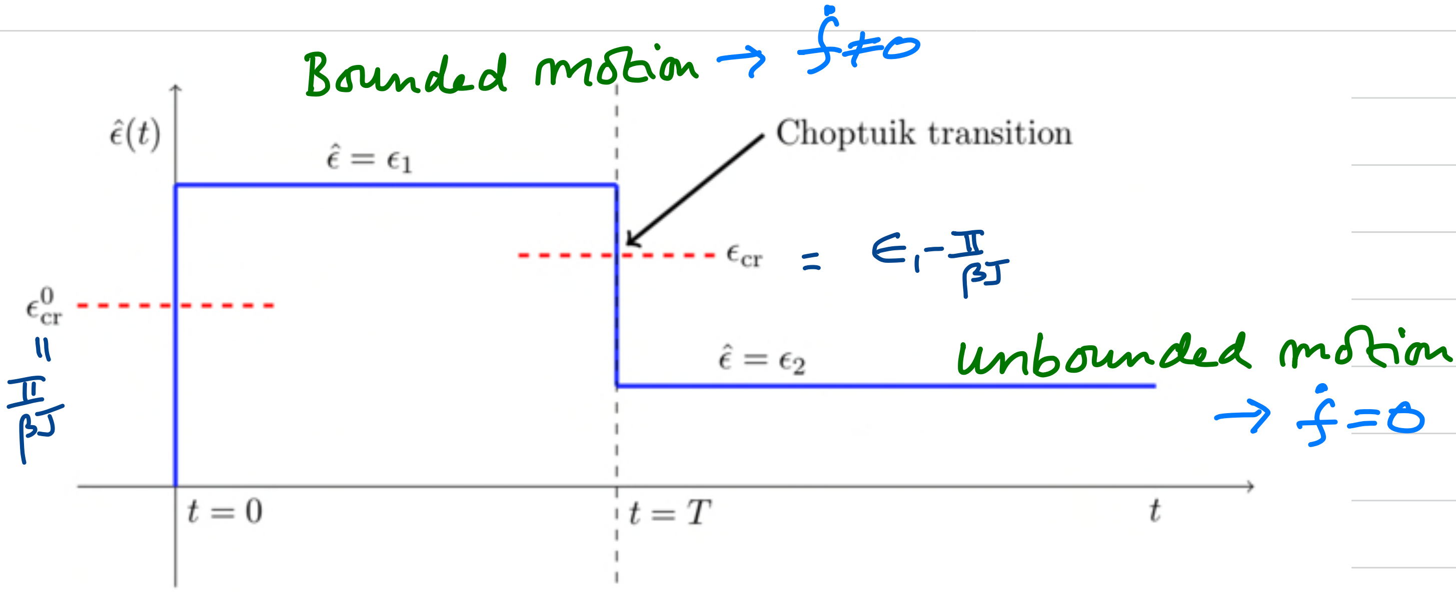
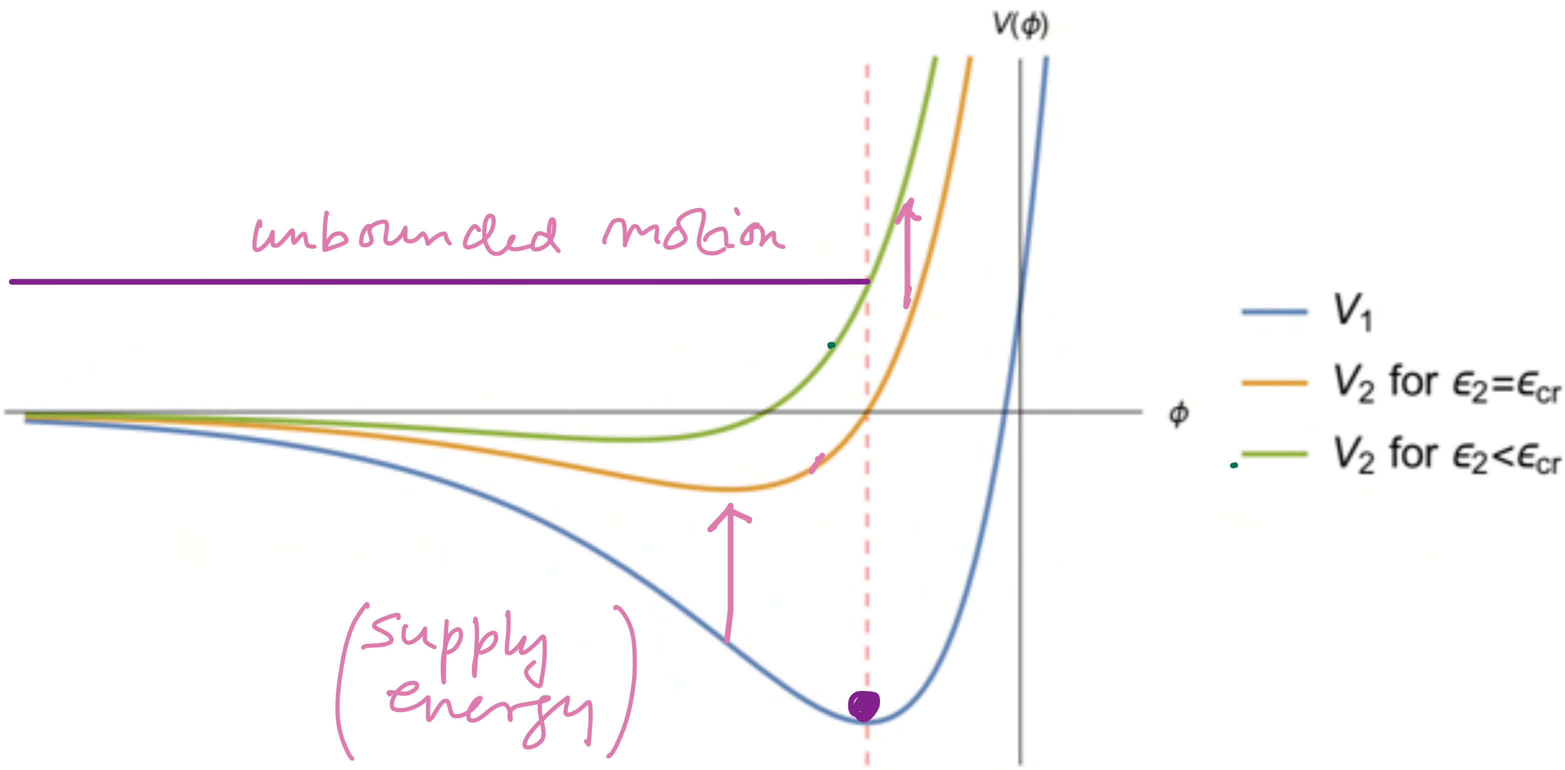
MAIN LESSON

by tuning the quench parameter ϵ,
we can pump out / in energy
into the system
and
remove or form a horizon,
depending on whether the
classical solution exhibits

$\dot{f} \neq 0$ (no horizon) or $\dot{f} = 0$ (horizon)

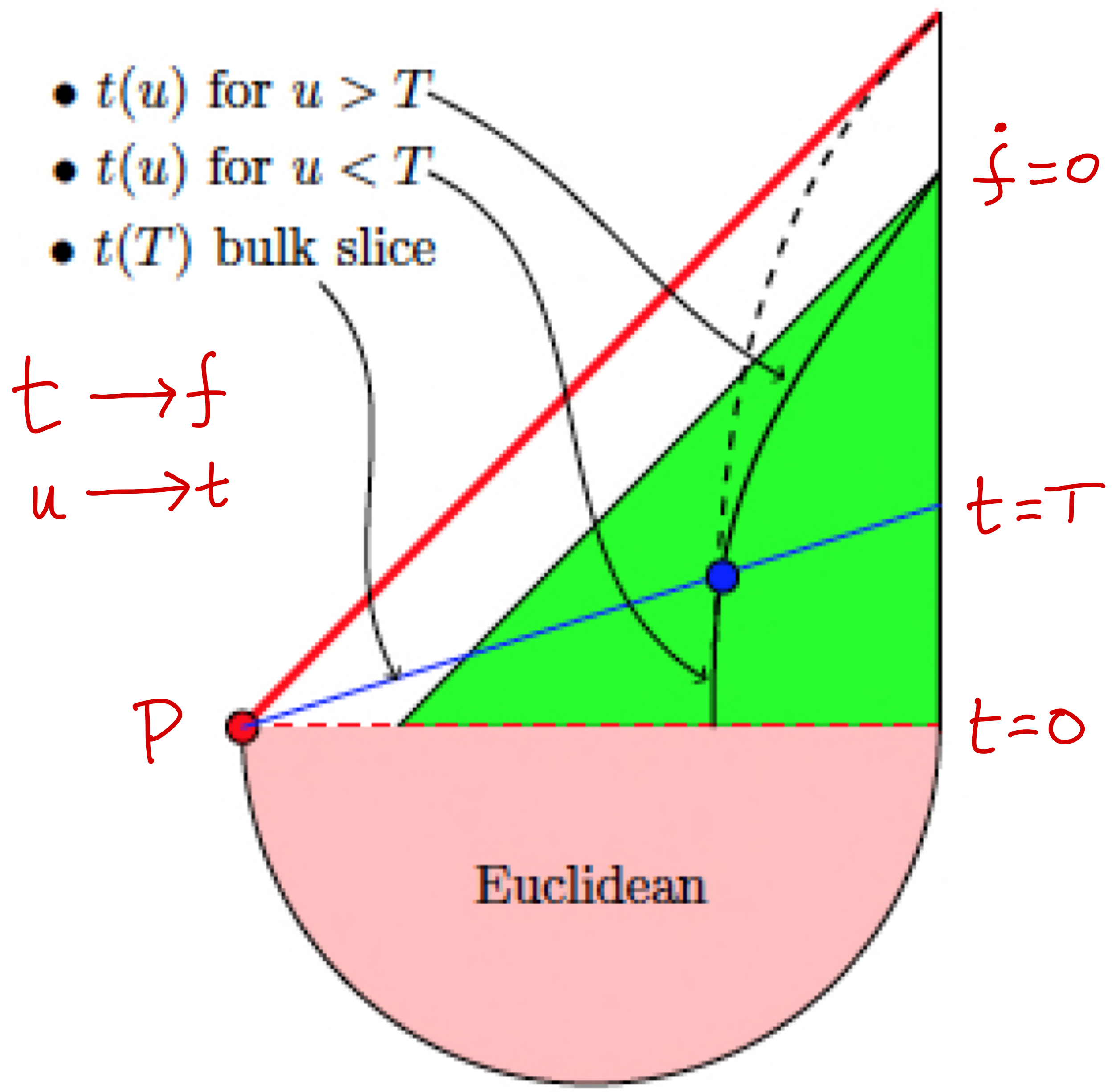
Tuning the $\epsilon(t)$ & $V(\phi)$





Detailed solutions for $\phi(t)$ for $0 \leq t \leq T$ (oscillatory) & $T \leq t < \infty$ (unbounded) are presented in JHEP 2019, 67 (2019).

Space-time picture



Asymptotic solution + BH temp.

$$\dot{f}(t) = e^{\phi(t)} \xrightarrow[t\to\infty]{} e^{-tC}$$

$$C = J\left[\left(\epsilon_1 - \frac{\pi}{\beta J}\right)\left(\epsilon_1 - \frac{\pi}{\beta J} - \epsilon_2\right)\right]^{1/2}$$

$$\epsilon_1 > \frac{\pi}{\beta J} \quad \text{and} \quad \epsilon_2 < \epsilon_1 - \frac{\pi}{\beta J}$$

From this asymptotic formula one can find the temperature of the new BH:

$$\boxed{\begin{aligned} T_{BH} &= \frac{2\pi}{J^2}\left[\left(\epsilon_1 - \frac{\pi}{\beta J}\right)\left(\epsilon_1 - \frac{\pi}{\beta J} - \epsilon_2\right)\right]^{1/2} \\ &\sim \left[\epsilon_2^{cr} - \epsilon_2\right]^{1/2} \end{aligned}}$$

which has a Choptuik type scaling law, with exponent $1/2$.

Conclusions and ongoing work

Recall our starting point:

$$A_s(t,\ell) = \int [df]\, e^{iS[f]}$$

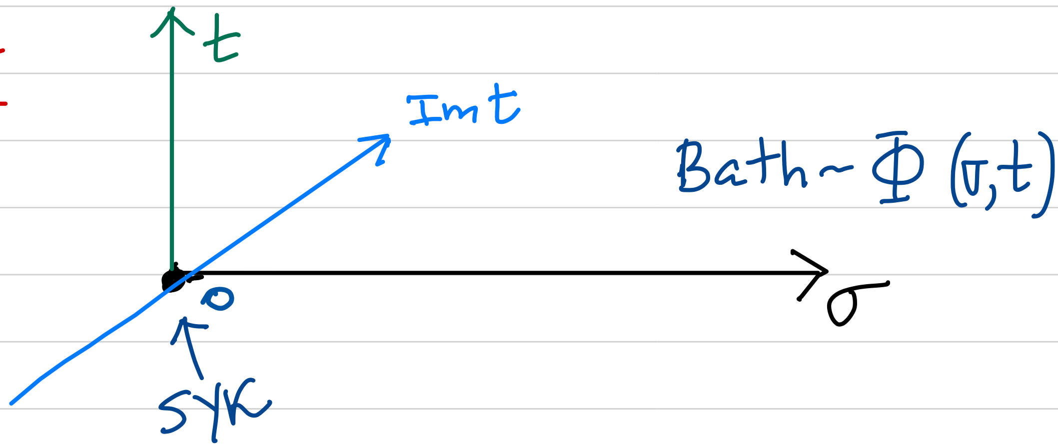
$$S[f] = -\frac{N\alpha_s}{J}\int_C dt\left[\underbrace{\{f,t\}}_{\text{SYK}} - \frac{J^2}{2}\epsilon(t)(f')^{\frac{1}{2}}\right]$$

Tuning the quench parameter $\epsilon(t)$, we can pump energy into the system to create a geometry with a horizon, a black hole with a temperature that can be calculated and

$$T \propto (\epsilon_c - \epsilon)^{1/2}$$

Question Can one study BH evaporation, in the SYK model by coupling it ~~to~~ a bath, where $E(t)$ is driven by dynamics?

Answer



The SYK + Bath coupling term:

$$-\frac{J^2}{2}\int \Phi(\sigma=0,t)\, f'(t)^{1/2}\, dt + S_{bath}[\Phi]$$

The 'bath' is a CFT living in the half plane $\sigma \geq 0$.

Solving for the bath $\Rightarrow$

$$-\frac{J^2}{2}\int dt\; f'(t)^{\frac{1}{2}} \left\langle \Phi(\sigma=0,t)\right\rangle$$

$$\left\langle \Phi(\sigma=0,t)\right\rangle = \int dt'\; G(0,t;0,t')\, f'(t')^{1/2}$$

$\Rightarrow$ NL eqn for $f(t)$

In progress : (A. Gaikwad, A. Kaushal, G. Mandal + SRW)

Related work : Chen, Qi, Zhang, 2003.13147 for eternal BH.

Thank you !