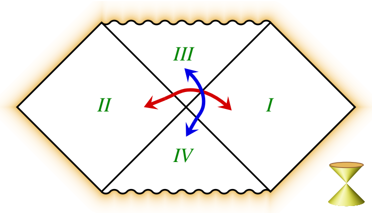
Antipodal Identification in black holes

G. 't Hooft

Abstract: Although string theories and holographic methods may be considered to suggest ideas about physics at the Planck scale, they are not necessarily needed to characterise the fundamental degrees of freedom for a black hole if it is much larger than the Planck size. In that case, conventional quantum field theory, combined with what is known about general relativity, contains enough information to identify the fundamental degrees of freedom for a black hole. And then, we see that the horizon does generate novelties, which can be studied directly.

We find that the entire Penrose diagram for the eternal black hole is the most appropriate background metric to be used. Locally, we only need to consider low energy particles in this background, but then something has to be done when these approach the past and future horizons. Using the gravitational back reaction, one finds that there is only one answer standing out, and it is surprising: regions II and IV have to be identified with the antipodes of regions I and III. An observer passing through a horizon will see particles at the antipodes going backwards in time, and also their energies are inverted. We show that all this can make sense, and leads to beautifully unitary scattering amplitudes for black holes, while surprising new alleys for further study could also be useful for strings and holography.



Gerard 't Hooft

Antipodal Identification in black holes

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Moscow May 7, 2020

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While holographic models, with or without duality, may be useful for creating abstract models, the real physical world may require more rigorous procedures to figure out its deeper laws.

In this talk I explain how more straightforward methods, with fewer ad hoc assumptions, may unveil new, surprising features, especially in the case of black holes.

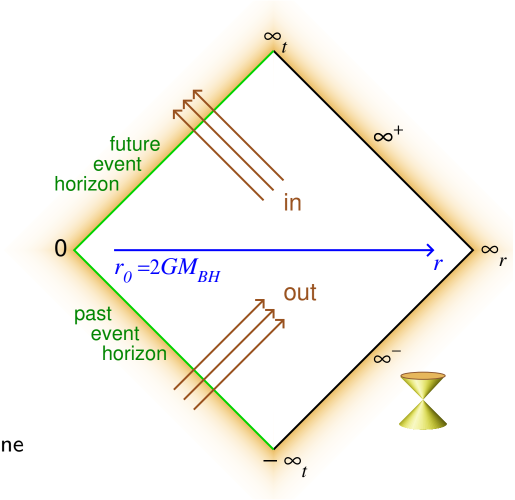
To describe a quantum system, we need:

- A *Hilbert space*, spanned by an orthonormal basis of elementary states,
- An indication as to what these basis elements mean physically: what are the *observables*?
- A unitary *evolution operator* that tells us how the system evolves.
- Unitarity of this operator demands that the states evolve entirely within this Hilbert space, so we must be sure we have the entire Hilbert space needed to describe the system (*"completeness"*)

This can be done for a black hole. The Hilbert space is derived from the system of quantised fields (including grav. fields) in the black hole background. But completeness requires that we modify the *boundary conditions.*

It all begins with the gravitational back reaction.

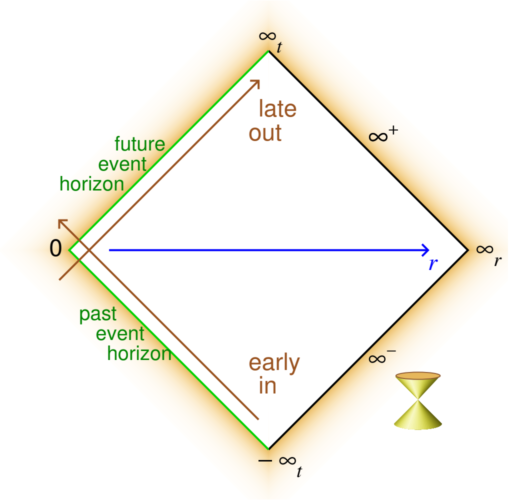
Black Hole



The Penrose diagram is space-time in coordinates such that the lightcone is standing upright everywhere.

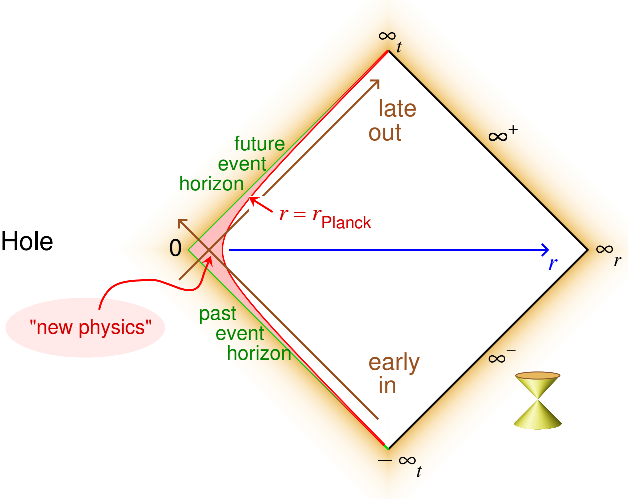
For the Schwarzschild metric, here is the part visible from the outside.

Black Hole



∞_t
late
out
∞^+
future
event
horizon
0
r
∞_r
past
event
horizon
early
in
∞^-
$-\infty_t$

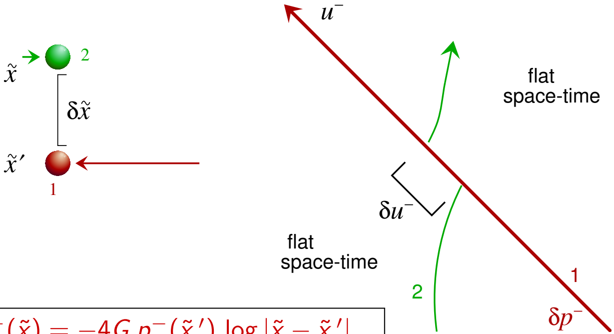
Black Hole



∞_t
late
out
∞^+
future
event
horizon
$r = r_{\text{Planck}}$
0
∞_r
r
"new physics"
past
event
horizon
early
in
∞^-
$-\infty_t$

To understand what happens at the Planck scale, we need the gravitational backreaction:

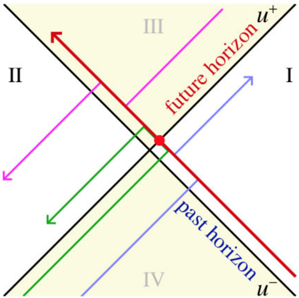
Lorentz boosting the light (or massless) particle gives the *Shapiro time delay* caused by its grav. field:



$$\delta u^-(\tilde{x}) = -4G\, p^-(\tilde{x}\,')\,\log|\tilde{x}-\tilde{x}\,'|\ .$$

P.C. Aichelburg and R.U. Sexl, J. Gen. Rel. Grav. **2** (1971) 303,
W.B. Bonnor, *Commun. Math. Phys.* **13** (1969) 163,
T. Dray and G. 't Hooft, *Nucl. Phys.* **B253** (1985) 173.

The gravitational back reaction: a given in-going particle (red line) causes all out-going particles (colored lines) to shift by the same amount, δu, which only depends on the angular variables (θ, φ), not on u.



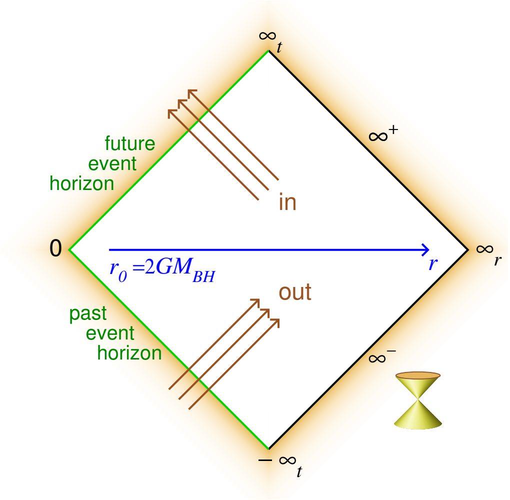
Note sign switches:

the momentum p^- of the particle coming in in region *II*, is *minus* the momentum coming in at *I*.

The variable u^- of the particles going out also switches sign compared to that of the particles going in.
The data are shifted right across the horizon

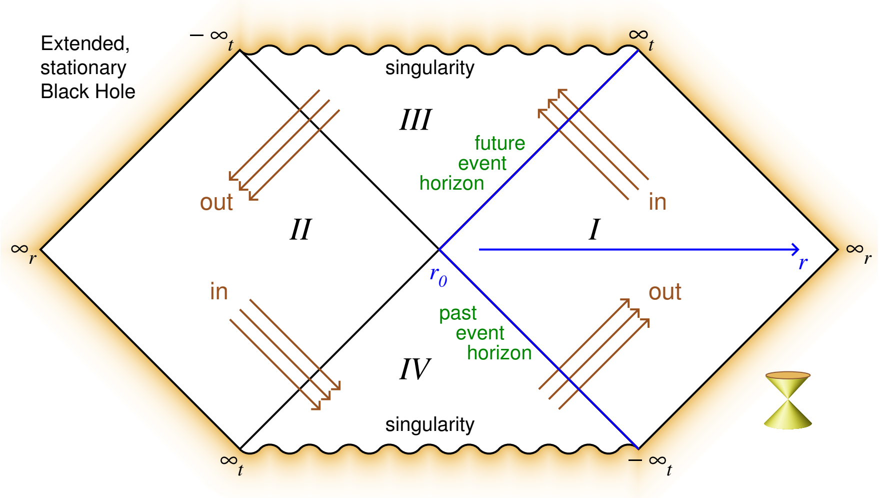
Same with past event horizon, by time reversal!

Black Hole



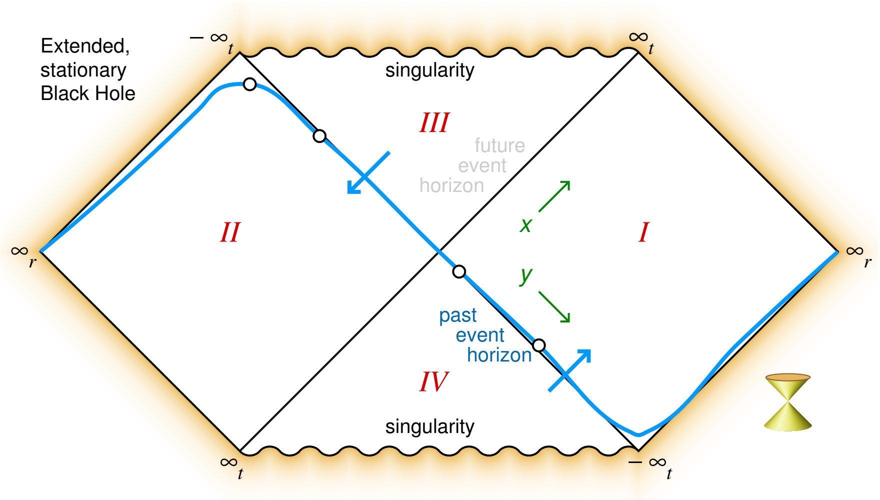
Starting point:

all quantum states of a black hole can be obtained by considering only soft particles (particles that are moving sufficiently slowly so as not to produce significant gravitational deformations), in the extended Penrose diagram for the eternal black hole.

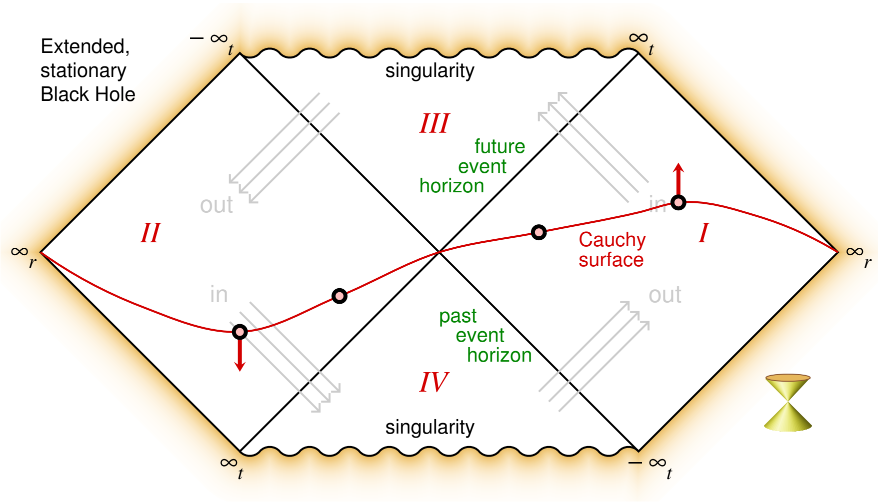


In regions *I* and *II*, *time* runs in opposite directions. Cauchy surfaces must be drawn from ∞_r in region *II* to ∞_r in region *I*.

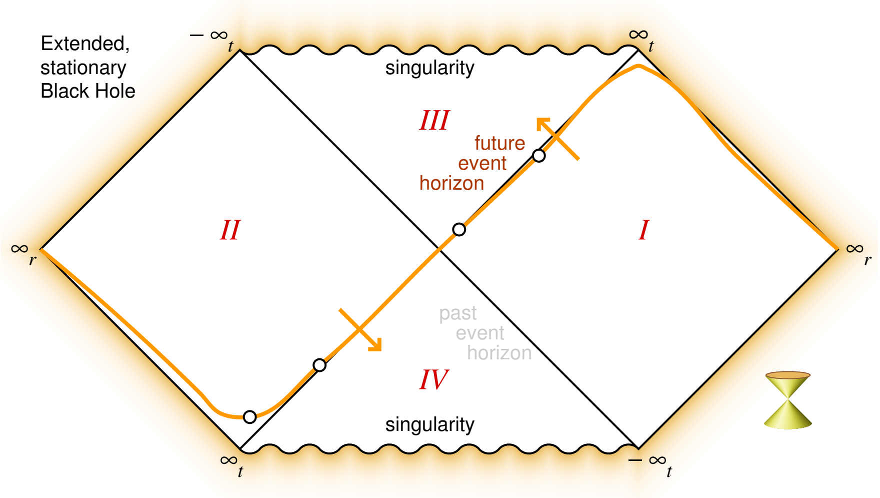
$$\frac{x}{y} = e^{2\tau} \quad ; \qquad \tau = t/4GM_{\rm BH}$$



The regions *I* and *II* are exact copies of one another.
For local observers, the Cauchy surface goes from down to up in both regions. For distant observers, the direction of time switches in region *II*.



All these Cauchy surfaces go through the origin – *so one might expect that regions I and II do not talk to one another*. Regions *II* and *IV* seem to be irrelevant as well. *But :* The gravitational back reaction *shifts* the data on the Cauchy surface across the horizon.



The gravitational shifts caused by the *early* in-particles and the *late* out-particles (see slides 8, 9), grow *exponentially* with time difference.

Hawking thought that, for the out-particles, this shift is irrelevant – *they are thermal anyway!*, so we may ignore the shifts caused by the early in-particles. *The early in-particles are then represented by including the collapsing matter in the metric*.

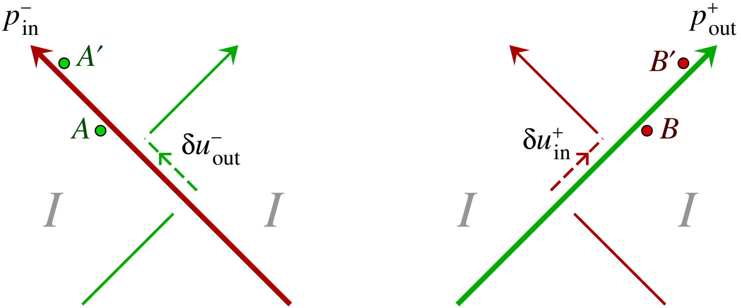
But how do we motivate ignoring the late out-particles?
For quantum mechanical pure states, these out-particles should cause the information going in to shift as well.

Most physicists - illogically - treat the in- particles quite differently from the out-particles.

But the microscopic laws of nature are PCT invariant. We should treat in- particles the same way as the out-particles.

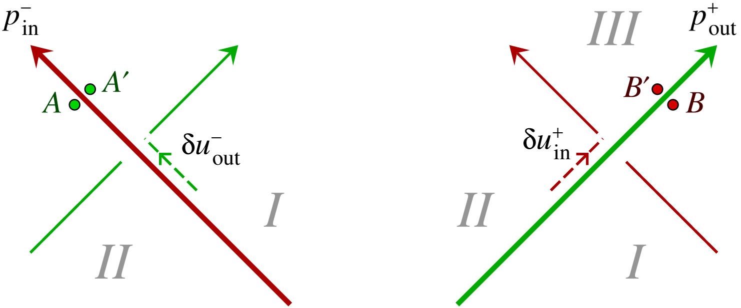
Which space-time points must be regarded as neighbors?

Standard GR formalism says that points are connected such that there is a delta distribution of curvature:



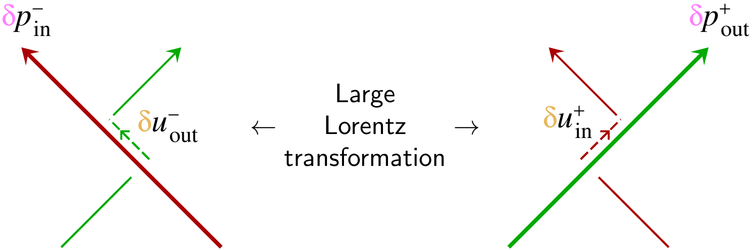
Which space-time points must be regarded as neighbors?

But the firewall transformation treats space-time as locally flat:



The firewall transformation

The positions of *all* out-particles are *identified* with the momenta of *all* in-particles, and *vice versa*, such that the commutation rules are preserved: $[u^{\pm}, p^{\mp}] = i$.



Thus, we count the out-particles not independently from the in-particles; instead, our states in Hilbert space are defined *either* by counting all in-particles, *or* all out-particles.

These states are related by the BH scattering matrix.

According to conventional physics, the in-particles leave their *footprints* on the out-particles. Now, we proposed to *identify* the out-particles with the in=particles' footprints!

We thus remove the effect of the early in- particles as well as the late out-particles, by re-defining the metric beyond them, which we can't see directly anyway. This is a cut-and-paste procedure for the metric. To be referred to as the *firewall transformation*. This procedure removes the firewalls - thus solving the "firewall problem".

Our cut-and-paste procedure totally changes the metric behind the horizons – there are no particles anymore that can cause the metric to depart from the eternal Penrose diagram.

This is why we decided to start from the empty / eternal Penrose diagram.

This procedure forces the in-particles to impart their information on the out-particles, by shifting their wave functions:
momentum in = position out and vice versa: $x \leftrightarrow p$.

The best way to count the particles is: count a particle
as in-going if $|p^-_{\rm in}| < M_{\rm Planck}$ and $|u^+_{\rm in} > L_{\rm Planck}$, and
as out-going if $|p^+_{\rm out}| < M_{\rm Planck}$ and $|u^-_{\rm out}| > L_{\rm Planck}$.

Then these particles always keep their momenta small, so that they do not cause space-time curvature, while their positions are always greater than $L_{\rm Planck}$ so that "ordinary physics" should apply to them.

However, replacing momentum by position has a subtle effect on *space-time topology.*

Thus to describe the evolution of *all* quantum states in the Schwarzschild geometry, we can now limit ourselves to the *maximal analytic extension* for the *eternal black hole*. And now we *only* allow soft particles in this eternal Penrose diagram.

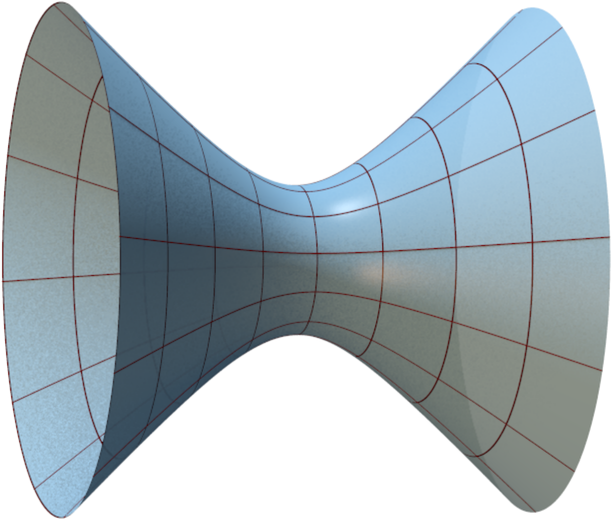
But there is still a problem:

In order to represent the entire universe exactly once in the particle wave functions, we *must* divide the universe in two, while keeping smoothness.

Only one solution: identify the points (r, t, θ, ϕ) in region *I* with $r, -t, \pi - \theta, \phi + \pi)$ in region *II*. The other regions, *III* and *IV*, undergo a similar identification, but they play no significant role.

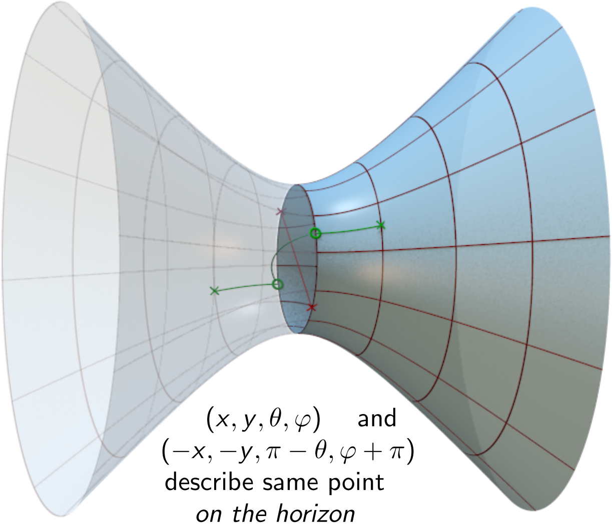
The antipodal identification:

II



I

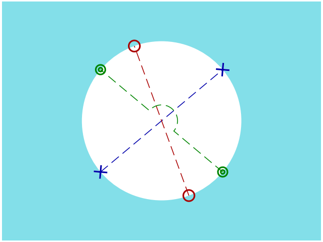
Regions *I* and *II* describe *different* points in the *same* universe.



(x, y, θ, φ) and
$(-x, -y, \pi - \theta, \varphi + \pi)$
describe same point
on the horizon

Black emptiness: blue regions are the accessible part of space-time; dotted lines indicate identification.

The white sphere within is *not* part of space-time. Call it a 'vacuole'.



At given time t, the black hole is a 3-dimensional vacuole. The entire life cycle of a black hole is a vacuole in 4-d Minkowski space-time: *an instanton*

N.Gaddam, O.Papadoulaki, P.Betzios (former Utrecht PhD students)

Space coordinates change sign at the identified points

– *and also time changes sign*

(Note: time stands still at the horizon itself).

This scheme only requires CPT symmetry, no other combinations of C, P and/or T.

Our philosophy is to use only region I to describe all the physics, for all transverse angles, while it connects smoothly points to their antipodes on the horizon. This then is a natural boundary condition, but we do have to take care, because near the horizon, particles connect to antiparticles *going backwards in time!*.

Question asked by Witten (personal communication):

How can that be, how do their quantum fields connect?

If time reverses, then

$$\phi(x,t) \leftrightarrow \phi(-x,-t), \qquad \text{but the commutation rules read}$$

$$[\phi(\vec{x},t),\, \dot{\phi}(\vec{x}\,',t)] = i\delta^3(\vec{x}-\vec{x}\,')\ .$$

How does i move to $-i$ across the boundary?

Indeed, time $\leftrightarrow -$ time, therefore also: Energy $\leftrightarrow -$ energy,

or more precisely Energy $\leftrightarrow (\Lambda -$ Energy), where Λ is the *largest amount of energy possible*.

Hamiltonian not only has a lowest eigenvalue ("vacuum state") but also a highest eigen value ("antivacuum").

This also happens in some advanced theories of quantum mechanics.

Should the 'antivacuum" not be highly curved?

No! We had decided to ignore the gravitational effect of all these "soft" particles".

The gravitational effects of particles passing through the horizon has now been accounted for by modifying the metric of distant past and distant future (the collapse and the final complete explosion are replaced by the eternal metric).

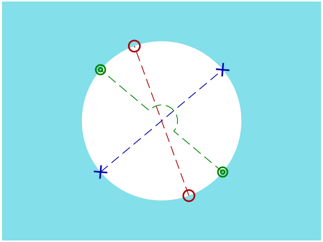
Many apparent paradoxes may be solved this way.

Important conclusion: "holography" and "AdS - CFT" do not give you the correct answers if one doesn't know what one is doing. They are not fool-proof.

Opening up (collapse) and closing in (final evaporation) of a black hole:

Black emptiness: blue regions are the accessible part of space-time; dotted lines indicate identification.

The white sphere within is *not* part of space-time. Call it a 'vacuole'.



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Entanglement of Hawking particles

The hartle-Hawking state,

$$|\, HH \,\rangle \;= C \sum_{E,n} e^{-\frac{1}{2}\beta E}\, |E, n\rangle_I \; |\Lambda - E,\, n\rangle_{II}$$

where II = antipode of I ,

is now a *pure* quantum state, where regions I and II are entangled.
It is *not* a thermal state.

Only if we do not look at states II, the states in I seem to form a perfect thermal mixture.

Only those General coordinate transformations are allowed that are one-to-one, so that no doubling takes place for the asymptotic regions.

Philosophy: what is needed is the evolution law over small time stretches.

If we have that, we can integrate the equations to get the large-time behaviour. Not the other way around!

During the small time interval, the black hole may be considered as eternal. This is why we consider the Penrose diagram of the stationary black hole.

The *imploding matter* and the *final explosive evaporation* do not play a role here. They are left out. Later we worry about how they can occur at other epochs of time.

During this interval we consider only excitations due to soft particles (these are the particles that do not (yet) affect the space-time metric as their grav. fields are weak).

Soft particles are soft for the local, freely falling observer.

THANK YOU

Expand in Spherical harmonics:

$$u^{\pm}(\Omega) = \sum_{\ell,m} u_{\ell m} Y_{\ell m}(\Omega) \ , \qquad p^{\pm}(\Omega) = \sum_{\ell,m} p^{\pm}_{\ell m} Y_{\ell m}(\Omega) \ ;$$

$$[u^{\pm}(\Omega),\, p^{\mp}(\Omega')] = i\delta^2(\Omega,\, \Omega') \ , \qquad [u^{\pm}_{\ell m},\, p^{\mp}_{\ell' m'}] = i\delta_{\ell\ell'}\delta_{mm'} \ ;$$

$$u^{-}_{\rm out} = \frac{8\pi G}{\ell^2+\ell+1} p^{-}_{\rm in} \ , \qquad u^{+}_{\rm in} = -\frac{8\pi G}{\ell^2+\ell+1} p^{+}_{\rm out} \ ,$$

$p^{\pm}_{\ell m}$ = total momentum in of ${}^{\rm out}_{\rm in}$-particles in (ℓ, m)-wave ,

$u^{\pm}_{\ell m}$ = (ℓ, m)-component of c.m. position of ${}^{\rm in}_{\rm out}$-particles .

Because we have linear equations, all different ℓ, m waves decouple, and for one (ℓ, m)-mode we have just the variables $u^{\pm}$ and $p^{\pm}$. They represent only one independent coordinate u^{+}, with $p^{-} = -i\partial/\partial u^{+}$.

The evolution law applies to every (ℓ, m) mode separately (in our approximation, there is no cross-talk). In each (ℓ, m) mode, the energy κ (seen by the distant observer) is separately conserved — but only when regions I and II are taken together: the states are *entangled* over I and II. So there is one wave function $\psi_\sigma(|u^+|)$ where $\sigma = I$ or II. The out-states are obtained from the in-states by Fourier transformation, which is unitary by construction:

The evolution equation, at given energy κ, is:

$$\psi^{\rm out} = \begin{pmatrix} F_+ & F_- \\ F_- & F_+ \end{pmatrix} e^{-i\kappa \log\left(8\pi G/(\ell^2+\ell+1)\right)} \psi^{\rm in} , \quad \text{where}$$

$$F_\pm(\kappa) = \int_0^\infty \frac{{\rm d}y}{y} y^{\frac{1}{2}-i\kappa} e^{\mp iy} = \Gamma(\tfrac{1}{2} - i\kappa)\, e^{\mp \frac{i\pi}{4} \mp \frac{\pi}{2}\kappa} .$$

Matrix $\begin{pmatrix} F_+ & F_- \\ F_- & F_+ \end{pmatrix}$ is unitary: $F_+F_-^* = -F_-F_+^*$ and $|F_+|^2 + |F_-|^2 = 1$.

The integration kernel vanishes for large values of the argument, so this interaction is approximately local in time. The (ℓ, m) waves do not spend much time in the black hole. the Hartle-Hawking particles do stay there forever.

The operators $u(\theta, \varphi)$ and $p(\theta, \varphi)$ change signs if we go from pode to antipode. therefore, only odd values of ℓ contribute. This is counter-intuitive, but remember that u and p are **not second-quantized**.

Note that the local freely falling observer will find it obvious that u and p switch sign if we interchange regions *I* and *II*.

The identification of our position- or momentum-states as elements of the Fock space of the Standard Model is highly non-trivial

(and needs to be studied further.)

The $\ell = 0$ "dust shell" is not a legal state here. One must consider all its myriads of dust particles separately.
The *energy* is defined in the regions *I* and *II* separately, and *that* can be in odd and even ℓ states.

Our procedure it totally invariant under time translations.
Total energy is exactly conserved,

Not conserved: black hole mass =
total energy minus energies of all distant particles.

The particles in their (ℓ, m) modes do not spend much time in the black hole, but the Hartle-Hawking background is there eternally.

During formation and during final evaporation, more particles are outside, so the black hole starts out and ends up very light.

With our, calculable and unitary, evolution operator the system became totally internally consistent.

But the relation between QFT Fock states and our spherical waves of momentum distribution requires further study.

Conjecture: high ℓ values distinguish different SM particles.

The Fourier transform in $x,\ p$ space is non-local:

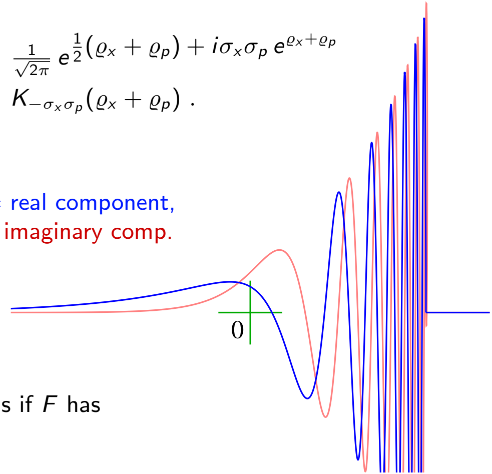
$$\langle x|p\rangle = \frac{1}{\sqrt{2\pi}}\, e^{ipx}$$

But if we write $x = \sigma_x\, e^{\varrho_x}$ and $p = \sigma_p\, e^{\varrho_p}$, where σ_x and σ_p are signs $\pm$, then the relation becomes:

$$\begin{aligned} \langle \varrho_x, \sigma_x | \varrho_p, \sigma_p \rangle &= \frac{1}{\sqrt{2\pi}}\, e^{\frac{1}{2}(\varrho_x + \varrho_p) + i\sigma_x\sigma_p\, e^{\varrho_x+\varrho_p}} \\ &= K_{-\sigma_x\sigma_p}(\varrho_x + \varrho_p)\ . \end{aligned}$$

Blue = real component,
Red = imaginary comp.

$K_+(x)$:



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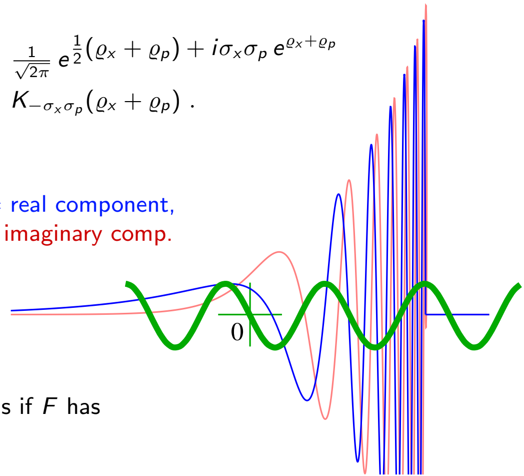

In practice it will appear as if F has a finite support.

So, a fast particle moving in will shift all particles on their way out; some of them will move from region *I* to *II* or back.

This is why we cannot ignore the particles of region II .

Furthermore, region *II* is an *exact copy* of region *I*. It has asymptotic regions. Therefore, it represents an entire universe.

What universe is that ?

Only one answer makes sense:

It is the other side of the same black hole.

This is a topological twist without singularities,

because nowhere in regions I or II, the radius is less than $2GM_{\rm BH}$.

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