Normal charge densities in quantum critical superfluids

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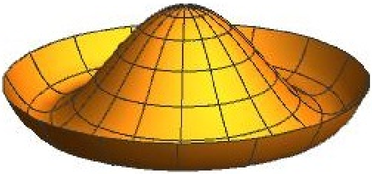
Plan of the talk

1. Brief review of superfluid effective theories (hydro).
2. Brief review of holographic superfluids.
3. Holographic computation of the normal density and main results.
4. Link to experiments on high T_c superconductors.

- Superfluidity arises from the spontaneous breaking of a U(1) symmetry – the condensate transports mass/charge without friction.

- The order parameter can be modeled by a complex scalar with Mexican hat potential, which acquires a vev.

- The vev of the condensate is given by the modulus, the phase is a gapless mode (no energy cost, linear dispersion relation) – the Goldstone boson.



- The long-wavelength, low-energy dynamics of superfluids are well-described by the Landau-Tisza hydrodynamic model.

- Consistent coupling of the Goldstone mode (superfluid phase) to the conserved densities of the system (external sources off):

$$\partial_\mu T^{\mu\nu} = 0\,, \quad \partial_\mu j^\mu = 0\,, \quad u^\mu \partial_\mu \varphi = \mu\,.$$

- Modified constitutive relations and thermodynamics compared to ordinary hydrodynamics (ideal order through this talk)

$$T^{\mu\nu} = (\epsilon_n + P) u^\mu u^\nu + P\eta^{\mu\nu} + \frac{\rho_s}{\mu}\partial^\mu\varphi\partial^\nu\varphi\,, \quad j^\mu = \rho_n u^\mu + \frac{\rho_s}{\mu}\partial^\mu\varphi\,,$$

$$\epsilon_n + P = Ts + \rho_n\mu\,, \quad \rho = \rho_n + \rho_s\,,$$

$$dP = sdT + \rho d\mu - \frac{\rho_s}{2\mu} d(\partial_\nu\varphi\partial^\nu\varphi + \mu^2)\,.$$

- Important consequences on the spectrum of hydrodynamic modes: apparition of a superfluid sound mode mixing the Goldstone and the usual ‘charge diffusion’ mode:

$$\omega_i = \pm c_s^2 q + O(q^2)$$

- Superfluid second sound mode:

$$c_s^2 = c_2^2 = \left(\frac{s}{\rho}\right)^2 \frac{\rho_s}{(sT + \mu\rho_n)(\partial[s/\rho]/\partial T)_\mu} .$$

- Superfluid fourth sound (holding the normal component still)

$$c_s^2 = c_4^2 = \frac{\rho_s}{\mu\left(\frac{\partial\rho}{\partial\mu}\right)_s}.$$

- The normal and superfluid densities (IR parameters) are not related in a simple way to the charge residing in the condensate (UV parameter).

- For instance, in ^{4}He, the condensate contains less than 10% of the total number of atoms.

- The normal density can be computed by a weakly coupled calculation [Chapter 2, Schmitt'15] assuming Galilean/Lorentz boosts:

$$ {}^4\mathrm{He}: \qquad \rho_s(T=0) = \rho(T=0)\,, \quad \rho_n(T=0) = 0 $$

  At $T = 0$, the system is completely superfluid and the Goldstone (superfluid 'phonon') governs its low-energy dynamics.

- At small T:

$$ \rho_n(T) = \frac{2\pi^2 T^4}{45 c^5} = \frac{s_{ph} T}{c^2} $$

In more details (not discussed during the talk)

- Assume a linear dispersion relation for the phonon:

$$\epsilon_q = cq$$

(Warning: as we shall see, $c_s \neq c$, so this actually assuming some underlying Galilean/Lorentzian boost symmetry with c the IR speed of light – see later)

- Assume bose statistics, and compute the phonon pressure ($d = 3$):

$$P_{ph} = -T \int \frac{d^3\mathbf{q}}{(2\pi)^3} \ln \left(\underbrace{1 - e^{-\epsilon_q/T}}_{f(\epsilon_q)} \right) = \frac{\pi^2 T^4}{90 c^3}$$

- The phonon entropy is

$$s_{ph} = \frac{\partial P_{ph}}{\partial T} = \frac{2\pi^2 T^3}{45 c^3}$$

- Now let's compute the normal density in the frame where the superfluid is at rest. The momentum density

$$\mathbf{g} = \rho_n \mathbf{v_n} + \rho_s \mathbf{v_s} \quad \Rightarrow \quad \mathbf{g} = \rho_n \mathbf{w}\,, \quad \mathbf{w} = \mathbf{v_n} - \mathbf{v_s}$$

- The momentum density of the normal density can also be written

$$\rho_n \mathbf{w} = \int \frac{d^3\mathbf{q}}{(2\pi)^3} \mathbf{q} f\left(\epsilon_q - \mathbf{q}\cdot\mathbf{w}\right)$$

- This leads to

$$\rho_n(\mathbf{w} \to 0) = \frac{2\pi^2 T^4}{45 c^5} = \frac{s_{ph} T}{c^2}$$

- We can plug these results in the expressions for the (non-relativistic) sound modes.

- Normal, 'first' sound

$$c_1^2 = \frac{\partial P}{\partial \rho} \underset{T \to 0}{\longrightarrow} c$$

- Superfluid second sound mode:

$$c_2^2 = \frac{s^2 T \rho_s}{\rho c_V \rho_n} = \frac{c^2}{3} = \frac{c^2}{d} \,.$$

This last result is the Landau prediction for the low temperature behaviour of second sound in d spatial dimensions.

- These results can be recovered more rigorously in the relativistic case thanks to Son's universal Quantum Effective Action formalism for relativistic superfluids [Son'02]

$$\mathcal{L} = P(X)\,, \qquad X = \partial_\mu\varphi\partial^\mu\varphi$$

where P turns out to be the thermodynamic pressure.

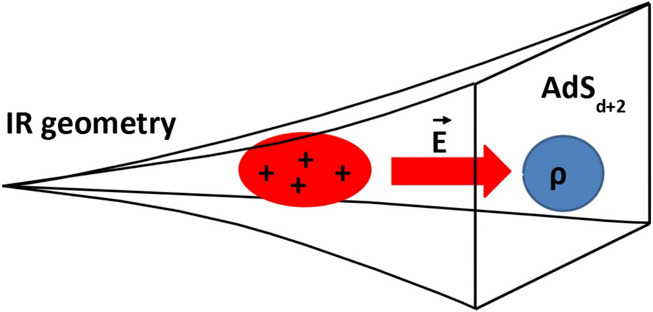
- Generalization to small nonzero temperature and addition of the normal fluid velocity and density by [Nicolis'11].

- Computation of the temperature dependence of the normal density from [Delacrétaz, Hofman and Mathys'19]

$$\rho_n = \frac{sT}{\mu c_{ir}^2}(1 - c_{ir}^2) \quad \text{(private communication)}$$

where c_{ir} is the effective light velocity in the IR.

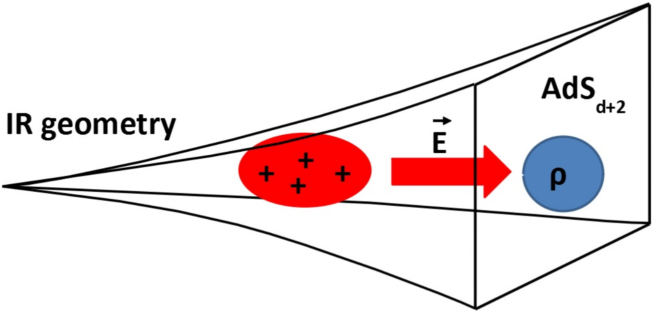
- To go beyond the EFT, a microscopic model is needed.
- In BCS superconductors, the normal density is computed to be exponentially suppressed at low temperatures.
- Other data points can be provided using holographic models of superfluids.



- A superfluid can be realized in the boundary by spontaneously breaking a U(1) symmetry. This was originally done [Gubser'08, Hartnoll, Herzog & Horowitz'08] by coupling a charged, complex scalar to gravity

$$S = \int d^{d+2}x\sqrt{-g}\left[R - \frac{1}{4}F^2 - |D\eta|^2 - V(|\eta|)\right].$$

- At low temperatures, η condenses close to the horizon, leading to a spacetime with a lump of charged scalar field sitting outside the horizon.



$$S = \int d^{d+2}x\sqrt{-g}\left[R - \frac{1}{4}F^2 - |D\eta|^2 - V(|\eta|)\right].$$

- The original solutions constructed by [Hartnoll, Herzog & Horowitz'08] were shown to obey the Landau-Tisza model of superfluid hydrodynamics [Herzog & Yarom'09, Sonner & Withers'10, Herzog & al'11, Bhattacharya & al'11].

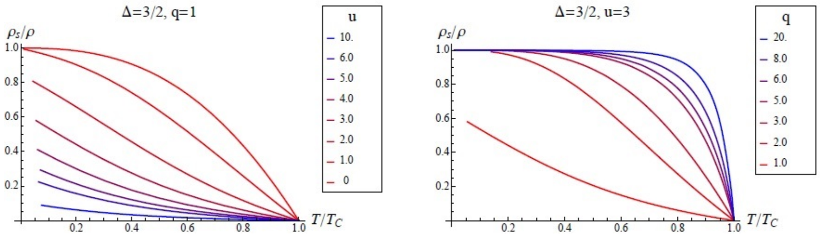
- We wish to compute the normal and superfluid densities in holographic superfluids. For this, we need to extract the one-point functions

$$\langle T^{tx}\rangle = (sT + \mu\rho_n)\, u_x + \frac{\rho_s}{\mu}\partial_x\varphi\,, \quad \langle j^x\rangle = \rho_n u_x + \frac{\rho_s}{\mu}\partial_x\varphi$$

together with $\partial_x\varphi$.

- This can be done by solving the coupled perturbation equations for δa_x, δg_{tx} at $\omega = q = 0$, which give access to the required vevs as well as $\partial_x\varphi$ after a gauge transformation, [Herzog & Yarom'09].

In $d = 3$, [Herzog & Yarom'09] found for instance



The $(q = 1, u = 0, 1)$ and $(u = 3, q = 2 - 20)$ do asymptote to unity as $T \to 0$, but the others do not and there $\rho_s(T = 0) < \rho(T = 0)$.

Why ?

Our strategy:

- At low frequencies, the hydro prediction for the current retarded Green's function at ideal order is

$$\omega \to 0 : \qquad G^R_{JJ}(\omega) = \frac{\rho_n^2}{sT + \mu\rho_n} + \frac{\rho_s}{\mu} + O(\omega)$$

- Holographically,

$$G^R_{JJ}(\omega) = \frac{\delta a_x^{(1)}}{\delta a_x^{(0)}}\,, \qquad \delta a_x = \delta a_x^{(0)} + u\delta a_x^{(1)} + O(u^2)$$

- The $\omega = 0$ term in $G^R_{JJ}(\omega)$ is given by the solution to the $\omega = 0$ δa_x eom which is regular at the horizon, see eg [Davison, Goutéraux & Hartnoll'15].

- So we will compute this regular solution in a small T expansion, which should then give access to ρ_n.

Warm-up: no condensate.

$$\left[C^{d/2-1}\sqrt{\frac{D}{B}}A_t^2\left(1-\frac{sT}{A_tR}\right)\left(\frac{\delta a_{\hat{x}}}{A_t}\right)' + sT\frac{D}{C}\left(\frac{\delta a_{\hat{x}}}{A_t}\right)\right]' = 0$$

$$\Big(ds^2 = -D(r)dt^2 + B(r)dr^2 + C(r)(dx^2+dy^2)\Big)$$

$$R(r) \equiv -\frac{C^{d/2}A_t'}{\sqrt{BD}}\,, \quad R(r) = R(r_h) = \rho$$

This suggests at we can expand at low T in powers of sT:

$$\mathcal{A} \equiv \frac{\mu}{a_{\hat{x}}^{(0)}}\frac{a_{\hat{x}}}{A_t} = \mathcal{A}_0 + (sT)\mathcal{A}_1 + (sT)^2\mathcal{A}_2 + \ldots$$

We wish to solve order by order imposing regularity at the horizon.

We find

$$\mathcal{A}_0 = 1$$

$$\mathcal{A}_1 = -\int_0^r \sqrt{\frac{B}{D}} \frac{1}{C^{d/2-1} A_t^2} \left[\frac{D}{C} + c_1\right] dr'$$

$$\mathcal{A}_2 = \int_0^r \sqrt{\frac{B}{D}} \frac{1}{C^{d/2-1} A_t^2} \left[\frac{D}{C} + c_2\right] dr' \int_0^{r'} \sqrt{\frac{B}{D}} \frac{1}{C^{d/2-1} A_t^2} \left[\frac{D}{C} + c_1\right] d\tilde{r}$$
$$- \int_0^r \sqrt{\frac{B}{D}} \frac{1}{C^{d/2-1} A_t^3 R} \left[\frac{D}{C} + c_1\right] dr'$$

$c_{1,2}$ must be fixed so that $\lim_{r_h} \delta a_x(r) \sim \lim_{r_h} A_t \mathcal{A}$ is regular. However, it is not guaranteed that it is consistent to do so order by order in sT, rather than directly on the resummed $\mathcal{A}$.

In the case at hand, it turns out to be consistent.

This leads to

$$Z \equiv \lim_{\omega \to 0} Re\left[G^R_{J_x J_x}(\omega, q=0)\right] = \frac{\rho}{\mu} - \frac{sT}{\mu^2} + O(sT)^2$$

consistent with the hydrodynamic expectation

$$Z = \frac{\rho_n}{sT + \mu\rho} \underset{sT \to 0}{\longrightarrow} \frac{\rho}{\mu} - \frac{sT}{\mu^2} + \dots$$

We can iterate at higher orders in sT ($\mathcal{A}_{i \geq 2}$) and the agreement persists.

Actually, in this case, a closed for expression had already been found [Davison, Goutéraux & Hartnoll'15]:

$$\delta a_x^{reg}(r) = \frac{sT + \rho A_t(r)}{sT + \mu\rho}$$

By expanding in sT, we recover the same results.

Now with a condensate:

$$\left[C^{d/2-1}\sqrt{\frac{D}{B}}A_t^2\left(1-\frac{sT}{A_tR}\right)\left(\frac{\delta a_{\hat{x}}}{A_t}\right)'+sT\frac{D}{C}\left(\frac{\delta a_{\hat{x}}}{A_t}\right)\right]'=\boxed{-(sT)\frac{2q^2\eta^2C^{d-1}A_t^2}{R^2}\left(\frac{\delta a_{\hat{x}}}{A_t}\right)'}$$

$$\left(ds^2=-D(r)dt^2+B(r)dr^2+C(r)(dx^2+dy^2)\right)$$

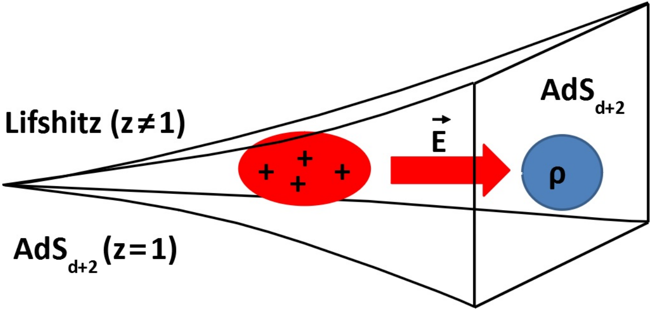
$$R(r)\equiv-\frac{C^{d/2}A_t'}{\sqrt{BD}}\,,\quad \lim_{r\to r_h}R=\rho_{in}\,,\quad \lim_{u\to\infty}R=\rho$$

No closed form expression available, we can only use the expansion in sT (or numerics).

We find

$$
\begin{aligned}
\mathcal{A}_0 &= 1 \\
\mathcal{A}_1 &= -\int_0^r \sqrt{\frac{B}{D}}\frac{1}{C^{d/2-1}A_t^2}\left[\frac{D}{C}+c_1\right]\,dr' \\
\mathcal{A}_2 &= \int_0^r \sqrt{\frac{B}{D}}\frac{1}{C^{d/2-1}A_t^2}\left[\frac{D}{C}+c_2\right]dr'\int_0^{r'} \sqrt{\frac{B}{D}}\frac{1}{C^{d/2-1}A_t^2}\left[\frac{D}{C}+c_1\right]d\tilde{r} \\
&\quad -\int_0^r \sqrt{\frac{B}{D}}\frac{1}{C^{d/2-1}A_t^3 R}\left[\frac{D}{C}+c_1\right]dr' \\
&\quad \boxed{+\int_0^r \sqrt{\frac{B}{D}}\frac{1}{C^{d/2-1}A_t^2}dr'\int_{r_h}^{r'} \sqrt{\frac{B}{D}}\frac{2q^2\eta^2 C^{d/2}}{R^2}\left[\frac{D}{C}+c_1\right]d\tilde{r}}\,.
\end{aligned}
$$

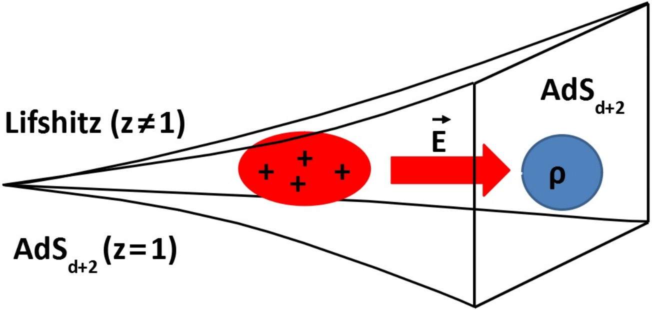
- At second order, whether the c_i can consistently be set to zero depends on the last integral in $\mathcal{A}_2$
  1. If the integral converges in the IR as $T \to 0$ ($r_h \to +\infty$), we can consistently set $c_2 = 0$.
  2. If the integral diverges in the IR, we need to keep both c_1 and $c_2 \neq 0$ to find a regular limit.
- This reveals that $\lim_{T\to 0} \rho_n(T)$ is controlled by the competition between two deformations of the groundstate, particle-hole symmetry breaking or $U(1)$ symmetry breaking ($\sim \eta^2/R^2$).
- So we need to understand the $T = 0$ groundstates of our model.



- By considering a quartic potential, [Gubser & Nellore'09, Horowitz & Roberts'09] showed that two types of IR geometries were allowed:

$$ds^2_{IR} = -\frac{L_t^2}{r^{2z}}dt^2 + \frac{L_{IR}^2 dr^2 + L_x^2 d\vec{x}^2}{r^2}$$

- Whether the AdS$_{d+2}$ or Lifshitz groundstate is selected depends on whether the gauge field is irrelevant at $T = 0$ close to the horizon or not [Gubser & Nellore'09].



$$\mathrm{AdS}_{d+2} \qquad ds^2_{z=1} = -\frac{L_t^2}{r^2}dt^2 + \frac{L_{IR}^2 dr^2 + L_x^2 d\vec{x}^2}{r^2}$$

- The time component of the gauge field in the IR AdS$_{d+2}$ geometry is a mode which backreacts on the metric as

$$\delta(ds^2) = ds^2_{z=1}\left(1 + \# r^\beta + \dots\right)$$

- $\beta < 0$: irrelevant mode, the IR AdS$_{d+2}$ is RG-stable.
- $\beta > 0$: relevant mode, the IR AdS$_{d+2}$ is RG-unstable. The flow is driven to the Lifshitz geometry with $z \neq 1$.

Return to the integral in $\mathcal{A}_2$:

- $1 \leq z < d+2$: condensate-dominated

$$\rho_n(T) = \frac{sT}{\mu c_{ir}^2}(1 - c_{ir}^2) + \ldots\,, \quad c_{ir} = \frac{L_t}{L_x} r_h^{1-z}$$

  This is the EFT (z=1) result, generalized for $1 \leq z < d+2$.

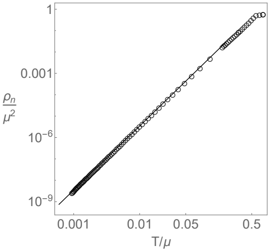
- $z > d+2$: particle-hole breaking dominated

$$\rho_n(T) = \rho_n^{(0)} + \ldots$$

  For sufficiently large Lifshitz exponent, the normal density no longer vanishes at $T = 0$.

- For all z, $\rho_{in}(T = 0) = 0$.

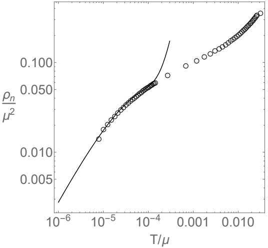
Numerical results for the low temperature behavior of ρ_n in $d = 2$



$z = 1 < d + 2$

$\rho_n \simeq \frac{sT}{\mu c_{ir}^2}(1 - c_{ir}^2) \sim T^3$

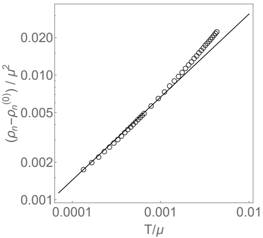
$s \sim T^2,\ c_{ir} \sim T^0$



$z = 2 < d + 2$

$\rho_n \simeq \frac{sT}{\mu c_{ir}^2}(1 - c_{ir}^2) \sim T$

$s \sim T,\ c_{ir} \sim T^{1/2}$



$z = 12 > d + 2$

$\rho_n \simeq \rho_n^{(0)} + \# T^{2/3}$

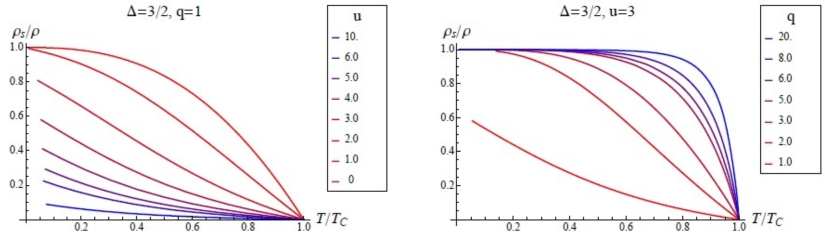
Summary so far:

- $\rho_n(T = 0) = 0$ in holographic phases where the condensate dominates over particle-hole breaking.

- The calculation reproduces the expected EFT result for phases with emergent Lorentz symmetry, and can be extended to Lifshitz-invariant phases with $z < d + 2$.

- However, for $z > d + 2$, the normal density is non-vanishing. Unrelated to the presence of a charged extremal horizon.

- Explains previous observations in earlier literature [Herzog & Yarom].



Open questions:

- In Lifshitz-invariant phases with $z < d + 2$:

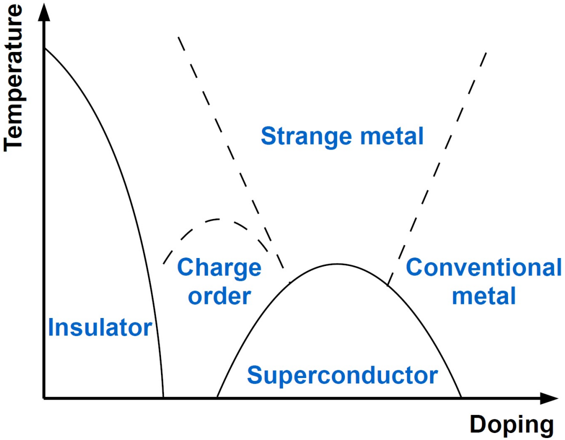
$$\rho_n(T) = \frac{1 - c_{ir}^2}{c_{ir}^2} \frac{sT}{\mu} + \dots, \quad c_{ir} \equiv L_t / L_x r_h^{1-z} \sim T^{1-1/z}$$

- This directly implies that the superfluid second sound mode vanishes as $c_2^2 \sim T^{2-2/z}$.

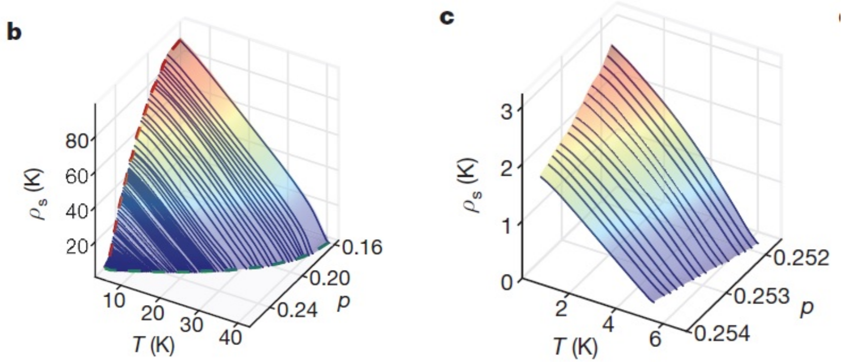
- But the starting point of the Quantum Effective Action is that the Goldstone governs the dynamics even at $T = 0$

- Generalization to Lifshitz phases?

Are there other systems that feature a non-vanishing normal density? Maybe...

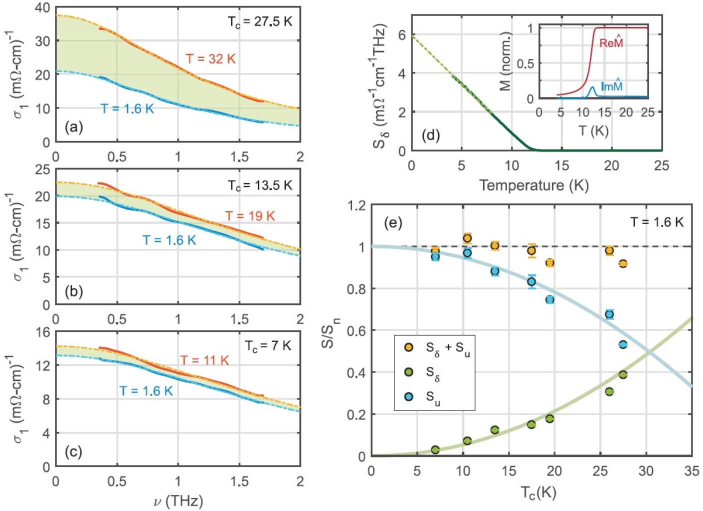


- In 2016, Bozovic et al. published a study of the superfluid density in very overdoped LSCO films.
- They belong to the family of cuprate superconductors which fall outside the BCS paradigm.



They reported two suprising features

- The superfluid density is anomalously low.
- It has a linear behaviour with temperature, while standard 'dirty' BCS theory predicts T^2.



- Then [Mahmood et al'18] measured the ac conductivity of these films and reported a very modest loss of spectral weight below T_c.
- They conclude that this implies that $\rho_n(T=0) \equiv \rho_n^{(0)} \neq 0$, once again at odds with BCS.

- To capture this behavior, consider a more general action [Adams, Crampton, Sonner & Withers'12]

$$S = \int d^{d+2}x\sqrt{-g}\left[R - \frac{Z(\phi)}{4}F^2 - |D\eta|^2 - \frac{1}{2}(\partial\phi)^2 - V(\phi, |\eta|)\right].$$

- We also want to consider more general groundstates

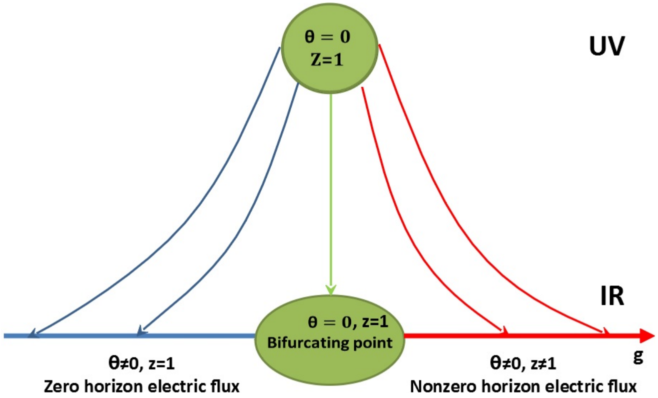
$$ds^2_{IR} = r^{\frac{2}{d}\theta}\left[-\frac{L_t^2}{r^{2z}}dt^2 + \frac{L_{IR}^2 dr^2 + L_x^2 d\vec{x}^2}{r^2}\right]$$

- They violate hyperscaling [Charmousis, Goutéraux et al'10, Goutéraux & Kiritsis'11, Huijse, Sachdev & Swingle'11]

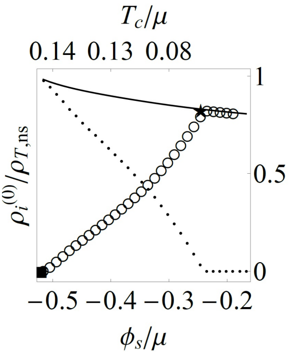
$$s \sim T^{\frac{d-\theta}{z}}$$

$$S = \int d^{d+2}x\sqrt{-g}\left[R - \frac{Z(\phi)}{4}F^2 - |D\eta|^2 - \frac{1}{2}(\partial\phi)^2 - V(\phi, |\eta|)\right].$$

- This holographic setup realizes the following scenario:



- The condensate always acts as an irrelevant deformation of the normal groundstate.



- Results qualitatively very similar to [Bozovic & al'16, Mahmood & al'18].
- Consequence of the quantum critical properties of the **underlying normal groundstate**.
- Suggests that in real systems, whether $\rho_n \to 0$ or not depends on the spectrum of deformations around the groundstates / the nature of interactions.

Thank you!