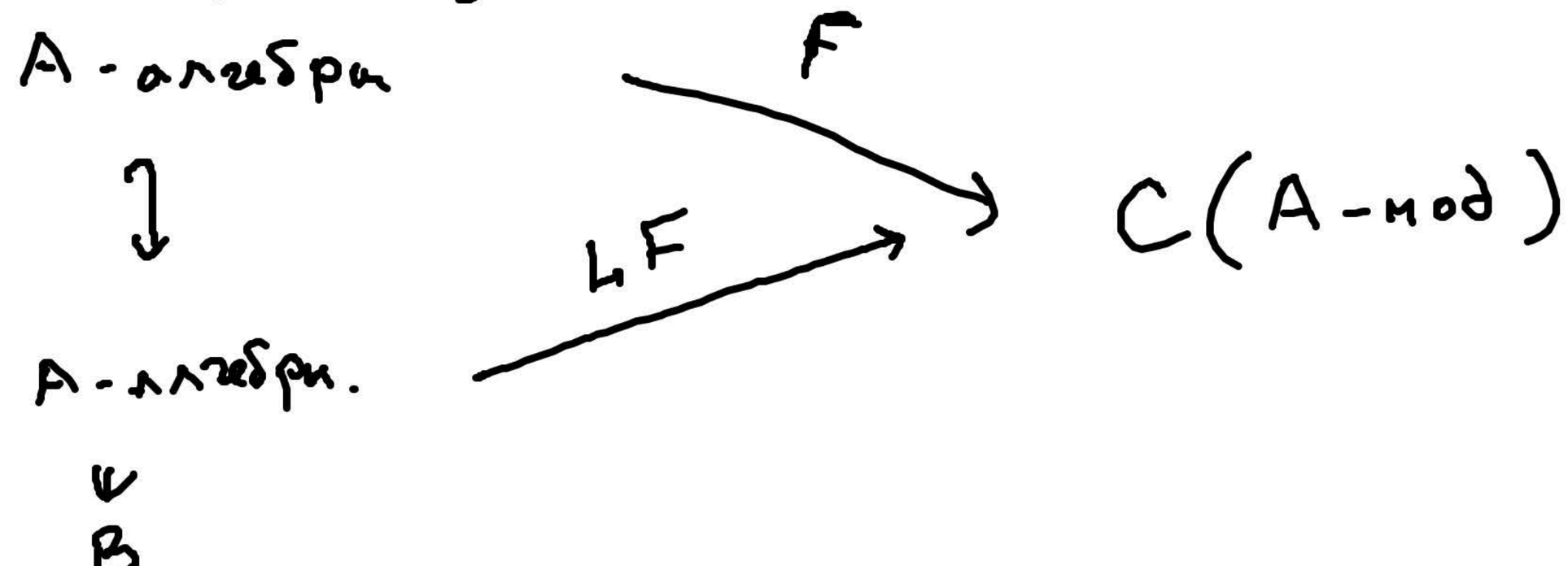
полиномиальные



$\Downarrow$

B

$$\rightrightarrows P_1 \rightrightarrows P_0 \twoheadrightarrow B$$

ациклические.

$$\to P_2 \to P_1 \to P_0 \to B$$

$$L_1F(B) = \mathrm{Tot}\left(F(P_\cdot)\right)$$

(2) $F(P) = (\Omega^{\cdot}_{P/A}, d)$.

$\angle F(B) = \angle \Omega_{B/A} = F^{\cdot} \angle \Omega_{B/A}$

$P = \theta_P \to \Omega_{P/A} \to \Omega^{\cdot}$ -

$\cup \qquad \cup \qquad \vee$

$0 \to \Omega^{1}_{P/A} \to \Omega^{2}$

$\angle \Omega^{1}_{P/A} = B$.

$$\rightrightarrows P_1 \rightrightarrows P_0 \to B$$

$$\mathbb{L}_{B/A} = \left(\cdots \to \Omega^1_{P_1/A} \to \Omega^1_{P_0/A} \right) \otimes \rightleftarrows$$

$$\left(\to \Omega_{P_1/A} \otimes B \to \Omega^1_{P_0/A} \otimes_{P_0} B \right)$$

Примери.

(а) $\mathbb{Q} \subset A$ $\quad L\Omega_{B/A} = A$

(б) $A = R[x]$ $\quad B = R[x]/(x) = R$

$$\Rightarrow R[x, x_1 \ldots x_n] \overset{\partial_0}{\underset{\rightarrow}{\rightrightarrows}} R[x, x_1 x_2] \;\rightrightarrows\; R[x, x_1] \overset{x_1 \to x}{\underset{x_1 \to 0}{\rightrightarrows}} R[x] \twoheadrightarrow B$$

$\partial_i(x_j) = x_j \qquad n \neq j \leq i$

$\partial_i(x_j) = x_{j-1} \qquad i < j$

$\partial_n(x_n) = 0$

<u>Св-ва.</u> ⓐ $H_0\, L_{B/A} = \Sigma'_{B/A}$.

ⓑ B/A гладкая $\quad L_{B/A} = \Omega'_{B/A}$.

ⓒ $A \to A'$, A, L

$$L_{B/A} \simeq L_{B \otimes_A A'/A'}$$

если $\operatorname{Tor}_i^A(B, A') = 0 \quad i > 0$.

ⓓ $A \to B \to C$

PD-оболочка (x) в $\mathbb{Z}[x]$.

$$\cdot \left(L\Omega_{\mathbb{Z}[x]/\mathbb{Z}[x]} \right)^{\wedge} \simeq \left(D_{\mathbb{Z}[x]}(x) \right)^{\wedge} \quad ?$$

$$@$$

$$\cdot \quad F^q/F^{q+1} \simeq (x)^{[q]}/(x)^{[q+1]}$$

Примери.

(a) $A = \quad B = A/(f)$, f не делител 0.

I

$L_{B/A} \simeq I/I^2$ [1].

$A = \mathbb{Z}[x] \to \mathbb{Z}[x]/(x) = B$

$\uparrow$

$\mathbb{Z}$

$L_{\mathbb{Z}/\mathbb{Z}}$

$\|$

$\to L_{\mathbb{Z}[x]/\mathbb{Z}} \to 0 \to L_{B/A}$

$$x \mapsto f$$

$$\mathbb{Z}[x] \longrightarrow A$$

$$A \overset{L}{\underset{\mathbb{Z}[x]}{\otimes}} L_{\mathbb{Z}/\mathbb{Z}[x]} \simeq L_{\mathbb{Z} \otimes_{\mathbb{Z}[x]} A / A} = L_{B/A}$$

$$\parallel$$

$$(f)/(f^2)[1] = I/I^2[1].$$

(δ) $B \longleftarrow A$ $\quad L_{B/A} = I/I^2[$

©

$$B = A[x_1 \dots x_n] / (f_1, \dots f_m) = I.$$

рег. пос-ть

$$B \overset{L}{\otimes}_{A[x_1, \dots x_n]} L_{A[x_1, \dots x_n]/A} \to L_{B/A} \to L_{B/A[x_1 \dots x_n]} \to$$

$$f \to df$$

(d)

$A \to B = A/(f)$ f не делитель 0.

$$L\Omega^{\wedge}_{B/A} = \left(A \overset{L}{\otimes}_{\mathbb{Z}[x]} D_{\mathbb{Z}[x]}(x)\right)^{\wedge} =$$

$$= D_A^{\wedge}(f)$$

(e) $$L\Omega^{\wedge}_{\mathbb{F}_p/\pi} = D^{\wedge}_{\mathbb{Z}_p[x]}(x) \Big/ {}^{\wedge}(x)$$

(d) $R/\mathbb{F}_p$

$R \overset{x \to x^p}{\rightleftarrows} R$.

$L_{R/\mathbb{F}_p} = 0$.

$L_{R/\mathbb{F}_p} \xrightarrow{F} L_{R/\mathbb{F}_p}$

$\| 0$

$\cdots \to P_1 \to P_0 \to R$

$\downarrow F \qquad \downarrow F \qquad \downarrow$

$\cdots \to P_1 \to P_0 \quad R$

$\cdots \to P_1 \to P_0 \quad R$

$$A_{inf}(\mathcal{O}_{\mathbb{C}_p}) = W\Big(\varprojlim_{Fr} \mathcal{O}_{\mathbb{C}_p}/p\Big).$$

$$\begin{array}{ccccc}
\mathbb{L}_{A_{inf}/\mathbb{Z}_p} & = 0 & & (\theta) \subset A_{inf} & \twoheadrightarrow \mathcal{O}_{\mathbb{C}_p} \\
 & & & \uparrow & \\
 & & & \mathbb{Z}_p & \\
\wedge & & \wedge & & \wedge \\
\mathbb{L}_{\mathcal{O}_{\mathbb{C}_p}/\mathbb{Z}_p} & = & \mathbb{L}_{\mathcal{O}_{\mathbb{C}_p}/A_{inf}} & = & (\theta)/(\theta^2)[1]
\end{array}$$

$$
\begin{array}{ccc}
(\theta) & & \\
\cap & & \\
A_{inf}\left(\mathbb{Z}_p[\mu_\infty]\right) & \longrightarrow & A_{inf}^{(\theta)} \\
\downarrow & & \downarrow \\
\mathbb{Z}_p[\mu_\infty] & \longrightarrow & \mathcal{O}_{\mathbb{C}_p}
\end{array}
$$

$$
\Rightarrow \quad \overline{\mathbb{Z}}_p \otimes_{\mathbb{Z}_p[\mu_\infty]} \mathbb{Z}_p[\mu_\infty]/\mathbb{Z}_p \quad \xrightarrow{\sim} \quad \mathbb{Z}_p/\mathbb{Z}_p
$$

$$\overline{\mathbb{Z}}_p \otimes \Omega^1_{\mathbb{Z}_p[\mu_\infty]/\mathbb{Z}_p} \hookrightarrow \Omega^1_{\overline{\mathbb{Z}}_p/\mathbb{Z}_p}$$

$$\begin{array}{ccc}
\overline{\mathbb{Q}}_p/\overline{\mathbb{Z}}_p(1) & & \\
\| & \xrightarrow{d\log} & \Omega^1_{\overline{\mathbb{Z}}_p/\mathbb{Z}_p} \\
\overline{\mathbb{Z}}_p \otimes \mu_\infty & & \\
\| & & \\
\varepsilon & \to & \dfrac{d\varepsilon}{\varepsilon}.
\end{array}$$

$\mathbb{Q}_p$

$$\Gamma \Omega_{A_{inf}/\mathbb{Z}_p} = \bullet \quad A_{inf}.$$

$$\Gamma \Omega_{A_{inf}/(\theta)/A_{inf}} = A_{crys}.$$

$$\begin{array}{ccc} \mathbb{Z}_p/\mathbb{Z}_p[x] & \longrightarrow & \mathbb{Z}_p[x] \\ \downarrow & & \downarrow \\ \mathbb{Q}_p/\mathbb{Q}_p[x] & \longrightarrow & \mathbb{Q}_p[x] \end{array}$$