Holographic confinement confronts lattice

Niko Jokela

UNIVERSITY OF HELSINKI

HELSINKI INSTITUTE OF PHYSICS

Key collaborators: C.Hoyos, J.M.Penín, A.Pönni, A.V.Ramallo, T. Rindlischbacher, K.Rummukainen, A.Salami, J.G.Subils, J.Tarrío

Holography: from high-energy physics to quantum information

virtually in Steklov Mathematical Institute Moscow – 19 October 2020

Outline

- Motivation
- 1. Bulk story: gravity side
  - Is entanglement a probe of confinement?
- 2. Boundary meets bulk
  - Bulk reconstruction from data
- 3. Boundary story: gauge field theory side
  - Simulating Yang-Mills to extract entanglement entropy
- Summary

Motivation

- Wish to understand confinement

  conformal symmetry must be broken

- Non-perturbative: defied analytic attempts. Numerical progress.
- Use non-AdS/non-CFT
- Tools at disposal:
  - Wilson etc loops
  - Entanglement entropy
  - C-functions
- Holography is never going to solve confinement, but

  we may learn valuable lessons

- Holography score card by comparison to YM simulations

Why entanglement entropy?

- Quantum information theory (related entanglement measures)
- Universal order parameter for quantum phase transitions
  [Kitaev-Preskill hep-th/0510092]
- "Measured" in cold atom systems
  [Islam et al. 1509.01160]
- $N_c \ll \infty$: Computable from the lattice
  [Buividovich-Polikarpov 0802.4247, ...]
- $N_c = \infty$: Holography
  - simple prescription via minimal surfaces
    [Ryu-Takayanagi hep-th/0603001,Hubeny-Rangamani-Takayanagi 0705.0016]
  - relationship to black hole entropy
- Claim from holography: entanglement probes confinement
  [Nishioka-Takayanagi hep-th/0611035,Klebanov-Kutasov-Murugan 0709.2140]

Is entanglement a probe confinement?

- Effective degrees of freedom:
  - deconfining phase: colorful (e.g. gluons) $\sim \mathcal{O}(N_c^2)$
  - confining phase: color singlets (e.g. glueballs) $\sim \mathcal{O}(1)$
- Derived quantities of EE capture the number of dofs
- Key idea: C-function constructed from EE acts as an order parameter for deconfinement at $N_c \sim \infty$
- Disclaimer:
  - in the presence of fundamentals, there is no local order parameter at finite-N_c [see however Cherman-Jacobson-Sen-Yaffe 2007.08539]
  - not equivalent to linear interquark potential at large separation [NJ-Subils 2010.tomorrow]

C-theorems

C-theorems state that $\exists$ a function $C(g_i, \Lambda)$ s.t.

- decreases under RG
- is stationary at fixed points
- morally counts dofs: at UV more than at IR

Assuming Lorentz invariance under RG:

- 1d: g-theorem: thermal free energy decreases [Affleck-Ludwig '90]
- 2d: Zamolodchikov's c-theorem, at fixed points CFT and hence c = central charge
- 3d: F-theorem (F=free energy on sphere), holographic proofs [Jafferis,Jafferis-Klebanov-Pufu-Safdi] [Casini-Huerta]
- 4d: a-theorem (a = anomaly) [perturbative proof Osborn, non-perturbative Komargodski-Schwimmer]
- >4d: only holographic proposals [Myers-Sinha,Liu-Mezei]

In anisotropic case, many proposals only from holography...

[...,Hoyos-NJ-Penin-Ramallo 2001.08218,...]

Definition of EE

- Take scalar QFT in $\mathbb{R}^{d+1}$ in vacuum state $|0\rangle$ and split

$$A = \mathbb{R}^d \times I_\ell \qquad , \qquad B = \mathbb{R}^d \times (\mathbb{R} - I_\ell)$$

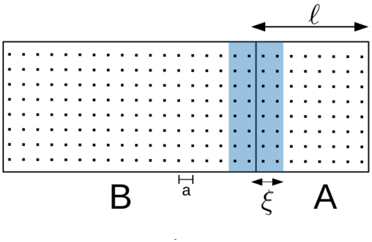
- Quantum entanglement (von Neumann) entropy

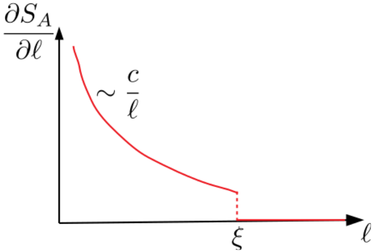
$$S_A = -\mathrm{Tr}\rho_\mathrm{A} \log \rho_\mathrm{A} \ , \ \text{reduced density matrix } \rho_\mathrm{A} = \mathrm{Tr}_\mathrm{B}|0\rangle\langle 0|$$

- EE for region A is entropy seen by an observer unable to access dofs in B

- Consider 1+1d example [Calabrese-Cardy hep-th/0405152,0905.4013, Holzhey-Larsen-Wilczek hep-th/9403108]

$$S_A(\ell) = \begin{cases} \frac{c}{3}\log\frac{\ell}{a} + const. & , \ \ell \ll \xi \\ \frac{c}{6}\log\frac{\xi}{a} & , \ \ell \gg \xi \end{cases}$$





- A possible proposal for a c-function would be

$$c(\ell) = 3\ell\frac{\partial S_A}{\partial \ell}$$

as it picks the central charge.

1. Bulk story

Holographic entanglement entropy

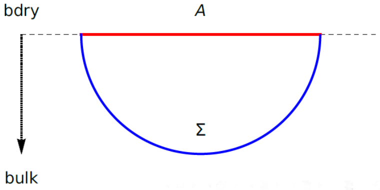
Given a spatial partition of the $(d+1)$-dim. spacetime on QFT:

[Ryu-Takayanagi hep-th/0603001,Hubeny-Rangamani-Takayanagi 0705.0016,. . .]

$$S_A = \min \frac{\mathrm{Vol}(\Sigma)}{4G_N^{10}} \ ,$$

where Σ is a co-dimension two bulk surface subject to

- Σ is homologous to A
- $\partial\Sigma = \partial A$



Holographic Wilson loop

- The Wilson loop operator is dual to the partition function of fundamental strings attached to the boundary
  [Rey-Yee hep-th/9803001, Maldacena hep-th/9803002]

$$\langle W(C)\rangle = \int_{\partial\Sigma = C} dX e^{-S_{string}(X)} \ .$$

- In the 't Hooft limit, sufficient to minimize the bosonic tree-level Nambu-Goto action.
- Potential for quarks separated by $L \ll \tau$ (temporal loop)

$$V(L) = \frac{S_{NG} - S_{NG}^{\parallel}}{\tau} \ .$$

Holographic EE and Wilson loops

- For simplicity, let us consider temporal Wilson loop and entanglement entropies for strips/slabs of width ℓ.
- For a $(d+1)$-dimensional scale-free theory dual to $AdS_{d+2} \times \mathcal{M}_{8-d}$, the finite parts are trivial to compute

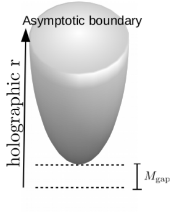
$$V(L) \propto 1/L$$
$$S_{RT} \propto 1/\ell^{d-1}$$

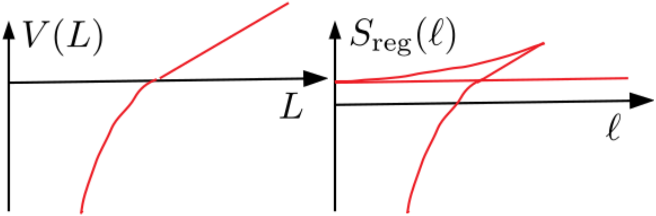
- Our interest, asymptotically free theories close to (super) Yang-Mills in 3d and 4d, dual to geometries induced by Dp-branes, $p = 2, 3$:

$$V(L)\big|_{p=2} \propto 1/L^{2/3}$$
$$S_{RT} \propto 1/\ell^{\frac{4}{5-p}}\big|_{p=2} = 1/\ell^{4/3}$$

Holographic confinement

- Common lore: geometries that are confining are cigar-like or have some “IR bottom”





$\Rightarrow$ both are sensitive to deconfinement transition

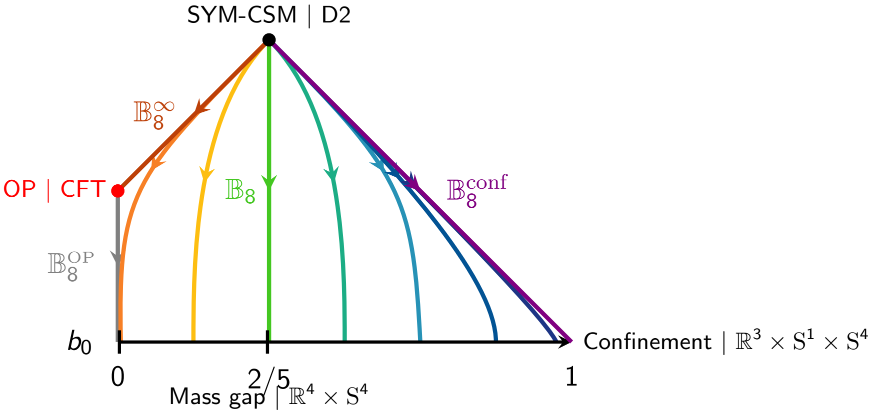
- We claim: not so fast!

[NJ-Subils 2010.tomorrow]

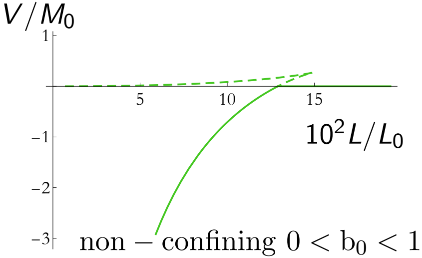
- Case study in a family of theories

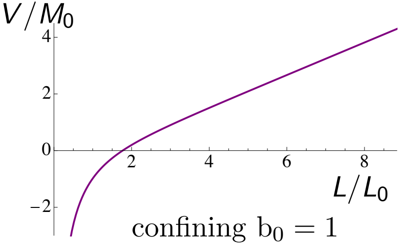
[Faedo-Mateos-Pravos-Subils 1702.05988]
[NJ-Subils 2010.tomorrow]

$$U(N)_k \times U(N+M)_{-k}$$



- By defining the holographic confinement as linear potential

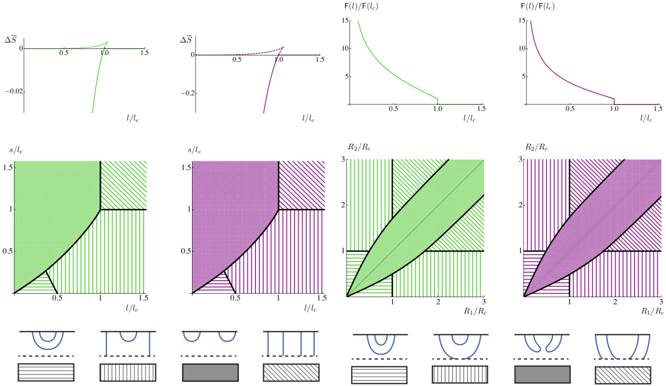




we can investigate whether entanglement measures can distinguish the two.

Holographic confinement

- No entanglement measure cannot distinguish confining theory from non-confining



- Entanglement is instead sensitive to mass gap:

$$\boxed{\textit{EE transition} \leftrightarrow \textit{finite correlation length } \xi}$$

2. Connecting boundary with bulk

Bulk reconstruction

- This is applied AdS/CFT in reverse
- Idea: take boundary measurements and attempt to “guess” what is bulk
- All examples of duality involve strongly coupled gauge field theories
- Need either lattice simulations or experimental measurements
    - they are *noisy*

  $\Rightarrow$ one cannot find precise dual geometry
- How to deal with inherently noisy data?

[NJ-Pönni 2007.00010]

Bulk reconstruction from EE

- Make as little assumptions on bulk as possible: use RT
  - can only reconstruct some components of the bulk metric
- Important point: data for $dS_{EE}/d\ell$ available from lattice
- Can be shown, for a given metric that:

$$\frac{dS_{RT}}{d\ell} = f(r_*) \ , \ \ r_* = \text{tip of RT surface}$$

- Naive approach would then to fit $f(r_*)$ and hence try to get the "best" metric
- A more sophisticated approach is to appreciate that the data is noisy:

$$\left\{ \left(\frac{1}{V} \frac{dS}{d\ell} \right)_i , \ell_i, \sigma_i \right\} \ , \ \ i \in 1, \dots, N$$

Bulk reconstruction from EE

- For simplicity, consider 5d bulk:

$$ds^2 = \frac{R^2}{z^2}\left(-\frac{b(z)}{a(z)^2}dt^2 + \frac{a(z)^2}{b(z)}dz^2 + d\vec{x}^2\right)\ ,\ b(z) = 1 - \left(\frac{z}{z_h}\right)^4$$

$$a(z) = 1 + \sum_{i=1}^{N_{basis}} a_i f_i(z) \qquad ,\ f_i(1) = f_i(0) = 0\ .$$

- Key point, by use of chain rule

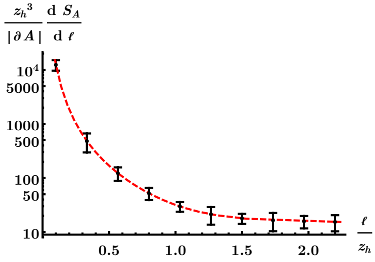
$$\frac{4G_N}{R^3}\frac{1}{V}\frac{dS_{RT}}{d\ell} = \frac{1}{z_*^3}$$

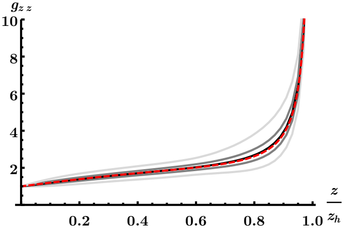
implies that the noisy data of $\partial_\ell S$ is really noisy data of $1/z_*^3$

- Use Hamiltonian Monte Carlo sampling to obtain posterior distributions

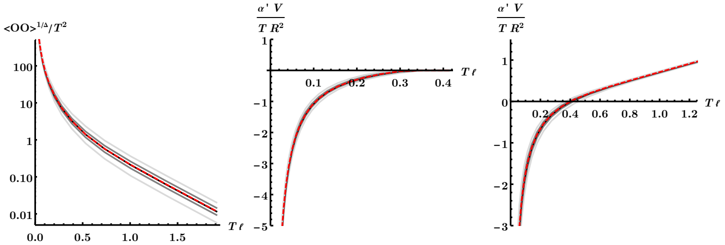
$$\left\{a_{1,\ldots,N_{basis}}, \frac{R^3}{4G_N}\right\}$$

- Test scenario by synthetic data





- Application to obtain predictions via AdS/CFT



Bulk reconstruction from EE

- The method works for any d
- Can be used for bulk reconstruction given any finite data
  - Entanglement entropy, mutual information, purification, ...
  - Wilson etc loops
  - Two-point functions
  - $\vdots$
- Enjoys other straightforward generalizations
  - UV can be changed e.g. to Dp asymptotics
  - IR horizon asymptotics can be replaced
- Code available at `https://github.com/arpn/hee_hmc`
- **Need real data!**

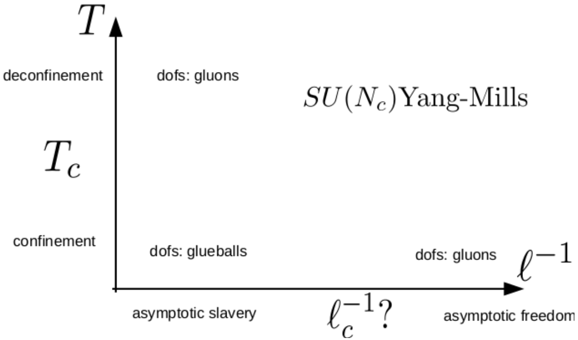
3. Boundary story

Existing results in the literature

Desire to have Yang-Mills lattice simulations to extract EE

- Relation of EE to free energy / path integral
  [Calabrese-Cardy'04&'05]
- Prescription for free energy measurements
  [Fodor'07,Endrödi-Fodor-Katz-Szabo'07]
- Pure glue SU(2) in 4d
  [Buividovich-Polikarpov'08]
- Pure glue SU(3) in 4d
  [Nakagawa-Nakamura-Motoki-Zakharov'09,Itou-Nagata-Nakagawa-Nakamura-Zakharov'11&'15]
- Pure glue SU(2,3,4) in 4d
  [Rabenstein-Bodendorfer-Buividovich-Schäfer'18]
- Pure glue SU(N_c) both in 3d and 4d
  [this work]

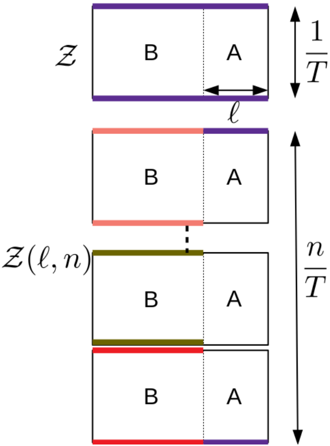
- Expected phase diagram for pure glue



- Can simulate any d or N_c, in practice
[NJ-Pönni-Rindlischbacher-Rummukainen-Salami work in progress]

$$3d \quad \text{and} \quad 4d \quad , \quad N_c \ll \infty$$

Replica method



- Entanglement entropy

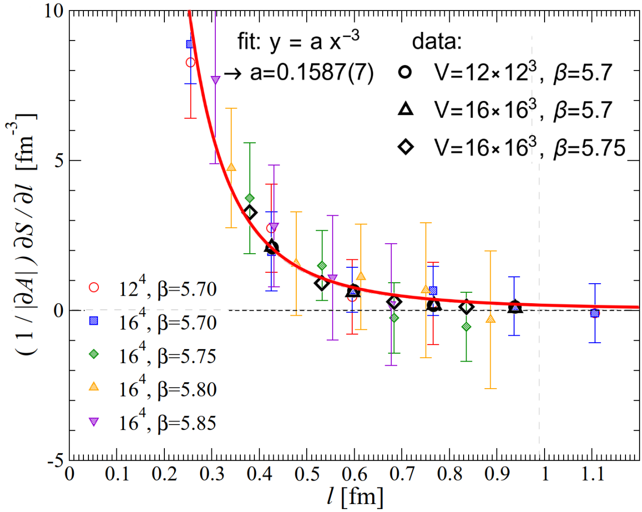
$$S_A = -\lim_{n\to 1} \partial_n \log \mathrm{Tr}_A \rho_A^n$$
$$= -\lim_{n\to 1} \partial_n \log \frac{\mathcal{Z}(\ell, n)}{\mathcal{Z}^n}$$

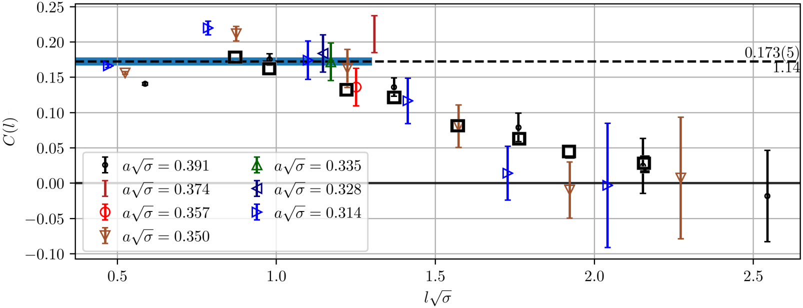
- Our interest

$$\partial_\ell S_A = \lim_{n\to 1} \partial_\ell \partial_n F(\ell, n)$$
$$\approx \frac{F(\ell + a, 2) - F(\ell, 2)}{a}$$

$SU(N_c = 3)$, 4d

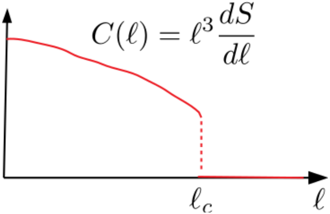
- Compares well with previous results in the literature [here shown Nakagawa-Nakamura-Motoki-Zakharov 1104.1011]
- Our statistical errors are invisible to eye (black markers).





vs. holography $S_A = \frac{N_c^2}{a^2} - \frac{N_c^2}{\ell^2}\left(1 + \log(\ell/a)?\right) + \ldots$

[Ryu-Takayanagi hep-th/0605073]



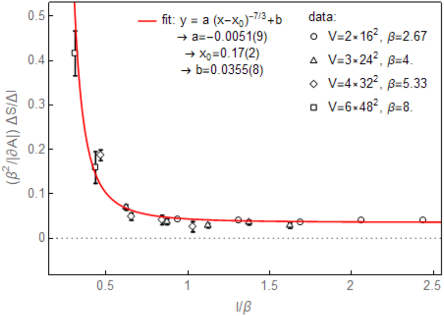
Simulating Yang-Mills theory

Points of interest

- $\ell_c \sim 0.8\mathrm{fm} \sim \Lambda_{\mathrm{QCD}}^{-1}$?
- finite-T: does $dS_A/d\ell \to S_{BH}$ as $\ell \to \infty$?
- Does the C-function scale as $N_c^2 - 1$?
  - replica number dependence
- $\vdots$
- shape dependence
- more cuts (mutual info), corners (cusp anomalous dimension)
- putting in fermions, or susy, is expensive, but other holographic tests can be done

$SU(N_c = 2)$, 3d

- New results for $SU(N_c)$ in 3d



- Aim to make a more direct comparison to D2-brane background
  - at UV (small ℓ) $dS/d\ell \sim \ell^{-7/3}$
  - at IR (large ℓ) $dS/d\ell \sim \mathrm{const.} = \mathrm{S_{BH}}$?

Summary

- Holo-entanglement does not probe confinement, but mass gap
- Lattice data for EE or Wilson loops can be generated $N_c \ll \infty$
- This data can be used to reconstruct the dual geometry
- Allows new predictions in same QFT w/ confidence intervals
- New era has begun

Precision holography!

Thank you!