Hydrodynamics and holography of the spin current

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Umut Gürsoy
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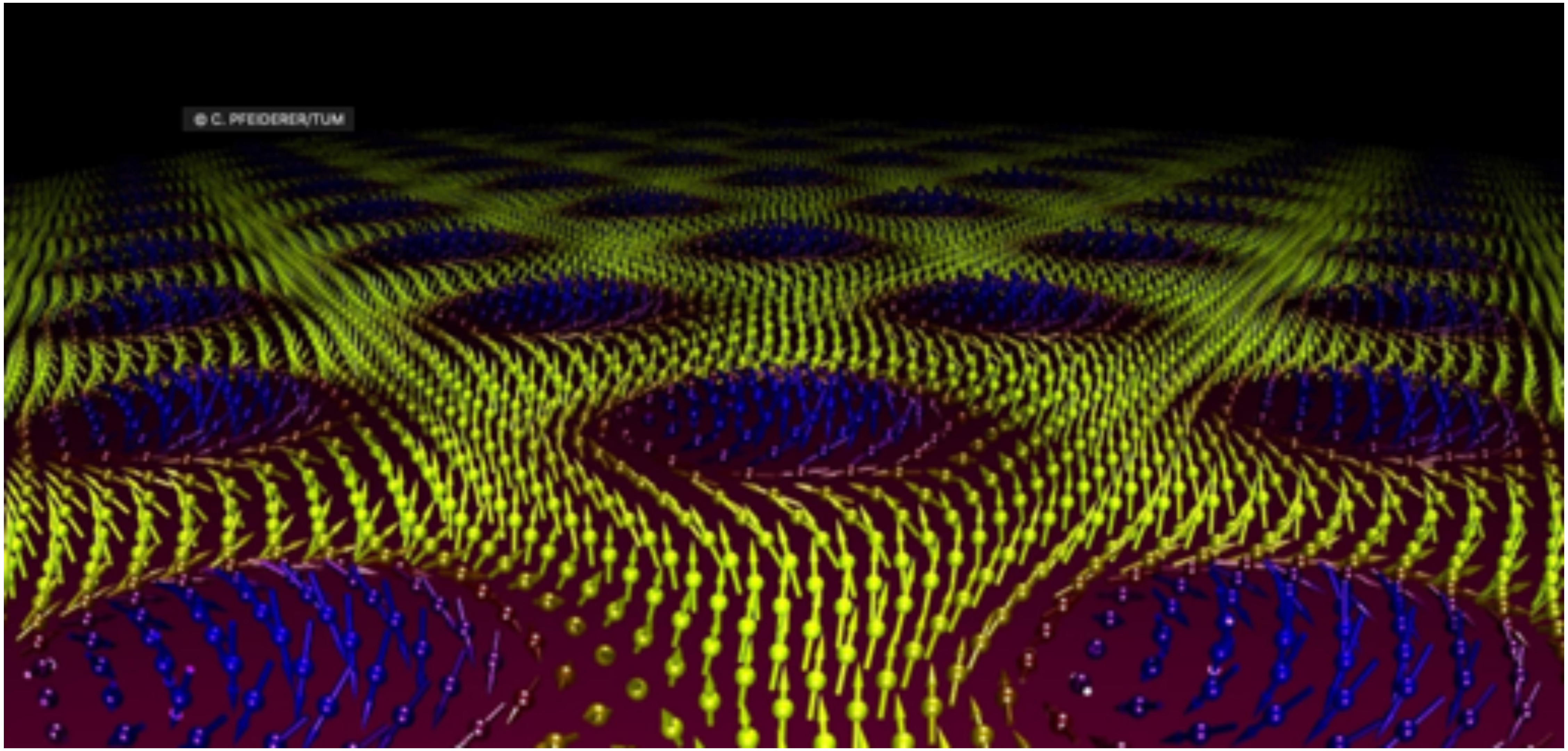
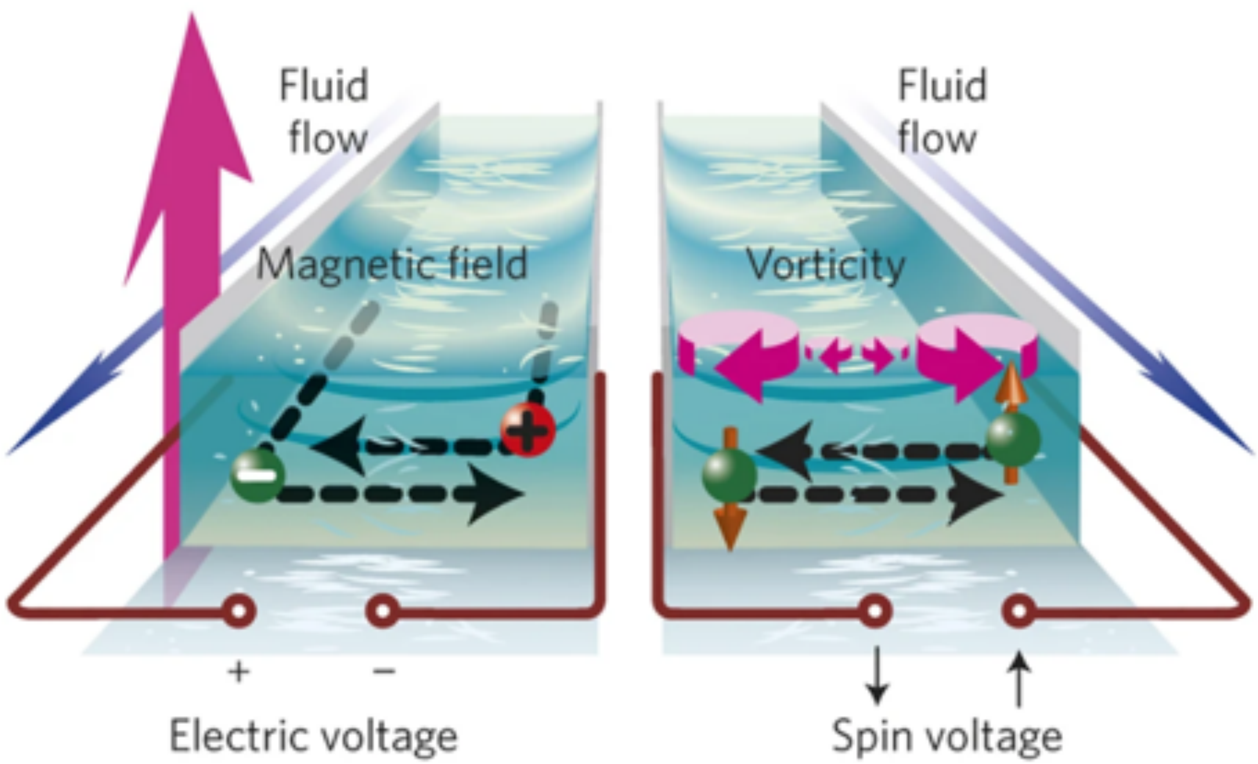
Based on: with Domingo Gallegos, arXiv:2004.05148
ongoing with Domingo Gallegos, Amos Yarom

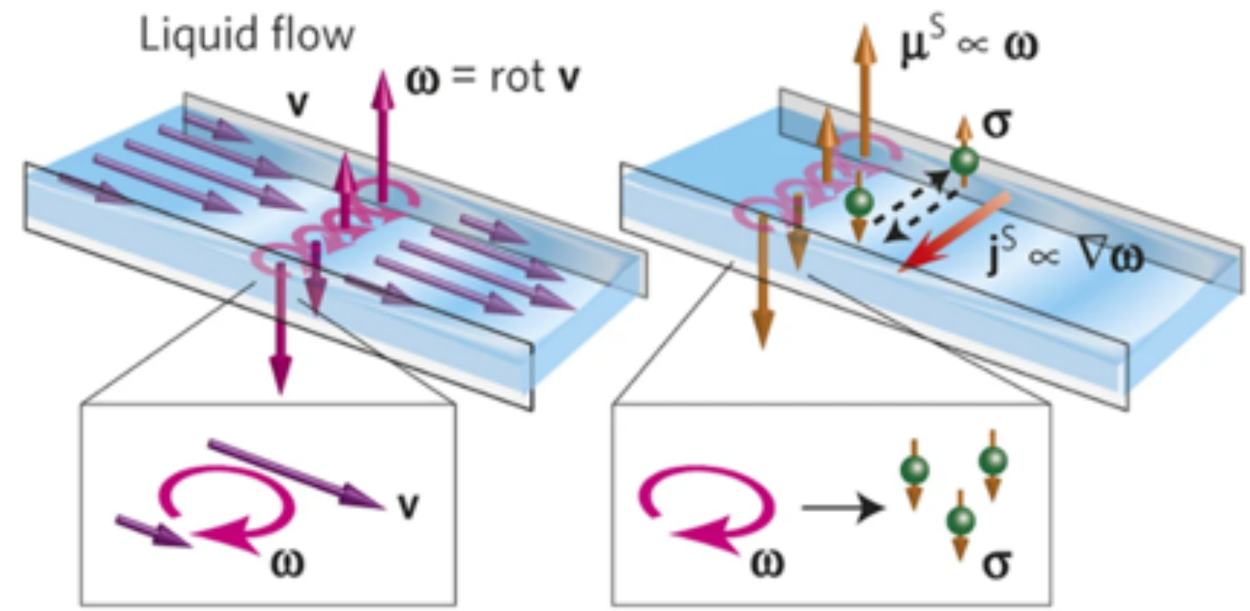
Spin transport



- Spintronics, liquid spintronics
- Graphene, Dirac/Weyl semimetals
- Astrophysics
- Quark gluon plasma

Liquid spintronics



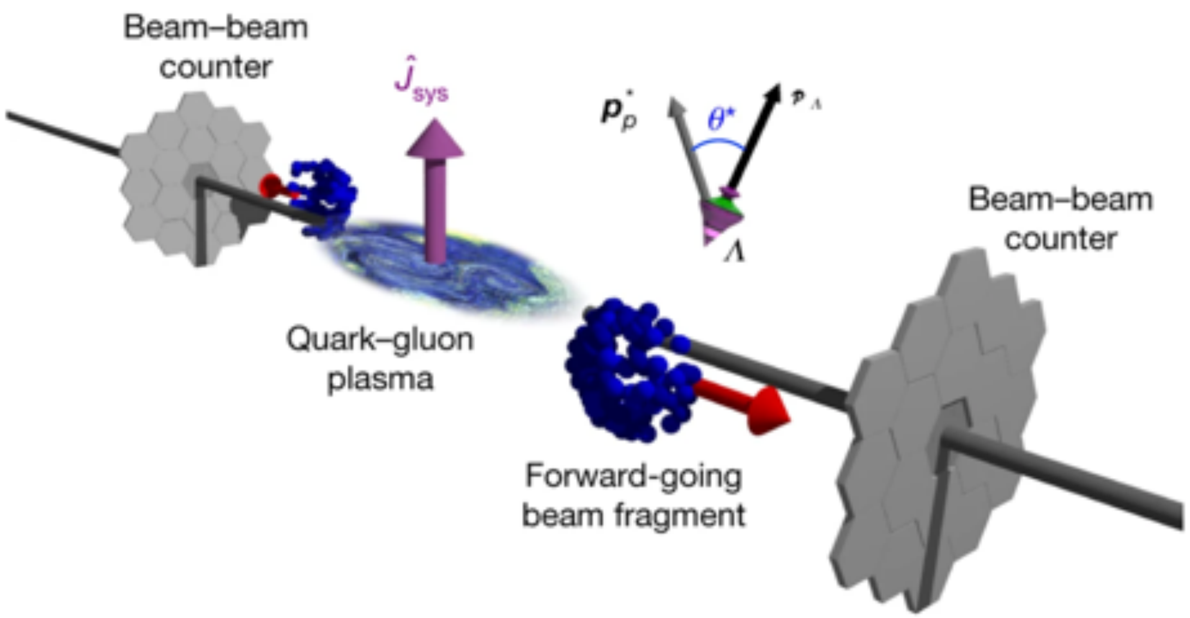


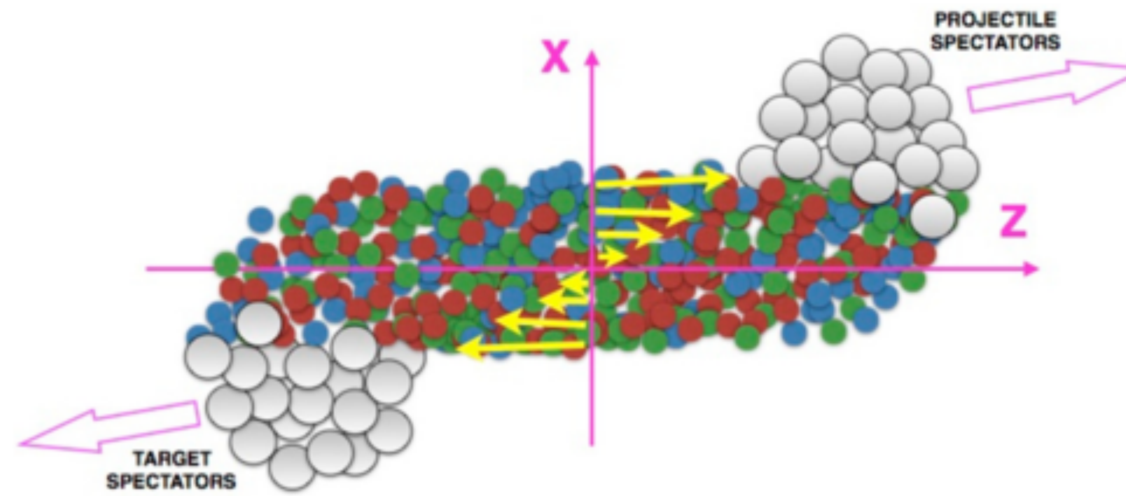
R. Takahashi et al. '16

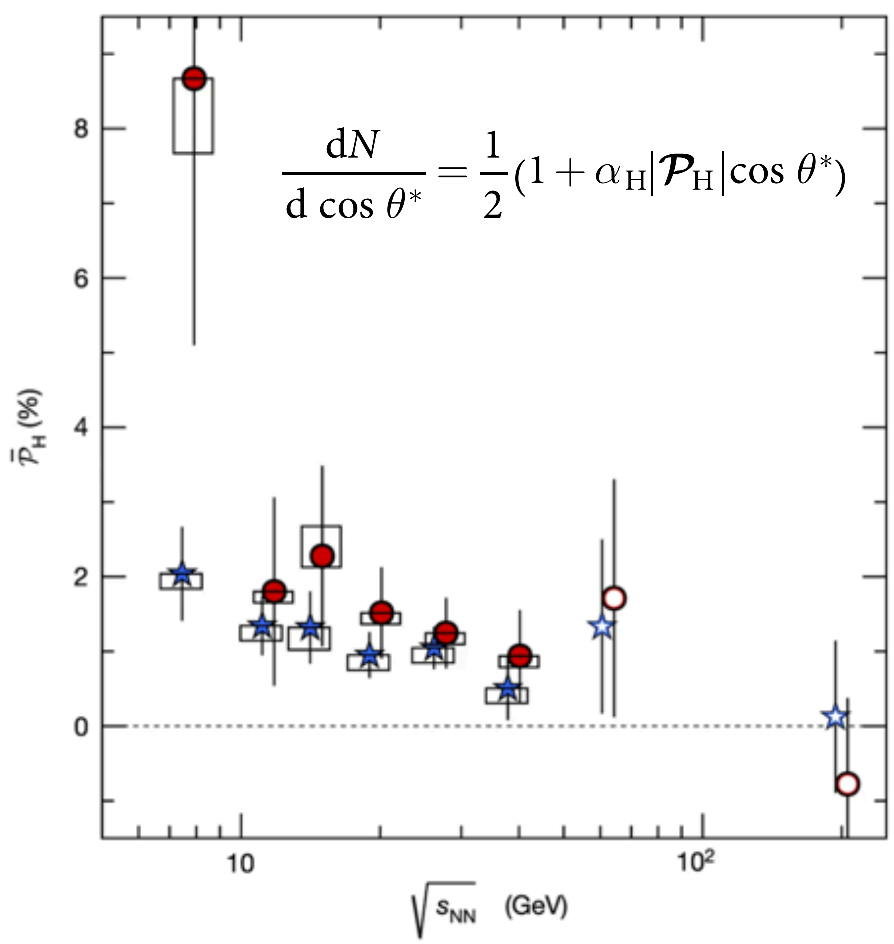
Generation of spin density by vorticity in liquid metals (Hg, GaInSn)

$\mu_s \propto \omega, J_s \propto \nabla\omega$

Quark gluon plasma







Global hyperon polarization at RHIC by spin-orbit coupling $\vec{S} \cdot \vec{J}$

QGP: most vortical fluid: $\omega \sim 10^{22}\ s^{-1}$

F. Becattini et al. ’17
STAR collaboration ‘19

Hydrodynamics with spin current

- All possible sources of spin density/current
- Relation of spin flow to energy/charge/chirality flow
- Classification of spin transport coefficients

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$\Longrightarrow$ construction of hydrodynamics with spin

$$T_{\mu\nu} \qquad\qquad S^{\lambda}_{\mu\nu}$$

Outline

- Definition of spin current
- Hydrodynamics with spin current and torsion
- Holographic description: first order gravity

  Lovelock-Chern-Simons theory
- Analytic blackhole solutions
- Spin transport coefficients
- Discussion

Ambiguity in the spin current

Total angular momentum

$$J^{\lambda\mu\nu} = x^\mu T^{\lambda\nu} - x^\nu T^{\lambda\mu} + S^{\lambda\mu\nu}$$

Conservation laws

$$\partial_\mu T^{\mu\nu} = 0\,, \qquad \partial_\lambda J^{\lambda\mu\nu} = 0$$

Preserved by

$$T'^{\,\mu\nu} = T^{\mu\nu} + \frac{1}{2}\partial_\lambda\left(\Phi^{\lambda\mu\nu} - \Phi^{\mu\lambda\nu} - \Phi^{\nu\lambda\mu}\right)\,,$$

$$S'^{\,\lambda\mu\nu} = S^{\lambda\mu\nu} - \Phi^{\lambda\mu\nu}$$

Belinfante-Rosenfeld

$$\Phi^{\lambda\mu\nu} = S^{\lambda\mu\nu}$$

Removes spin current completely, renders energy-momentum tensor symmetric, compatible with GR

Spin in effective action

Consider quantum field in a nontrivial Lorentz representation

$$e^{iW[e,\omega]} = \int D\Psi e^{iI[e,\omega,\Psi]}$$

Variations define the energy-momentum and spin current

$$T^{\mu\nu} = \frac{\delta W}{\delta e^a_\mu} e^\nu_a, \qquad S^\lambda_{ab} = \frac{\delta W}{\delta \omega^{ab}_\lambda}$$

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Dependent in the absence of torsion:

$$T^a = de^a + \omega^a_b \wedge e^b$$

$\Longrightarrow$ spin current unambiguous when sourced by torsion

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Easier to formulate in terms of contorsion:

$$\omega^{ab}_\mu = \mathring{\omega}^{ab}_\mu + K^{ab}_\mu, \qquad \mathring{\omega} \sim \partial e$$

Lowest order source of spin flow in flat space e = 1

Hydrodynamic equations

Require invariance of W[e,ω] under

1. Diffeomorphisms

$$\delta_\xi e^a = \mathcal{L}_\xi e^a \,, \qquad \delta_\xi \omega^{ab} = \mathcal{L}_\xi \omega^{ab}$$

2. Local Lorentz transformations

$$\delta_\lambda e^a = -\lambda^a{}_b e^b \,, \qquad \delta_\lambda \omega^{ab} = D\lambda^{ab}$$

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Relativistic hydrodynamics with spin current

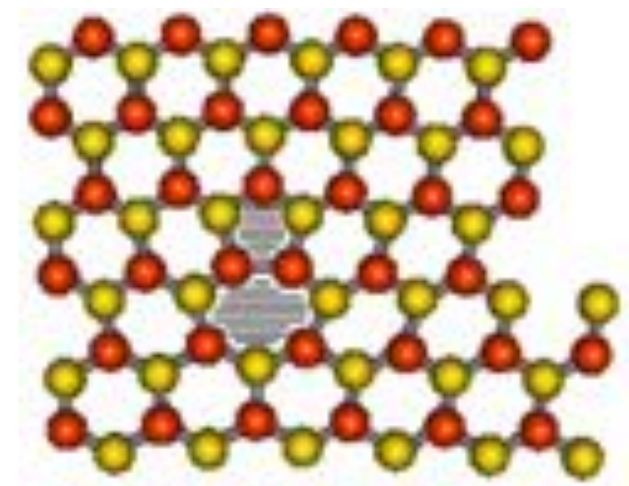
$$\mathring{\nabla}_\mu T^{\mu\nu} = R^{\rho\sigma\nu\lambda} S_{\lambda\rho\sigma} - T_{[\rho\sigma]} K^{\rho\sigma\nu} \,,$$ analogous to EM work j・E

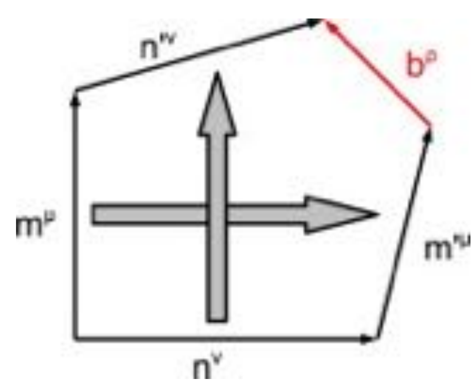
$$\mathring{\nabla}_\lambda S^\lambda{}_{\mu\nu} = T_{[\mu\nu]} - 2S^\lambda{}_{\rho[\mu} K^\rho{}_{\nu]\lambda}$$ antisymm. EMT generates S

Torsion in condensed matter

Kondo ’52; Katanaev, Volovich ’92; Furtado, Moraes ’99; Mesaros, Sadri, Zaanen ’09; de Juan, Cortijo, Vozmediano ‘10

e.g. dislocations in graphene





$$m'^{\lambda} - n'^{\lambda} = \left[\Gamma^{\lambda}_{\mu\nu} - \Gamma^{\lambda}_{\nu\mu}\right] m^{\mu} n^{\nu} = T^{\lambda}_{\nu\mu} m^{\mu} n^{\nu}$$

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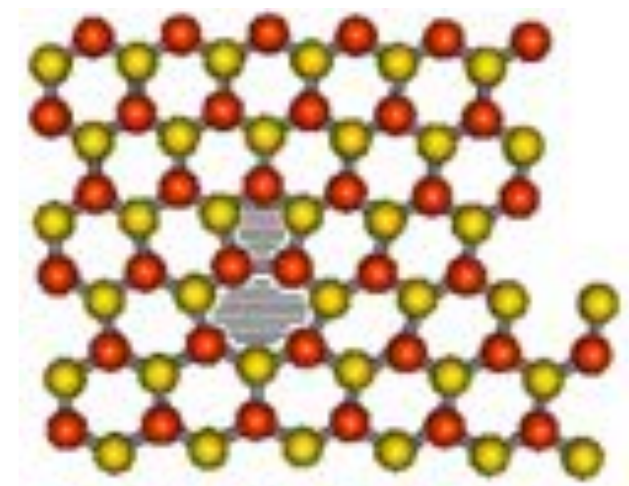

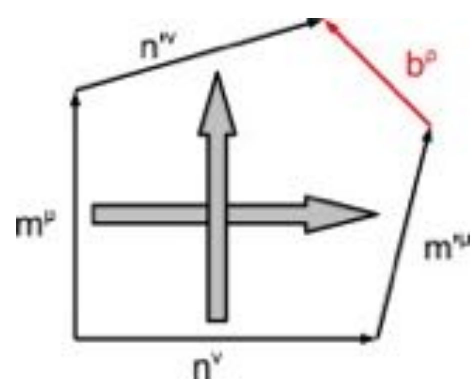

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Torsion = continuum description of dislocations in the background

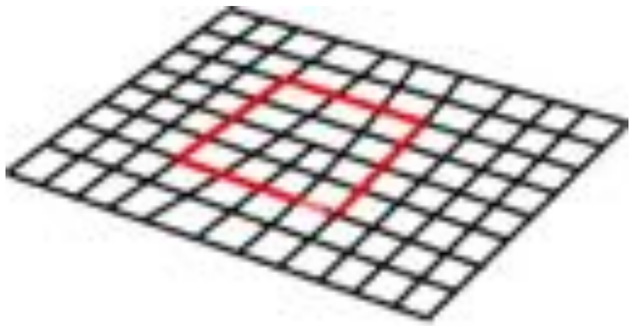
Classification of torsion sources

Assume flat 3+1D metric with nontrivial torsion $T^a_{\mu\nu} = (\omega^a_b)_\mu \wedge \delta^b_\nu$

In 3D: ω^{ab}_i → loop plane
→ Burger's vector

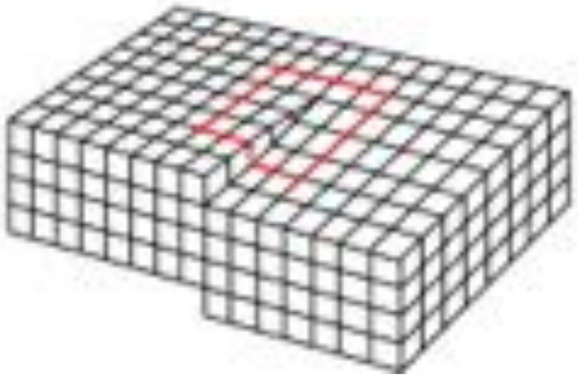
$\Longrightarrow$ 9 degrees of freedom

$i \parallel (ab)$



edge dislocation (6 d.o.f)

$i \perp (ab)$



screw dislocation (3 d.o.f)

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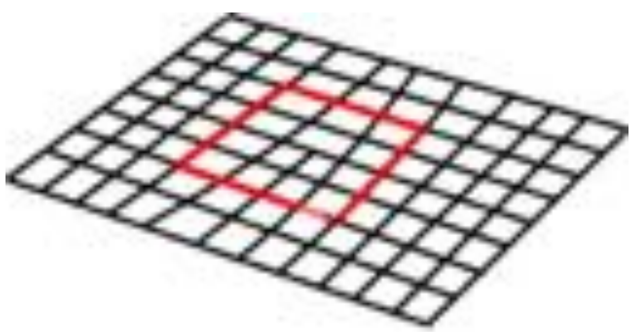
$i \parallel (ab)$ $\qquad\qquad$ $i \perp (ab)$



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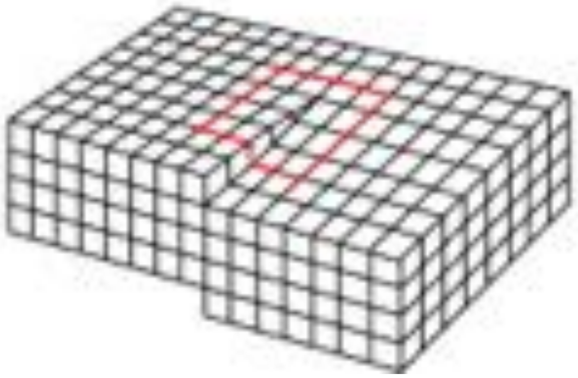

screw dislocation (3 d.o.f)

In 4D: ω_μ^{ab} $\Longrightarrow$ Wilson-edge (6 d.o.f.) + Wilson-screw (6 d.o.f)
+ Polyakov-edge (6 d.o.f) + Polyakov-screw (6 d.o.f)

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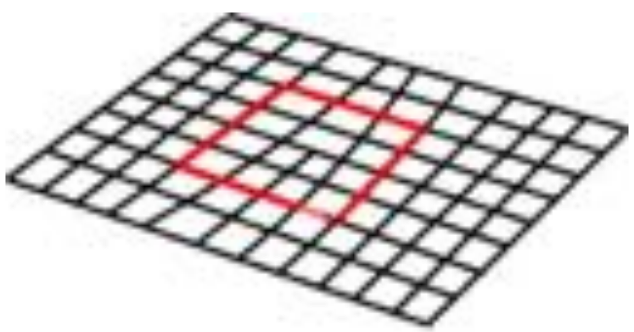

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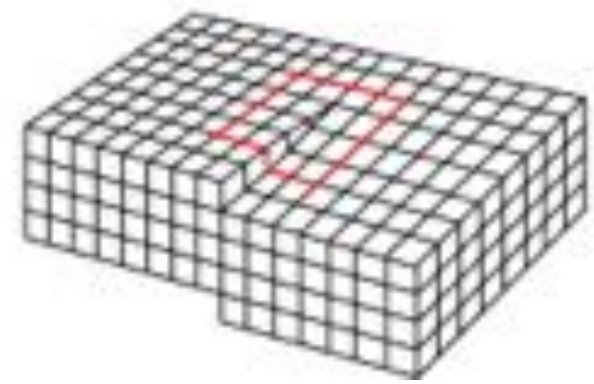

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Not a Lorentz invariant decomposition

Classification of torsion sources

Lorentz invariant decomposition: $\omega^{ab} = \omega^{ab}_{\text{vector}} + \omega^{ab}_{\text{axial}} + \omega^{ab}_{\text{tensor}}$

$$(\omega^{ab}_{\text{vector}})_\mu = \delta^a_\mu \tilde{V}^b - \delta^b_\mu \tilde{V}^a \qquad (\omega^{ab}_{\text{axial}})_\mu = \epsilon^{abc}{}_d\, \tilde{A}_c\, \delta^d_\mu$$

In hydro with flow velocity u^μ further decompose: Gallegos, UG ,'20

$$\omega^{ab}_\beta = e^a_\mu e^b_\nu \Big[-\epsilon^{\mu\nu}{}_{\rho\sigma} u^\rho \left(\mu_A \delta^\sigma_\beta + \mathcal{C}^\sigma_\beta - \mathcal{A}^\sigma_1 u_\beta \right) + 2 \left(\mu_V u^{[\mu} - \mathcal{V}^{[\mu}_2 \right) \Delta^{\nu]}_\beta$$
$$+ 2u^{[\mu} \epsilon^{\nu]}{}_{\rho\sigma\beta} u^\rho \mathcal{A}^\sigma_2 - 2u^{[\mu} \left(\mathcal{V}^{\nu]}_1 u_\beta + \mathcal{H}^{\nu]}_\beta \right) \Big]$$

1 scalar, 1 pseudo-scalar, 2 transverse vectors, 2 axial transverse vectors,
2 traceless-transverse symmetric tensors

Constitutive relations

Energy-momentum tensor:

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - p\Delta^{\mu\nu} + \underbrace{\bar{q}^\mu u^\nu + u^\mu q^\nu}_{\text{heat currents}} + \underbrace{\pi^{\mu\nu}}_{\substack{\text{symmetric} \\ \ni \text{ shear}}} + \underbrace{\tau^{\mu\nu}}_{\substack{\text{intrinsic} \\ \text{torque}}}$$

Spin current:

$$\begin{aligned} S^{\lambda\mu\nu} = {} & u^\lambda \left(u^\nu n_V^\mu - u^\mu n_V^\nu + \epsilon^{\mu\nu}{}_{\rho\sigma} u^\rho n_A^\sigma \right) + \rho_A \epsilon^{\lambda\mu\nu\rho} u_\rho + \rho_V \left(\Delta^{\lambda\nu} u^\mu - \Delta^{\lambda\mu} u^\nu \right) \\ & + N^{\lambda\mu} u^\nu - N^{\lambda\nu} u^\mu - \epsilon^{\mu\nu\rho\sigma} u_\rho \bar{N}_\sigma^\lambda + \Delta^{\lambda\nu} \bar{n}_V^\mu - \Delta^{\lambda\mu} \bar{n}_V^\nu \\ & + \epsilon^{\lambda\mu}{}_{\rho\sigma} u^\nu u^\rho \bar{n}_A^\sigma - \epsilon^{\lambda\nu}{}_{\rho\sigma} u^\mu u^\rho \bar{n}_A^\sigma \end{aligned}$$

Gallegos, UG ,'20

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Spin Source	μ_V	μ_A	$\mathcal{V}_1^\mu$	$\mathcal{A}_1^\mu$	$\mathcal{V}_2^\mu$	$\mathcal{A}_2^\mu$	$\mathcal{H}^{\mu\nu}$	$\mathcal{C}^{\mu\nu}$
Spin current	ρ_V	ρ_A	n_V^μ	n_A^μ	$\bar{n}_V^\mu$	$\bar{n}_A^\mu$	$N^{\mu\nu}$	$\bar{N}^{\mu\nu}$
Degrees of Freedom	1	1	3	3	3	3	5	5

Gallegos, UG ,'20

Spin transport from effective action

Define spin chemical potential:

$$\mu^{ab} = u^{\mu}\omega_{\mu}^{ab} = u^{a}\mathcal{V}_1^{b} - u^{b}\mathcal{V}_1^{a} - \epsilon^{abcd}u_c\mathcal{A}_{1d}$$

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Hydrostatics from effective action:

$$W = \int d^4x \sqrt{-g} P[T, \mu^i] + \sum_i \sigma_i(T) \mathcal{S}^i + \cdots$$

$$\mu^i = \{\mathcal{V}_1 \cdot \mathcal{V}_1 \,, \mathcal{A}_1 \cdot \mathcal{A}_1 \,, \mathcal{V}_1 \cdot \mathcal{A}_1\} \qquad S^i = \{\mathcal{V}_1 \cdot \omega \,, \mathcal{V}_1 \cdot a \,, \mathcal{A}_1 \cdot \omega \,, \mathcal{A}_1 \cdot a\}$$

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Polarization by spin chemical potential, vorticity and acceleration

$$\begin{aligned} S^{\lambda}_{ab} &= u^{\lambda}\left\{q_1\mathcal{V}_{1[a}u_{b]} + q_2\epsilon_{abcd}u^{c}\mathcal{A}_1^{d} + q^{3}\tilde{\mu}_{ab}\right\} \\ &\quad + u^{\lambda}\left\{\sigma_1\omega_{[a}u_{b]} + \sigma_2 a_{[a}u_{b]} + \sigma_3\Omega_{ab} + \sigma_4\epsilon_{abcd}a^{c}u^{d}\right\} \end{aligned}$$

Bjorken flow

Boost invariant expansion in z-direction: $u^\tau = 1$

No spin chemical potential and vorticity

Bjorken flow

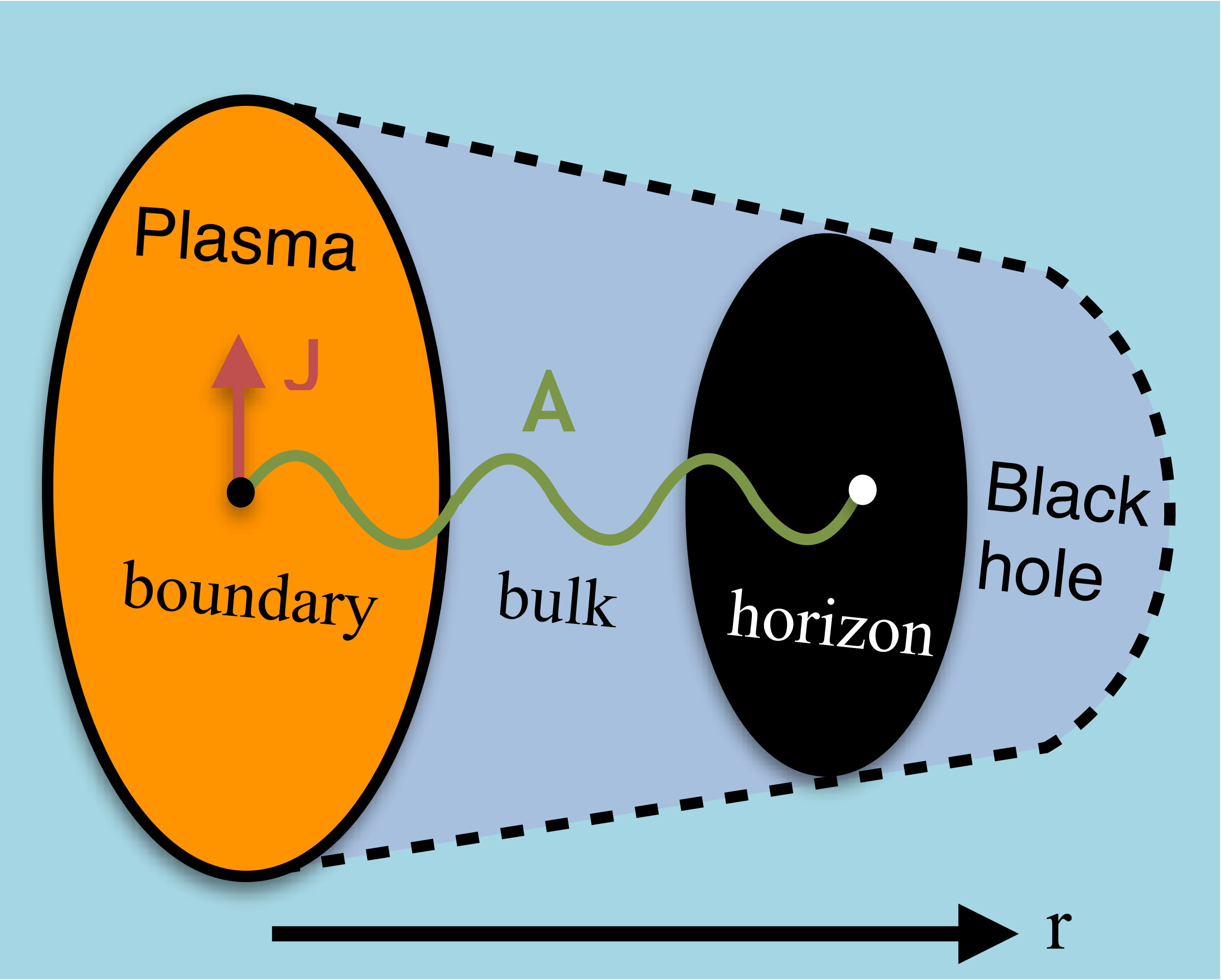
Boost invariant expansion in z-direction: $u^\tau = 1$

No spin chemical potential and vorticity

Acceleration polarizes the fluid in the z-direction (in the lab frame):

$$S^{\tau}_{xy} = \sigma_4(T)\frac{\sinh 2y}{2\tau}$$

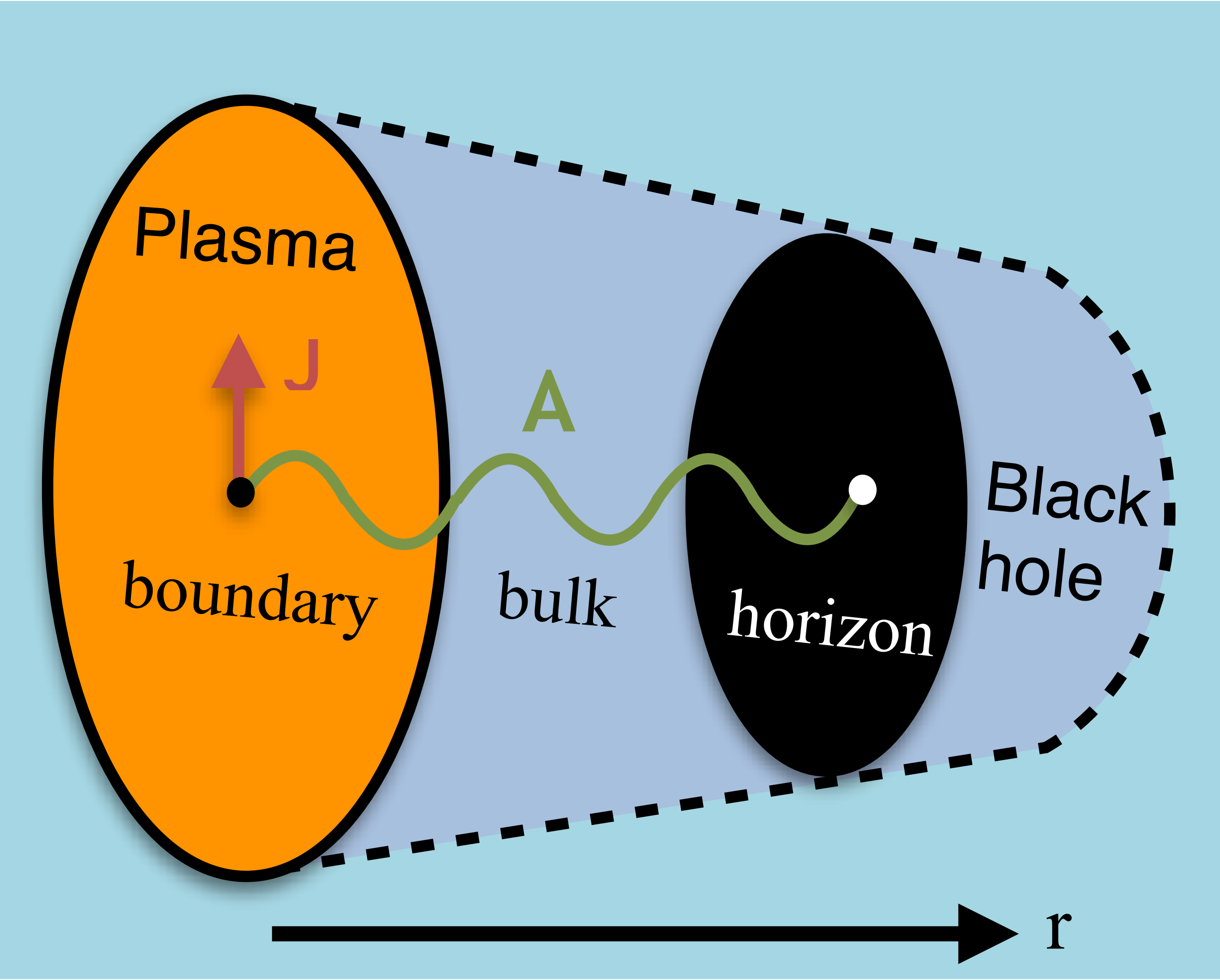
Holography



- Current J is mapped onto fluctuations on the horizon
- Generic properties of near horizon geometry

  $\Rightarrow$ universal transport e.g. $\eta/s = 1/4\pi$ Policastro, Son, Starinets `01
  Kovtun, Son, Starinets `04

Holography



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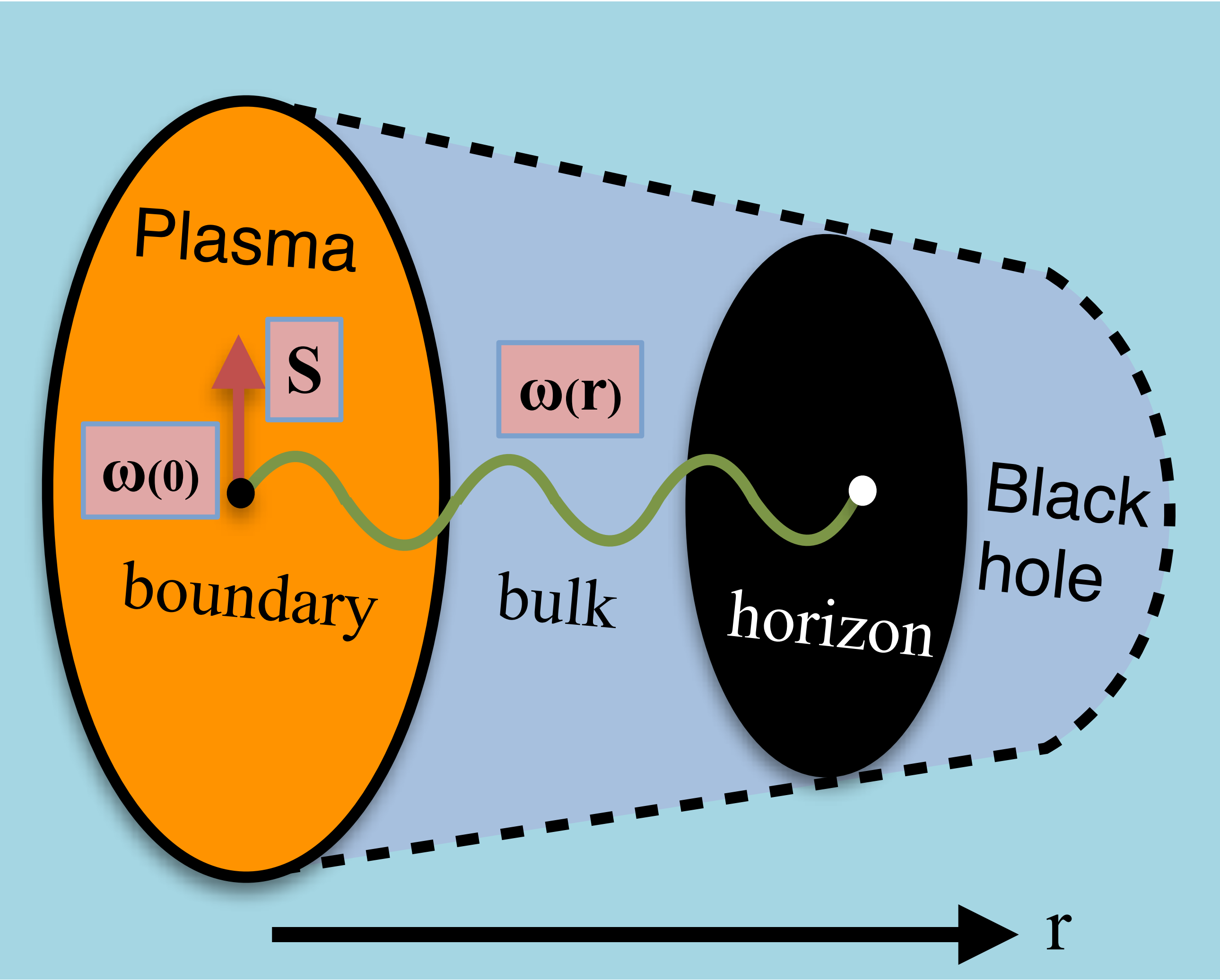
Similar universality in spin transport?

Holography of spin flow



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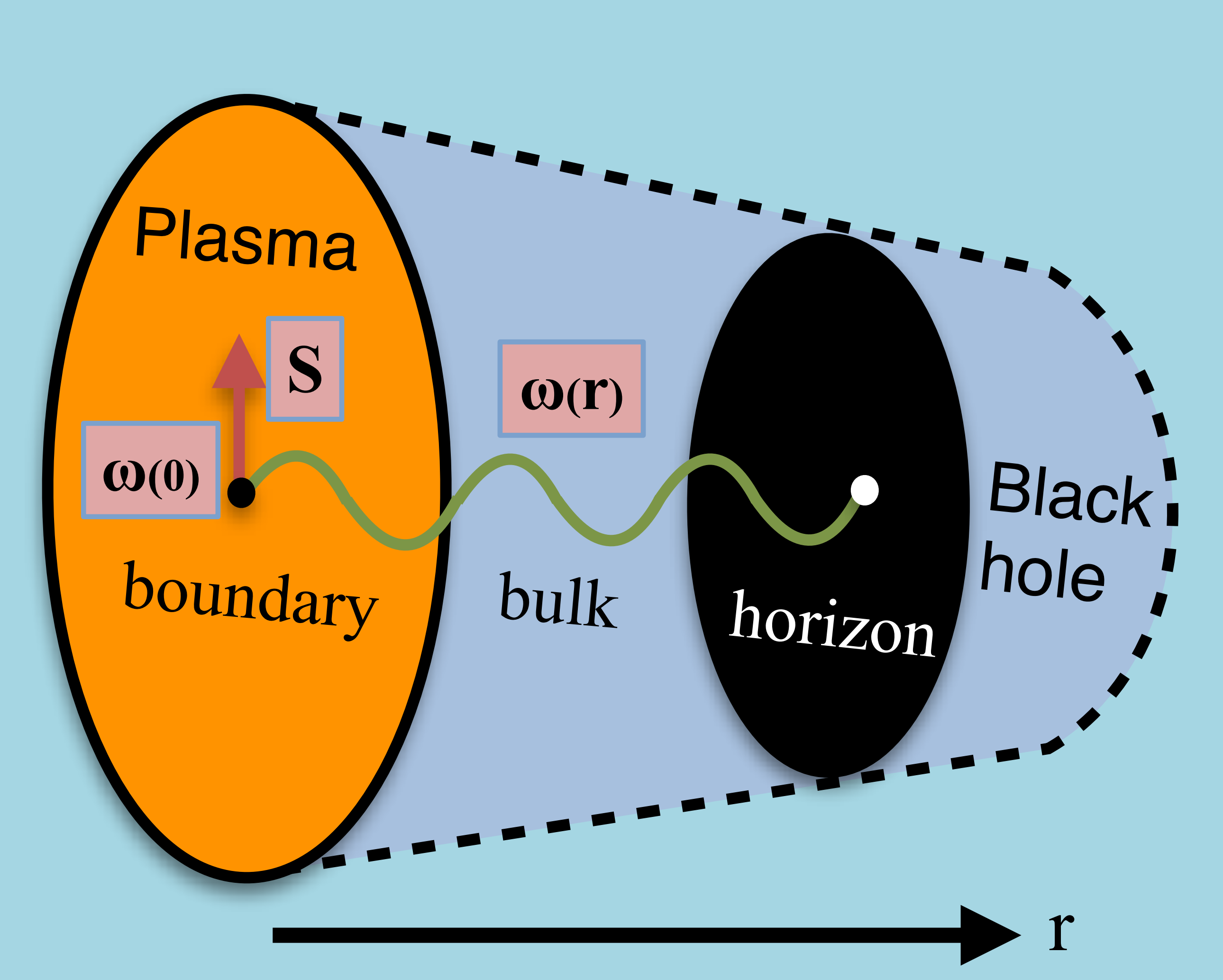

- Need a first order formulation with independent e^a and ω^{ab}
- Practical to have analytic black-hole solutions

Holographic description

Toy model: Lovelock-Chern-Simons AdS-gravity in 5D

Chamseddine ’89; Banados, Garay, Henneaux ’96; Zanelli 05;
Banados, Miskovic, Theisen ’06; Cvetkovic, Miskovic, Simic ‘17

$$S = \int \mathrm{Tr}\left\{\mathcal{F}\wedge\mathcal{F}\wedge\mathcal{A} - \frac{1}{2}\mathcal{F}\wedge\mathcal{A}\wedge\mathcal{A}\wedge\mathcal{A} + \frac{1}{10}\mathcal{A}\wedge\mathcal{A}\wedge\mathcal{A}\wedge\mathcal{A}\wedge\mathcal{A}\right\}$$

based on the G = SO(4,2) group algebra

- CS gravity for non-Abelian G, D $\geq$ 5 has local degrees of freedom unlike 3D
- Boundary symmetry $\simeq$ WZW_4 conformal theory with G

Nair, Schiff ’90; Losev, Nekrasov, Moore, Shatashvili ‘95

Lovelock-Chern-Simons

Relation to Einstein-Cartan in 5D: $\mathcal{J}_{A6} = P_A, \mathcal{J}_{AB} = J_{AB}$

$$\mathcal{A} \equiv \hat{e}^A P_A + \frac{1}{2}\hat{\omega}^{AB} J_{AB} \,,$$

$$\mathcal{F} = \hat{T}^A P_A + \frac{1}{2}\left(R^{AB} + \hat{e}^A \hat{e}^B\right) J_{AB}$$

First order gravity in 5D:

$$S_{LCS} = \kappa \int_{M_5} \epsilon_{ABCDE} \left[\underbrace{\hat{R}^{AB}\hat{R}^{CD}\hat{e}^E}_{\text{Gauss-Bonnet with } \lambda_{GB}=1/4} + \frac{2}{3}\underbrace{\hat{R}^{AB}\hat{e}^C\hat{e}^D\hat{e}^E}_{\text{Ricci}} + \frac{1}{5}\underbrace{\hat{e}^A\hat{e}^B\hat{e}^C\hat{e}^D\hat{e}^E}_{\text{Cosmological Constant}} \right]$$

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$\Longrightarrow$ Standard AdS solution for vanishing torsion

New solutions with propagating torsion Gallegos, UG ,'20

Charges and thermodynamics

Holographic renormalization in first order gravity: Banados, Miskovic, Theisen '06

$$\beta\mathcal{F} = \lim_{\epsilon\to 0}\left[S_{\text{on-shell}}(\epsilon) - V(\epsilon)\right]$$

counterterm action

Entropy as a charge from the Lee-Wald formula at horizon:

$$\mathcal{S} = 4\pi\int_{M_3^h}\epsilon_{ABCDF}\left(\hat{R}^{CD}\hat{e}^F + \frac{1}{3}\hat{e}^C\hat{e}^D\hat{e}^F\right)n^{AB}\,, \qquad n^{AB} \equiv D^{[A}\xi^{B]}$$

Total mass of the black hole: Gallegos, UG ,'20

$$M = M \equiv M_0 - \mu_I Q^I = \int_{M_3} d^3x\left[n_\mu\xi_\nu T^{\mu\nu} + \frac{1}{2}n_\lambda S^\lambda{}_{\mu\nu}\omega^{\mu\nu}{}_\alpha\xi^\alpha\right]$$

Satisfy the Smarr relation and the first-law:

$$\mathcal{F} = M_0 - T\mathcal{S} - \mu_I Q^I\,, \qquad d\mathcal{F} = -\mathcal{S}dT - Q^I d\mu_I\,,$$

Solution I: single axial

Thermodynamics:

$$\varepsilon = 96\pi^4 \kappa T^4 \left(1 + \frac{\mathcal{A}_2^2}{6\pi^2 T^2}\right) ,$$

$$p = -32\pi^4 \kappa T^4 \left(1 + \frac{\mathcal{A}_2^2}{6\pi^2 T^2}\right) ,$$

$$\pi^{\mu\nu} = 16\pi^2 \kappa T^2 \left(\Delta^{\mu}_{(\rho}\Delta^{\nu}_{\sigma)} - \frac{1}{3}\Delta^{\mu\nu}\Delta_{\rho\sigma}\right) \mathcal{A}_2^{\rho}\mathcal{A}_2^{\sigma}$$

Spin current:

$$S^{\lambda\mu\nu} = -32\pi^2 \kappa T^2 \left(u^{\lambda}\epsilon^{\mu\nu\alpha\beta} + \epsilon^{\lambda\alpha\beta[\mu}u^{\nu]}\right) u_{\alpha}\mathcal{A}_{2\beta}$$

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- Conformal fluid with axial vector spin source
- Non-trivial spin current *even for symmetric stress tensor!*

Solution II: Vector-axial-tensor

Thermodynamics:

$$p = -\epsilon/3 = -32\pi^4 \kappa T^4 \left(1 + \frac{\mathcal{C}^{\alpha\beta}\mathcal{C}_{\alpha\beta} - 2\mathcal{V}_2^2 - 6\mu_A^2}{12\pi^2 T^2}\right)$$

$$q^\mu = -16\pi^2 \kappa T^2 \left(\mathcal{C}^\mu_\alpha \mathcal{A}_1^\alpha - 2\mu_A \mathcal{A}_1^\mu + \epsilon^{\mu\alpha\rho\sigma} u_\alpha \mathcal{A}_{1\rho} \mathcal{V}_{2\sigma}\right) , \bar{q}^\mu = 0$$

$$\pi^{\mu\nu} = -16\pi^2 \kappa T^2 \left(C^{\mu\alpha} C^\nu_\alpha + \mathcal{V}_2^\mu \mathcal{V}_2^\nu - \frac{1}{3}\left(\mathcal{C}^\alpha_\beta \mathcal{C}^\beta_\alpha + \mathcal{V}_2^2\right) \Delta^{\mu\nu} - \mu_A \mathcal{C}^{\mu\nu}\right)$$

$$\tau^{\mu\nu} = 16\pi^2 \kappa T^2 u_\rho \mathcal{V}_2^\beta \epsilon^{\mu\nu\rho\sigma} \left(\mathcal{C}^\sigma_\beta + \mu_A \delta^\sigma_\beta\right)$$

Spin current:

$$S^{\lambda\mu\nu} = 32\pi^2 \kappa T^2 \left[\mu_A \epsilon^{\lambda\mu\nu\alpha} u_\alpha - u_\alpha \epsilon^{\lambda\alpha\beta[\mu} \left(\mathcal{C}^{\nu]}_\beta + u^{\nu]} \mathcal{A}_{1\beta}\right) + \Delta^{\lambda[\mu} \mathcal{V}_2^{\nu]} + 2u^\lambda u^{[\mu} \mathcal{V}_2^{\nu]}\right]$$

Solution II: Vector-axial-tensor

Thermodynamics:

$$p = -\epsilon/3 = -32\pi^4 \kappa T^4 \left(1 + \frac{\mathcal{C}^{\alpha\beta}\mathcal{C}_{\alpha\beta} - 2\mathcal{V}_2^2 - 6\mu_A^2}{12\pi^2 T^2}\right)$$

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$$S^{\lambda\mu\nu} = 32\pi^2 \kappa T^2 \left[\mu_A \epsilon^{\lambda\mu\nu\alpha} u_\alpha - u_\alpha \epsilon^{\lambda\alpha\beta[\mu} \left(\mathcal{C}^{\nu]}_\beta + u^{\nu]} \mathcal{A}_{1\beta}\right) + \Delta^{\lambda[\mu} \mathcal{V}_2^{\nu]} + 2u^\lambda u^{[\mu} \mathcal{V}_2^{\nu]}\right]$$

- Conformal fluid with spin sources
- Non-trivial spin current *even for symmetric stress tensor*
- Non-vanishing intrinsic torque and energy currents

Solutions at first order

Fluid-gravity correspondence: O(∂) transport to corrections to background

Bhattacharyya, Hubeny, Minwalla, Rangamani '08

Boosted black brane with corrections:

$$ds^2 = \frac{dr^2}{r^2 g^2} + r^2 \left[-f^2 u_\mu u_\nu + h^2 \Delta_{\mu\nu} + j \left(2hc_I \, u_\mu v^I_\nu - 2fb_I \, u_\mu v^I_\nu + h \, d_I t^I_{\mu\nu} \right) \right] dx^\mu dx^\nu$$

$\{c_I v^I_\mu, b_I v^I_\mu, d_I t^I_{\mu\nu}\}$ are linearly independent combinations of u^μ, T, ω^{ab}_μ

For simplicity we only consider $\mu_A \neq 0$ and $\mathcal{V}^\mu_2 \neq 0$

Transport for $\mu_A \neq 0$

Energy transport:

$$\bar{q}^\mu = -\frac{64\pi^4\kappa T^4}{\mu_A}\omega^\mu\,,$$

$$q^\mu = -\frac{64\pi^4\kappa T^4}{\mu_A}\omega^\mu\left(1-\frac{\mu_A^2}{2\pi^2T^2}\right)\,,$$

$$\pi^{\mu\nu} = 0\,,$$

$$\tau^{\mu\nu} = -16\pi^2\kappa T^2\epsilon^{\mu\nu\alpha\beta}u_\alpha\partial_\beta\mu_A$$

Spin transport:

$$J_A^\mu = 32\pi^2\kappa T^2\left(\mu_A u^\mu + \omega^\mu\right)\,.$$

Transport for $\mu_A \neq 0$

Energy transport:

$$\bar{q}^\mu = -\frac{64\pi^4 \kappa T^4}{\mu_A}\omega^\mu\,,$$

$$q^\mu = -\frac{64\pi^4 \kappa T^4}{\mu_A}\omega^\mu \left(1 - \frac{\mu_A^2}{2\pi^2 T^2}\right)\,,$$

$$\pi^{\mu\nu} = 0\,,$$

$$\tau^{\mu\nu} = -16\pi^2 \kappa T^2 \epsilon^{\mu\nu\alpha\beta} u_\alpha \partial_\beta \mu_A$$

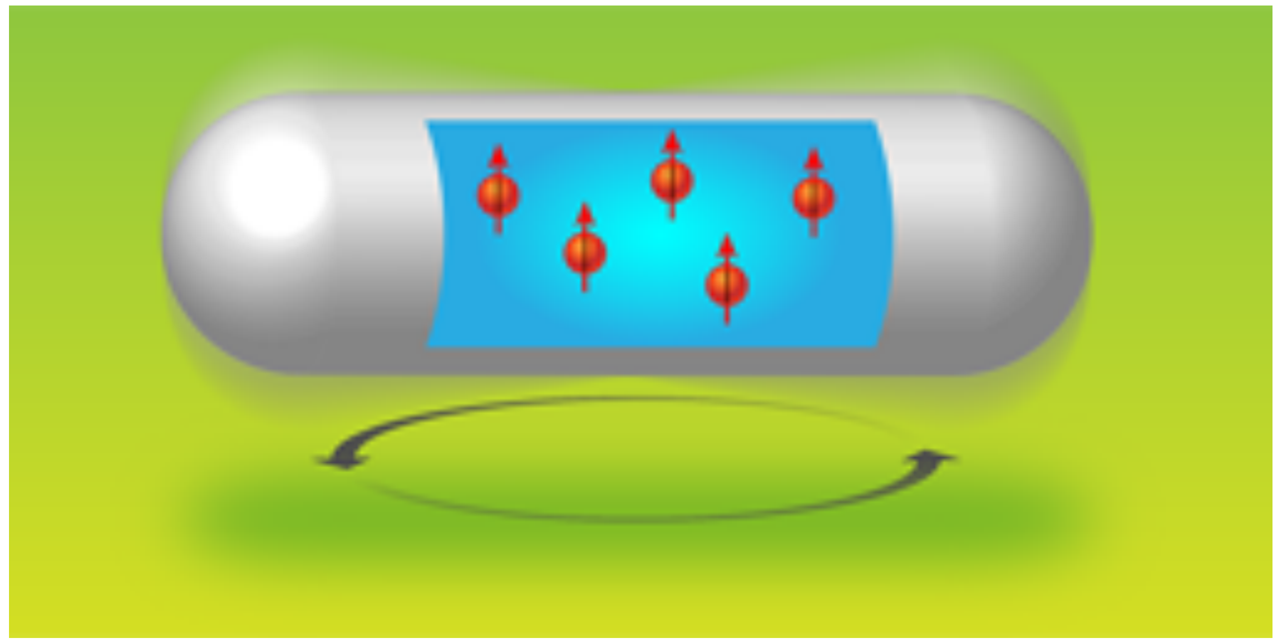
Spin transport:

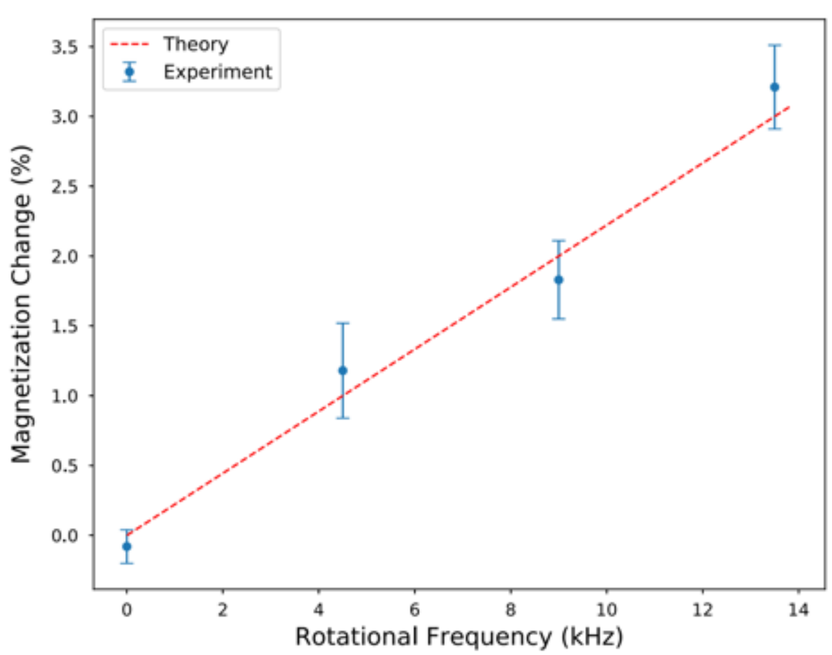
$$J_A^\mu = 32\pi^2 \kappa T^2 \left(\mu_A u^\mu + \omega^\mu\right)\,.$$

- Transport similar to chiral vortical separation effect: *generation of chiral imbalance by vorticity.*
- Here also an intrinsic torque
- More akin to "*chiral torsional effect*" Imaki and Yamamoto, '19

The Barnett effect

Magnetization of an uncharged body by rotation S. Barnett, 1915





$$J_A^\mu = 32\pi^2\kappa T^2\left(\mu_A u^\mu + \omega^\mu\right) .$$

Nuclear Barnett effect

Arabgol and Sleator '19

Transport for $\mathcal{V}_2^\mu \neq 0$

Novel possibilities for spin transport:

$$J_V^\mu = \frac{32}{3}\pi^2 T^2 (\nabla \cdot u) u^\mu \,,$$
$$\bar{J}_V^\mu = 32\pi^2 \kappa T^2 \mathcal{V}_2^\mu \,,$$
$$\bar{J}_A^\mu = 16\pi^2 \kappa T^2 \left(\eta^{\mu\nu} - \frac{\mathcal{V}_2^\mu \mathcal{V}_2^\nu}{\mathcal{V}_2^2} \right) \omega_\nu$$

- Spin current from divergence
- “Vortical-Hall” spin current

Summary

- Strong demand to develop relativistic hydrodynamics with spin
- Torsion acts as a source for the spin current
- Derived the hydro equations and constructed constitutive relations
- Novel effects from hydrodynamic effective action
- Lovelock-Chern-Simons provides a fruitful toy theory
- Charazterization of spin transport coefficients from holography

Outlook

- A systematic study of relativistic spin transport:

  Entropy generation, Onsager relations, etc.

- Analysis at weak coupling $\Longrightarrow$ universal transport coefficients?

- Derivation in more general holographic theories: e.g. supergravity

- Observable effects of the novel spin transport?