En route to a self-consistent AdS/QCD model in a magnetic background

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Talk during "Frontiers of holographic duality-2" (Steklov Mathematical Institute, online, Moscow), 27/10/2020

Based on

- H. Bohra, D. Dudal, A. Hajilou and S. Mahapatra, Phys.Lett.B 801 (2020) 135184
- H. Bohra, D. Dudal, A. Hajilou and S. Mahapatra, [arXiv:2010.04578 [hep-th]].
- Work in progress.

Overview

Quantum chromodynamics (at zero temperature)

2 crucial "effects"

- confinement: no free color charges (quarks/gluons), but only color-neutral states (mesons, baryons including proton and neutron, glueballs, hybrids)

- issue of chiral symmetry breaking: QCD action has chiral symmetry (very light quarks)
  $\Rightarrow$ dynamically this symmetry gets broken
  $\Rightarrow$ quarks get large effective masses, which in return "feed" the mass of the hadrons.
- Why so difficult: inherently nonperturbative problems: effects in e^{-1/g^2}

Quantum chromodynamics

2 crucial strong coupling effects

- These are in fact “old problems”, but still not fully understood, let stand alone formally proven!
- What happens at (very) high temperature, relevant for quark-gluon plasma (QGP)(RHIC or ALICE heavy ion collision, early universe)?

  With high, we mean $T \sim 200$ MeV $\sim 10^{12}$ K $\sim 100000\times$ the temperature in the Sun’s core.
- Thermal energy can (and does) destroy confinement bonds.
- Nonetheless, $g^2(T) \gg 1$, thus still strongly coupled physics at phase transition.
- Similar comments for high density QCD, also relevant for compact stellar objects.

Computation in QCD

How to extract physical information from QCD?

- QCD $=$ strongly coupled/nonperturbative physics is important
- hard to handle analytically
- several options:
  1. model building I: e.g. AdS/QCD aka Holographic QCD.
  2. model building II: e.g. effective models like NJL models, ...
  3. computer simulations: lattice QCD
  4. quantize the theory and try to get as good information as possible by variety of techniques (e.g. Dyson-Schwinger)

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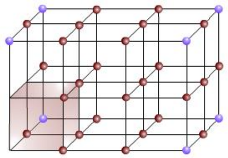
  2. model building II: e.g. effective models like NJL models, ...
  3. computer simulations: lattice QCD
  4. quantize the theory and try to get as good information as possible by variety of techniques (e.g. Dyson-Schwinger)
  5. compare with experiment and/or lattice data the intermediate and final results

Computation in QCD

Lattice simulations

- Lattice QCD: gauge invariant expectation values can be computed without gauge fixing (space-time is discretized).

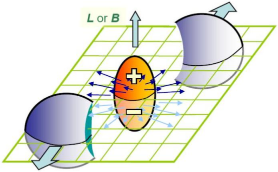


- Use Monte Carlo simulations with partition function $\int [dA] e^{-S_{YM}}$
- The numerical estimates are in good agreement with experimental data
- Comment: Real-life QCD (with massless/massive dynamical quarks) is however not that easy to simulate. . .
- Also difficult for real-time applications (transport properties, decays etc) or at finite density: sign (oscillatory) problem!

Overview

Why study strong magnetic fields?

- Experimental relevance: appearance in QGP after a (non-central) heavy ion collision (order $eB \sim 1-15m_\pi^2$) (Skokov, Tuchin, Kharzeev, McLerran, Deng, Huang ...)



$B+$ its anisotropy $\Rightarrow$ unexpected experimental features?

- Interesting new anomalous (transport) effects (chiral magnetic effect CME, chiral magnetic wave CMW), with analogues in condensed matter physics.
- no sign problem for finite B simulations!

Why study strong magnetic fields?

Lifetime$_{\text{constant } B} \sim 10$ fm McLerran, Skokov, NPA929 (2014); Tuchin, PRC88 (2013) .

Lifetime$_{QGP} \sim 1 - 10$ fm $\rightarrow$ incentive to take $\vec{B} = B\vec{e}_z$ constant (ignoring "spatial decay" as well!) $B_{LHC} \sim 0.2 - 0.3$ GeV^2

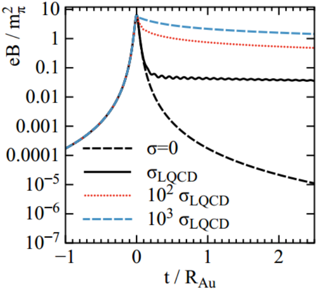


Figure: McLerran, Skokov, NPA929 (2014)

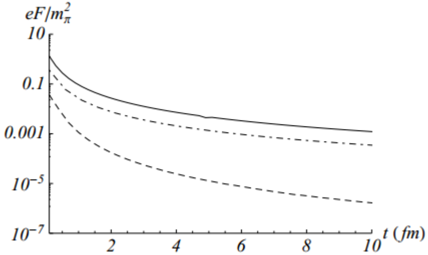
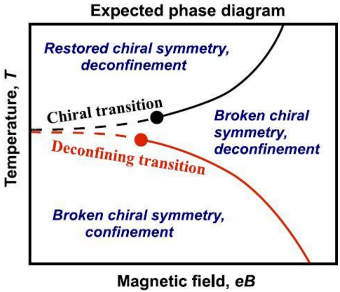


Figure: Tuchin PRC88 (2013)

Phase transitions in a magnetic field: magnetic catalysis?

Deconfinement transition T_{crit} and/or chiral transition T_{chiral} depending on B? If so, how?



Expected (chiral) behaviour: strong $\vec{B} \Rightarrow$ dynamics squeezed along $\vec{B}$, $\sim$ effectively a 2D theory, known to exhibit magnetic catalysis (Miransky et al).

Lattice QCD (our experiment under these circumstances): *inverse* magnetic catalysis

Unquenched lattice data: $T_{chiral}(eB) = T_{crit}(eB) \equiv T(eB)$ goes down.)

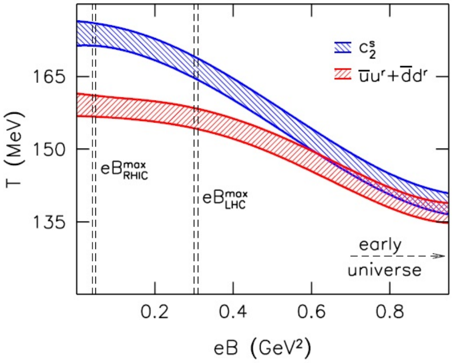


Figure: Bali et al, JHEP 1202 (2012); PRD86 (2012).

Any (in)direct experimental evidence for $\vec{B}$ in QGP?

- no direct evidence of $\vec{B}$.
- CME/CMW: possibly, but signal still consistent with zero, albeit with large errors. See Zhao and Wang, Prog. Part. Nucl. Phys. 107 (2019). Also “theoretical doubts”, see Rybalka, Gorbar, Shovkovy, Phys. Rev. D 99 (2019) (overdampening effect in the quark-gluon plasma destroys CME/CMW).
- Relation to anisotropy in direct photon production? (elliptic flow) [PHENIX Collaboration], Nucl. Phys. A 931 (2014) 1189; [ALICE Collaboration], J. Phys. Conf. Ser. 446 (2013) 012028.
- Relation to hyperon polarization? [STAR], Phys. Rev. C 98, 014910 (2018)

(De)confinement order parameter in Euclidean space

- Theoretical (formal) view on (de)confinement: VEV of Polyakov loop (breaking or not of $\mathbb{Z}_N$ center "symmetry").

$$\langle P \rangle = \langle \mathcal{P} e^{\int_0^\beta dt A_0} \rangle \sim e^{-F_q T}$$

- $\beta =$ inverse temperature, $F_q =$ free energy of a quark.
- Center transformation $=$ gauge transformation that is periodic-in-t up to a center transformation z_n. Leaves QCD action invariant in absence of quarks, but $\langle P \rangle \to z \langle P \rangle$
- Preserved symmetry iff $\langle P \rangle = 0$, that is, if $F_q = \infty$, that is, one cannot create a free quark: confinement!
- Deconfinement phase transition at T_c where $\langle P \rangle \neq 0$.
- In presence of quarks, only pseudo-order parameter (cross-over). So study of $\langle P \rangle$ still viable, also accessible via lattice.
- $\langle P \rangle$ is experimentally not so useful to "measure" onset of deconfinement.

Polyakov loop from lattice QCD

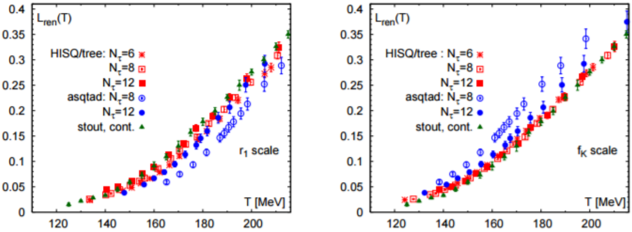


Figure: Bazavov et al, PRD85 (2012). T_c is inferred from the inflection point.

A complementary view

- Phenomenological view on deconfinement: QCD bound states are "melted away".
- Prototypical example: charm bound state, J/ψ.

  Its suppression known to be a good signal for the plasma phase (Matsui/Satz PLB178 (1986))

  Known to only melt "beyond deconfinement"; lattice spectral function analysis Asakawa/Hatsuda PRL92 (2004)

One can investigate how B influences the J/Ψ melting! See e.g. Braga's talk

A complementary view

- An important ingredient in the J/Ψ phenomenology is the linear potential (string tension) between the heavy constituents. Confining potential becomes B-dependent, elements of anisotropy $\rightarrow$ backlash on quarkonia properties!
- Various studies on the market on quarkonia in B-field Bonati et al, Dudal & Mertens, Strickland et al, Braga et al. . . .

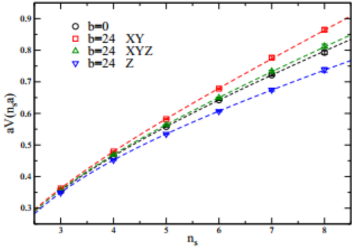


Figure: D'Elia et al, PRD89 (2014), PRD94 (2016) (lattice) + Chernodub, Mod.Phys.Lett.A 29 (2014) (pheno): static quark potential stronger in $\perp$ direction than in $\|$ one

So what will we be interesting in during this talk and how we'll look into these issues?

- String tension between heavy quarks in terms of $\vec{B}$ (see also Slepov's talk).
- Deconfinement and chiral transition.

$\rightarrow$ Via self-consistent holographic (AdS/QCD) model.

- Long(er) term goals: a full holographic study of elliptic flow, or the lifetime of the magnetic field, or ..., is not feasible/realistic. But certain (crucial) ingredients for such other studies are e.g. the EM conductivity, the heavy quark diffusion rates, the magnetization, etc. These are inherently non-perturbative QCD quantities, sometimes hard to access with lattice tools, but accessible (to some extent) by AdS/QCD.

  Let us first try to construct a reliable dual background, inspired by lattice QCD input in circumstances where the latter is available.

Overview

How to make magnetic predictions in holographic QCD models?

- Confinement is modeled in/described by background metric (e.g. AdS with a cut-off)

  Cut-off scale is needed to break conformal invariance, to provide a QCD scale.
- Quark physics is mostly modeled in via probe branes/effective (DBI) actions ($\rightarrow$ "quenched QCD": no dynamical quarks)

  (exception is V-QCD, Jarvinen et al, JHEP 03 (2012) and follow-up works.)
- problematic to capture all magnetic field effects: B can only couple to neutral glue/geometric background if charged quark dynamics is taken into account!
- In general: deconfinement transition $\leftrightarrow$ interpreted as Hawking-Page transition in dual picture [free energy comparison]. Checks with Polyakov loop.

(original) Soft wall model

Karch, Katz, Son, Stephanov, PRD74 (2006)

low T confined phase

$$ds^2 = \frac{L^2}{r^2}\left(-dt^2 + d\mathbf{x}^2 + dr^2\right), \qquad e^{-\Phi} = e^{-cr^2},$$

$r = 0$ (the QCD boundary) $\ldots\infty$.

high T deconfined phase (black hole)

$$ds^2 = \frac{L^2}{r^2}\left(-f(r)dt^2 + d\mathbf{x}^2 + \frac{dr^2}{f(r)}\right), \qquad e^{-\Phi} = e^{-cr^2}$$

$f(r) = 1 - r^4/r_h^4$. $r = 0\ldots r_h$; $T = \frac{1}{\pi r_h}$.

(original) Wall metric

Erlich, Katz, Son, Stephanov PRL95 (2005)

Soft wall action

$$S \propto \int d^5x\sqrt{-g}e^{-\Phi}\mathrm{tr}\left[F^2_{L,\mu\nu} + F^2_{R,\mu\nu} + |DX|^2 - m_5^2|X|^2\right]$$

(Einstein-Hilbert + Gibbons-Hawking boundary part not written)

Hard wall

dilaton $\Phi = 0$, but AdS cut hardly at $z = z_c$. The rest is the same.

A few (well known) properties

Field content, chiral dynamics, deconfinement

- A$\sim$ (gauge invariant) operators on the boundary (mesons)
- X = bifundamental, $X \sim \langle \bar{q}q \rangle$.
- String tension $\sigma_{QCD} \propto 1/r_0$ in hard wall. (No area law in this simple soft wall!)
- Linear behaviour $m_n \sim n$ in soft wall. Not in hard wall!
- Few parameters fixed by matching on QCD states at $T = 0 \Rightarrow$ decent results also for other states.
- QCD-desired chiral behaviour (GMOR relation etc).
- Deconfinement phase transition: $T_c \sim 122$ MeV (hard) or $T_c \sim 191$ MeV (soft) (Herzog, PRL98 (2007)).
- Neither hard or soft wall solve the bulk Einstein EOMs. (though, more involved AdS/QCD models do, evidently at the cost of complication, already for $B = 0$. but then also linear confining potential etc possible.)

Step 1: Einstein-Maxwell-Dilaton action

$$S_{EM} = -\frac{1}{16\pi G_5}\int \mathrm{d}^5x\sqrt{-g}\left[R - \frac{f_1(\phi)}{4}F_{(1)MN}F^{MN} - \frac{f_2(\phi)}{4}F_{(2)MN}F^{MN} - \frac{1}{2}\partial_M\phi\partial^M\phi - V(\phi)\right].$$

- We use $U(1)\times U(1)$, dual to baryon number current and to EM current.

  [Simplification of course, we are currently generalizing to QCD appropriate $U_b(1)\times SU_V(2)\times SU_A(2)$]

  [In theory: magnetic field can couple to mesons, but in $U(1)$ setting no charged mesons. Using the (unbroken) $U(1)'s$ in $U_b(1)\times SU_V(2)\times SU_A(2)$ to introduce appropriate vector potential and charged mesons. But this is not yet finished.]
- We use 1 $U(1)$ to couple a constant B-field to the theory, the other one to describe meson fluctuations.
- $B_{phys}\propto\frac{B}{L}$; we will keep using B.

Step 2: potential reconstruction method

- Dilaton potential $V(\phi)$ not yet fixed.
- We will parameterize the metric, introduce a few Ansätze and construct the (on-shell) potential corresponding $V(z)$ to the choices.

  This is a potential reconstruction approach, see also Alanen et al, Phys. Rev. D 80, 126008 (2009); Aref'eva, Slepov et al.
- A posteriori, we will check if that potential meets some physical requirements.

Step 3: the Ansätze

$$ds^2 = \frac{L^2 S(z)}{z^2}\left[-g(z)dt^2 + \frac{dz^2}{g(z)} + dy_1^2 + e^{B^2 z^2}\left(dy_2^2 + dy_3^2\right)\right],$$
$$\phi = \phi(z), \ \ A_{(1)M} = A_t(z)\delta^t_M, \ \ F_{(2)MN} = B dy_2 \wedge dy_3 \,.$$

We set

$$f_1(z) = \frac{e^{-cz^2 - B^2 z^2}}{\sqrt{S(z)}}\,.$$

then vector meson spectra on linear Regge trajectories for $B = 0$, with $m_n^2 = 4cn$. $c = 1.16$ GeV^2 by matching with lowest lying heavy meson states J/Ψ and Ψ'.

$$S(z) \ \ = \ \ e^{2A(z)}$$

A common choice is $A(z) = -az^2$ (see also original wall model or Andreev, Phys.Rev.D 73 (2006) with $a = 0.15$ GeV^2 to match deconfinement temperature $T_{crit} \approx 0.27$ GeV with $N \to \infty$ lattice extrapolation Lucini et al, JHEP 0401 (2004).

Step 4: solving the Einstein, ϕ and A_t-EOMs

with boundary conditions

$$g(0)=1 \text{ and } g(z_h)=0 \text{ (black hole)} \quad \text{or } g(z)\equiv 1 \text{ (thermal AdS)},$$
$$A_t(0)=\mu \text{ and } A_t(z_h)=0,$$
$$S(0)=1,$$
$$\phi(0)=0\,.$$

The dilaton in particular:

$$\phi(z)=\frac{\left(9a-B^2\right)\log\left(\sqrt{6a^2-B^4}\sqrt{6a^2z^2+9a-B^4z^2-B^2}+6a^2z-B^4z\right)}{\sqrt{6a^2-B^4}}$$
$$+z\sqrt{6a^2z^2+9a-B^2\left(B^2z^2+1\right)}-\frac{\left(9a-B^2\right)\log\left(\sqrt{9a-B^2}\sqrt{6a^2-B^4}\right)}{\sqrt{6a^2-B^4}}\,.$$

$\rightarrow B^4\leq 6a^2$ to ensure $\phi\in\mathbb{R}$.

Physicalness of the potential

- $\phi \to 0$ for $z \to 0$ (so conformal limit in the UV)
- $V(\phi) = -\frac{12}{L^2} + \frac{\Delta(\Delta-4)}{2}\phi^2 + \ldots$, $2 < \Delta < 4$ (so expected AdS limit for $\phi \to 0$ and BF bound is OK)
- $V(z) \leq V(0)$ (so Gubser stability criterion is also met)
- $V(\phi)$ is (almost) a genuine function of just ϕ, in that sense there is (almost) no dependence on T (or z_h), μ or B. This can be inferred from the following figures.

Physicalness of the potential

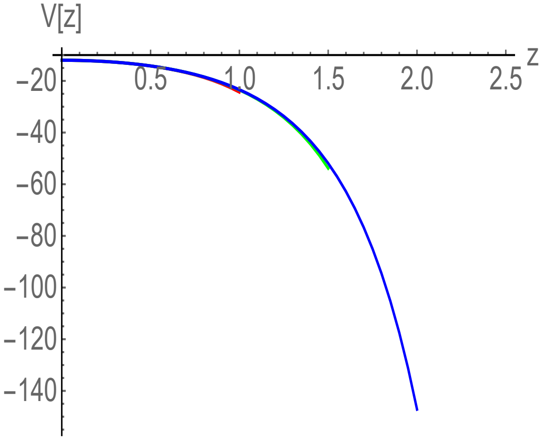


Figure: The variation of potential as a function of z for different z_h. Here $\mu = 0$ and $B = 0$ are considered. Here red, green and blue curves correspond to $z_h = 1$, $z_h = 1.5$ and $z_h = 2$ respectively.

Physicalness of the potential

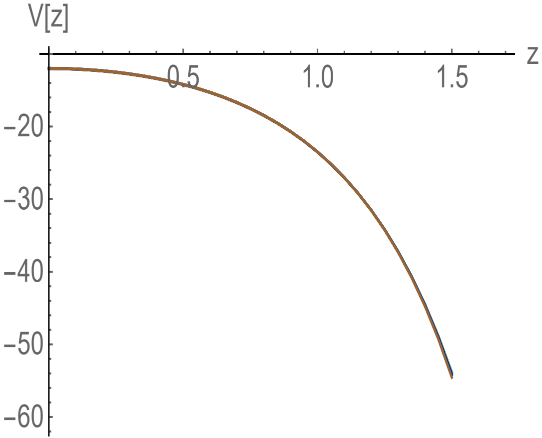
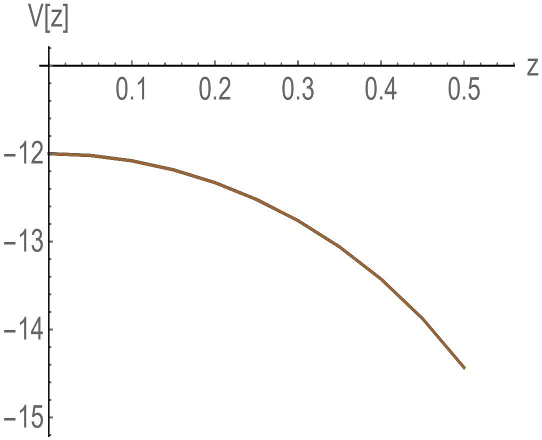


Figure: The variation of potential with different values of μ. Here $B = 0$ and $z_h = 1.5$ are considered. Red, green, blue and brown curves correspond to $\mu = 0, 0.2, 0.4$ and 0.6 respectively.

Physicalness of the potential



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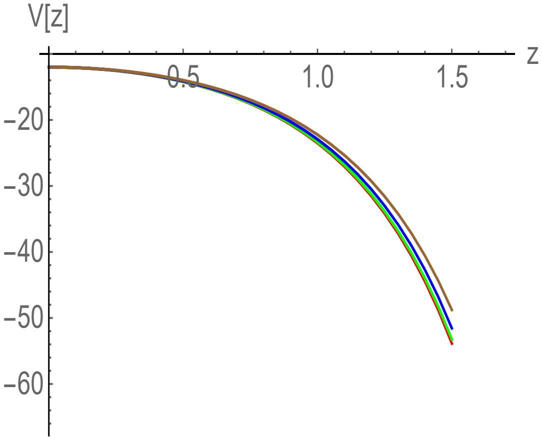
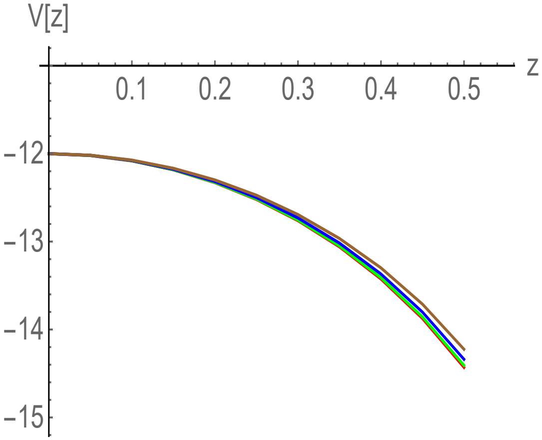


Figure: The variation of potential with different values of B. Here $\mu = 0$ and $z_h = 1.5$ are considered. Red, green, blue and brown curves correspond to $B = 0$, 0.1, 0.2 and 0.3 respectively.

Physicalness of the potential



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Overview

A few words on QCD

Motivation for this work

Holographic QCD in a magnetic field: model setup

Holographic QCD in a magnetic field: deconfinement and string tensions

Holographic QCD in a magnetic field: chiral sector

Thermodynamics and deconfinement transition

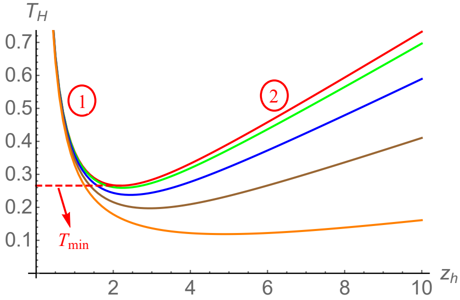


Figure: Temperature T as a function of horizon radius z_h for various values of the magnetic field B and $\mu = 0$. Here red, green, blue, brown and orange curves correspond to $B = 0$, 0.15, 0.30, 0.45 and 0.6 respectively. In units GeV. (1) is a stable, “large” black hole, (2) is an unstable, “small” black hole phase. Below T_{min}, there is no black hole. But there is a Hawking-Page transition from thermal AdS to (1) for $T > T_{HP} > T_{min}$.

Thermodynamics and deconfinement transition

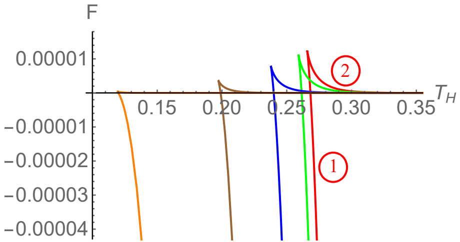
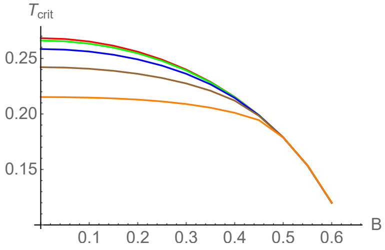


Figure: Free energy F as a function of temperature T for various values of the magnetic field B and $\mu = 0$. Here red, green, blue, brown and orange curves correspond to $B = 0, 0.15, 0.30, 0.45$ and 0.6 respectively. In units GeV. (1) is a stable, "large" black hole, (2) is an unstable, "small" black hole phase.

Deconfinement transition and inverse magnetic catalysis



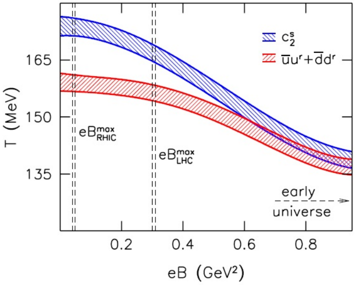
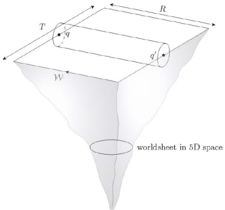


Figure: The variation of thermal AdS–black hole phase transition critical temperature T_{crit} with magnetic field B for various values of chemical potential μ. Here red, green, blue, brown and orange curves corresponds to $\mu = 0.0, 0.3, 0.6, 0.9$ and 1.2 respectively. In units GeV. Qualitatively consistent with lattice data. We do not have the inflection point). Remember, we are **mimicking** magnetized QCD: no (dynamical) fermions to “explain” B-dependent metric.

Wilson loop and string tension



$$\mathcal{F}(\ell, T) = T S_{NG}^{on-shell}(\ell, T)$$

where

$$S_{NG} = \frac{1}{2\pi\ell_s^2}\int d\tau d\sigma \sqrt{-\det\ G_s}$$

$$T_s = \frac{1}{2\pi\ell_s^2};\ G_s = \text{induced metric}$$

- The dual of the Wilson loop (rectangle $R \times T$) VEV comes from the free energy (via Nambu-Goto action) of an open string connecting the quark q with antiquark $\bar{q}$, with separation $R \equiv \ell$ Maldacena, PRL (1998), Brandhuber et al, PLB (1998).
- String frame: $(g_s)_{MN} = e^{\sqrt{\frac{2}{3}}\phi} g_{MN}$ and $A_s(z) = A(z) + \frac{1}{\sqrt{6}}\phi(z)$.
- Regularization/renormalization around $z = \varepsilon$, minimal subtraction of divergences, following Ewerz et al, JHEP **1803**, 088 (2018).

Wilson loop and string tension: parallel case

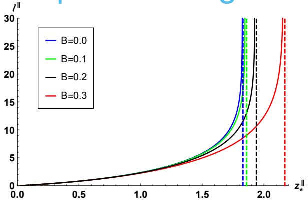


Figure: $\ell^{\parallel}$ as a function of $z_*^{\parallel}$ in the thermal AdS background for different (small) magnetic fields and $\mu = 0$. In units GeV. There is a kind of "dynamical wall" disallowing deeper penetration into the bulk (related to $A_s(z)$ having a minimum)

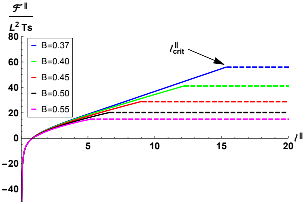


Figure: $\mathcal{F}^{\parallel}$ as a function of $\ell^{\parallel}$ in the thermal AdS background for different (large) magnetic fields and $\mu = 0$. In units GeV.

- $q, \bar{q}$ pair parallel/orthogonal to $\vec{B}$.

  Two possible string-configurations: $\cup$ hanging down until $z = \ell^*$ (connected) and 2 straight lines (disconnected).
- Depending on B, the connected or disconnected configuration has lower energy.

Wilson loop and string tension: discussion

- very similar results for the perpendicular orientation (I'll skip here for now).
- Peculiarity: we are mimicking QCD all too well: for larger B, there is a string breaking (disconnected config wins). But we do not have dynamical (light) quarks to make this possible.
- This can be averted by slightly adapting the form factor $A(z) = -az^2 - dB^2z^5$

  Then

  $$\mathcal{F}_{discon} = \frac{L^2}{\pi \ell_s^2} \int_\varepsilon^\infty dz \left[z^4 + \frac{2B^2z^3}{15d} + \ldots \right] = +\infty \text{ whilst } \mathcal{F}_{con} < +\infty$$

Deconfinement with new form factor $A(z) = -az^2 - dB^2 z^5$

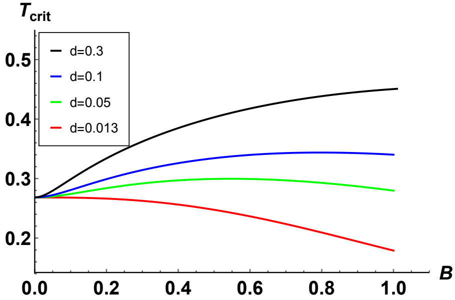
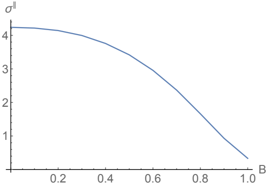
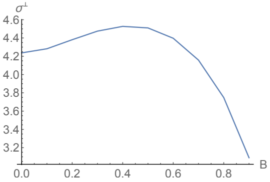


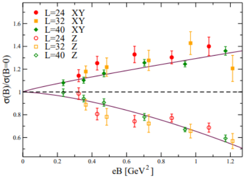
Figure: Deconfinement transition temperature in terms of magnetic field for the case $A(z) = -az^2 - d\,B^2 z^5$. Here we set $\mu = 0$. In units GeV.

- Depending on value of d, (inverse) magnetic catalysis.
- We took $d \approx 0.013$ GeV^2, to have largest $B_{max} = 1.02$ GeV and inverse catalysis.

String tensions with new form factor $A(z) = -az^2 - dB^2z^5$







- Both cases can be fitted perfectly well with a Cornell potential

$$\frac{\mathcal{F}_{con}^{\|,\perp}}{L^2 T_s} = -\frac{\kappa^{\|,\perp}}{\ell^{\|,\perp}} + \sigma_s^{\|,\perp}\ell^{\|,\perp} + C^{\|,\perp}$$

  We saw that the Coulomb strength $\kappa^{\|,\perp}(B) \approx \kappa(B=0)$, consistent with lattice data of D'Elia et al., Phys.Rev.D 89 (2014).

- For small B qualitatively OK with lattice ("less/more confinement" in parallel/perpendicular direction). For larger B, it remains to be seen what happens.

Overview

A few words on QCD

Motivation for this work

Holographic QCD in a magnetic field: model setup

Holographic QCD in a magnetic field: deconfinement and string tensions

Holographic QCD in a magnetic field: chiral sector

Adding flavour in probe brane approximation

Inspired by Colangelo et al, Eur.Phys.J.C 72 (2012), we add

$$S_{chiral} = \frac{N_c}{16\pi^2} \int \mathrm{d}^5 x \sqrt{-g}\, \mathrm{Tr}\left[|DX|^2 - m_5^2 |X|^2\right] .$$

- X is a $N_f \times N_f$ matrix-valued complex field in the bifundamental representation of $SU(N_f)_L \times SU(N_f)_R$, dual to $\langle \bar{\psi}^\alpha \psi^\beta \rangle$; $m_5^2 L^2 = -3$; $D_\mu X = \partial_\mu X - iA_{L,\mu} X + iX A_{R,\mu}$

  $SU(N_f)_L \times SU(N_f)_R$ invariance.
- Consider degenerate flavours by $X(z, x^\mu) = X_0(z) \mathbf{1}_{N_f} e^{i\pi(z,x^\mu)}$. In this approximation, the condensate can be extracted by solving the X-field equation of motion.
- Important difference: we do not need to add "phenomenological" $e^{-\phi}$ to implement confinement (already part of the metric).

The chiral condensate itself

- We must solve (in black hole background)

$$X_0''(z)+X_0'(z)\left(-\frac{3}{z}+2B^2z+\frac{g'(z)}{g(z)}+3A'(z)\right)+\frac{3e^{2A(z)}X_0(z)}{z^2g(z)}=0$$

with UV boundary expansion (cf. gauge-gravity dictionary, $d=1$ source m_q and $d=3$ chiral operator)

$$X(z)=m_qz+\sigma z^3+m_qnz^3\ln\sqrt{a}z+\mathcal{O}(z^4)$$

- m_q is input (bare quark mass), σ is numerically computed (e.g. via shooting method).
- In most works, σ is used as an avatar for chiral condensate itself.

The chiral condensate itself

We will however be a bit more careful. Following Dudal et al, Phys.Rev.D 93 (2016), we find

$$\langle\bar{\psi}\psi\rangle_{B,T} - \langle\bar{\psi}\psi\rangle_{B=0,T=0} = \frac{N_c}{2\pi^2}\left[\sigma(B,T) - \sigma(B=0,T=0)\right] + \frac{N_c m_q}{8\pi^2}B^2$$

via the partition function Z and gauge-gravity duality:

$$\langle\bar{\psi}\psi\rangle = \frac{1}{Z}\frac{dZ}{dm_q} = -\frac{d}{dm_q}\left(\frac{N_c}{16\pi^2}\int d^5x\sqrt{-g}\,\left[\partial_\mu X\partial^\mu X + m_5^2X^2\right]\right)$$
$$= \frac{N_c}{8\pi^2}\left(\frac{m_q}{\epsilon^2} + 4m_q n\log\sqrt{a}\epsilon + 4\sigma + B^2 m_q + 5m_q n - 3am_q\right)$$

The subtraction renders a finite result, irrespective of B and/or T, completely analogous to lattice procedure to renormalize the chiral condensate. Including $e^{-\phi}$ destroys this property. We take $\langle\bar{\psi}\psi\rangle_{B=0,T=0} = 0.0194\ \mathrm{GeV}^3$ ($N_f = 1$ lattice estimate from DeGrand, PoS LAT2006).

The chiral condensate itself

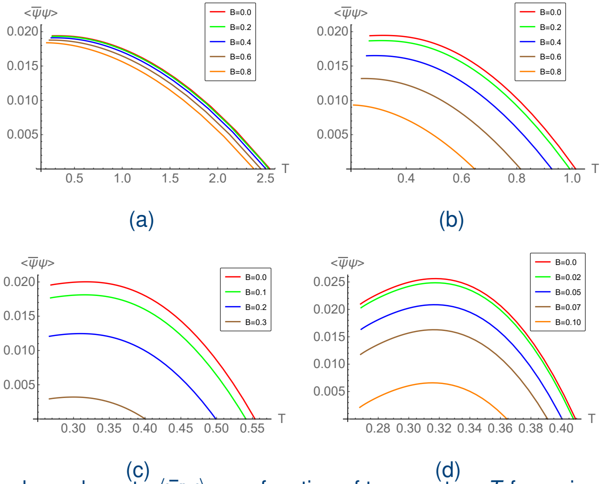


Figure: Quark condensate $\langle\bar{\psi}\psi\rangle$ as a function of temperature T for various values of the quark mass m_q. The upper left, upper right, lower left and lower right figures correspond to $m_q = 0.01, 0.1, 1.0$ and 10.0 respectively. In units GeV.

Inverse magnetic catalysis

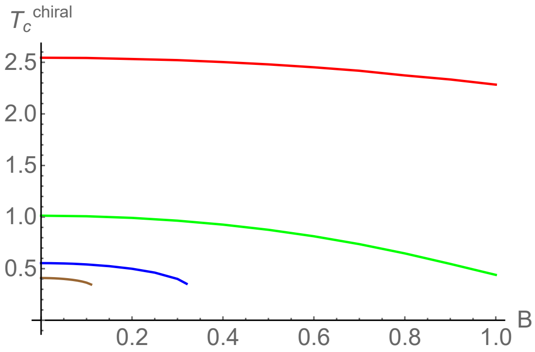


Figure: The variation of the chiral critical temperature with respect to magnetic field B for different quark masses. Here red, green, blue and brown curves correspond to $m_q = 0.01$, 0.10, 1.0 and 10.0 respectively. In units GeV.

Unfortunate feature of these type of models

- σ basically regulates the condensate, but $\sigma \propto m_q$. This is QCD-unlike!

  Situation handled in e.g. Gherghetta et al, Phys.Rev.D 79 (2009) by using a phenomenological Ansatz for $X_0(z)$ and a $V(X)$-potential, unfortunately still with a geometry not solving the Einstein-EOMs.
- We are now generalizing the potential reconstruction method to get a fully backreacted model, combining the phenomenological Ansatz for $X_0(z)$ with a corresponding potential $V(X)$.

Further improvements and applications

- Quark-gluon plasma transport properties, bringing earlier results of Dudal and Mertens to the level of the self-consistent background model.
- Charmonia spectra in magnetic field, including anomalous mixing with η_c.
- Can we embed the improved charmonia modelling of Braga et al into a self-consistent gravity model?

The End.

Spasiba!