Logic & foundations
A personal perspective

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Preamble

My thanks to the organizers for the invite.

The topic is close to my heart. It richly deserves

- a broad sweep of the glorious past,
- examining the less glorious present, and
- discussing the uncertain future.

But time is short and my scholarship is limited. Hence:

1. a personal perspective and
2. replacing systematic exploration with a few illustrations.

The goal

To spur a discussion on the role of foundations in logic.

If you find the talk is provocative, you are not wrong.
It is intended to be so.

Epigraph 1

When the roots are deep,
there is no reason to fear the wind.

Chinese proverb

Epigraph 2

There does not exist a category of science
to which one can give the name applied science.
There are sciences and the application of science,
bound together as a tree and the fruit it bears.

Louis Pasteur

Putting it more bluntly

It's not enough to know your worth;
you still need to be in demand.

Michael Zhvanetsky

Russian original:

Мало знать себе цену;
надо ещё пользоваться спросом.

My own roads to foundations, in

- First exposure: Congruent triangles
- Ural State University: Algebra, and logic, and gig jobs
- Jerusalem logic seminar
- University of Michigan

  What's an algorithm?
- Microsoft smorgasbord

  Software specification, verification, testing
  Access control, security, cybersecurity

  ⋮

  Quantum computing

Fertile Crescent: Alphabet, ≈ 1800 BCE

Continuum of sounds ⟶ few sounds. An astounding idea, a sacred result, imperfect but good enough execution. A triumph of syntax over semantics.

	Proto-Canaanite	Early Phoenician	Greek		Proto-Canaanite	Early Phoenician	Greek
ʾ			A	l			Λ
b			B	m			M
g			Γ	n			N
d			Δ	s			Ξ
h			E	ʿ			O
w			Y	p			Π
z			Z	ṣ			M
ḥ			H	q			
ṭ			Θ	r			P
y			I	š			Σ
k			K	t			T

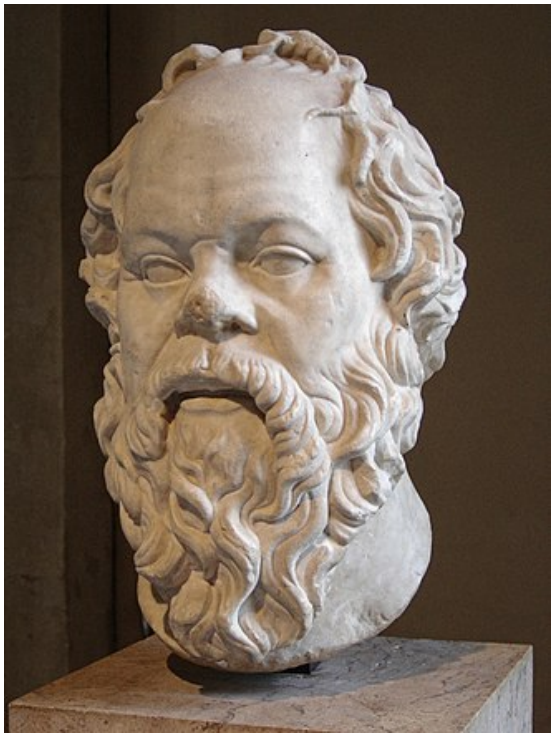
Phoenician	Paleo-Hebrew	Hebrew letter (Dfus)	English name
		א	Aleph
		ב	Bet
		ג	Gimel
		ד	Dalet
		ה	He
		ו	Waw
		ז	Zayin
		ח	Heth
		ט	Teth
		י	Yodh
		כ/ך	Kaph
		ל	Lamedh

		מ/ם	Mem
		נ/ן	Nun
		ס	Samekh
		ע	Ayin
		פ/ף	Pe
		צ/ץ	Tsade
		ק	Qoph
		ר	Resh
		ש	Shin
		ת	Taw

Ancient Greece: What is knowledge?

Plato's Theaetetus
(Θεαιτητος) ≈ 369 BCE

D1: Knowledge is perception
D2: Knowledge is true belief
D3: Knowledge is true belief with an account



A marble head of Socrates
in the Louvre (from Wikipedia)

Symbolic algebra

In 1591 “Introduction to the analytic arts” Franciscus Vieta used vowels for unknowns and consonants for constants.

François Viète (1540-1603)
(from Wikipedia)

Infinitesimal calculus

Isaac Newton 1643–1727

Gottfried Wilhelm Leibniz

1646–1716

Modern age of logic and foundations started as a big bang

Here are some contributors to the foundations of mathematics:

Bernard Bolzano (1781-1848),
Nikolai Lobachevsky (1792-1856),
Lejeune Dirichlet (1805-1859),
George Boole (1815-1864),
Karl Weierstrass (1815-1897),
Richard Dedekind (1831-1916),
Georg Cantor (1845-1918),
Gottlob Frege (1848-1925),
Giuseppe Peano (1858-1932),
David Hilbert (1862-1943),
Ernst Zermelo (1871-1953),
Bertrand Russell (1872-1970),
Leopold Löwenheim (1878-1957),
Hermann Weyl (1885-1955),
Thoralf Skolem (1887-1963),
Abraham Fraenkel (1891-1956),
Emil Post (1897-1954),
Alfred Tarski (1901-1983),
Andrei Kolmogorov (1903-1987),
John von Neumann (1903-1957),
Kurt Gödel (1905-1978),
Alan Turing (1912-1954)

Hermann Weyl 1885–1955

Constructivist

1918 "The continuum," most of classical calculus, predicatively, without proof by contradiction, infinite sets; a bet with George Pólya

1921 "On the new foundational crisis of mathematics," admitted numbers in a perpetual state of becoming, "that is the revolution!"

Pragmatic scientist

Some books: 1918 "Space, time, matter," 1926 "Theory of groups and quantum mechanics," 1939 "The classical groups," 1949 "Philosophy of mathematics and natural sciences," 1952 "Symmetry"

As we grow older
The world
becomes stranger,
the pattern
more complicated

T.S. Eliot

John von Neumann 1903–1957

Collected works, Pergamon Press, 1961-1963

- Vol 1 Logic, theory of sets and quantum mechanics
- Vol 2 Operators, ergodic theory and almost periodic functions on a group
- Vol 3 Ring of operators
- Vol 4 Continuous geometry and other topics
- Vol 5 Design of computers, theory of automata and numerical analysis
- Vol 6 Theory of games, astrophysics, hydrodynamics and meteorology

1930

Roughly, 1950s–1980s

For a while after WW2, logic was rather popular and foundational.

- The first computers were designed on a logic foundation.
- Symbolic artificial intelligence dominated for quite a while other AI approaches like statistical and cybernetics.
- Logic programming was quite a fad.
- Non-standard analysis.
- Forcing revolutionized set theory.

But then things started to change.

Roughly, 1990s to today

While the need for logic foundation research never was greater, less and less attention is given to foundational issues, even in areas like set theory.

By and large, logic factions are slowly fizzling out at top Mathematics and Philosophy departments.
For example, the Math Dept of ETH Zurich goes from having Zermelo, Bernays, Specker, etc. to having no logicians at all.

There are many logicians in computer science but not much foundational work is done there either. In particular, the golden age of logic in AI is behind us.

Future as it may be

Foundational logic research fades away. Significant logic areas become parts of mathematics or computer science.
As T.S. Eliot said in "The Hollow Men,"

Between the potency
And the existence
Between the essence
And the descent
Falls the Shadow
⋮
This is the way the world ends
Not with a bang but a whimper.

Better but challenging future

- **More comprehensive logic research**
  To me, logic is a science of convincing arguments.
  It does not reduce to the deductive mathematical logic.
  Mathematics may be the last science that needs outside deduction experts because mathematicians themselves are experts in deduction.
- **More comprehensive foundation research**
  Throughout ages, logicians made great contributions to the foundations of various sciences including mathematics but also natural sciences.

Inductive logic

Already Aristotle mentioned it in addition to deductive logic.
There has been some progress on the subject:

- Bayesian inference
- Popper's falsifiability
  (plus generalizations depending on the form of the original claim)
- Solomonoff's induction inference

Yet, inductive logic remains a challenge.

Judicial logic

Wikipedia's Legal Information article mentions computational models of argumentation and decision-making, of evidential reasoning, etc. "A variety of formalisms have been used, including propositional and predicate calculi; deontic, temporal and non-monotonic logics."

I am skeptical because of past lessons.

- Many-valued logics
- Logic and databases

Thinking fast and slow

by Daniel Kahneman

What is the logic of fast thinking?

Knowledge, information, privacy

What is knowledge?

We have more data than Plato did but limited progress; a common definition of knowledge as justified true belief is close to the 3rd definition in Theaetetus. One interesting complication is probabilistic knowledge.

What is information?

How is knowledge different from information?

What is privacy?

From "Inverse privacy": infons, tangible infons, personal infons, personal infoset, etc.

Beyond physics

"If people do not believe that mathematics is simple,
it is only because they do not realize how complicated life is."

John von Neumann

Can foundational logic research be useful in neuroscience and biology in general?

Social sciences seem to have no simple laws. Can foundational logic research be useful there? We should try. Maybe we can analyze the degrees of spin, and similarly for fake news.

Back to philosophy?

To an extent yes, but:

- Enable the virtuous circle of incremental-improvement.
  Recall Thomas Kuhn's story on electricity.
- Have applications in mind.
  Recall how the concept of set became prevalent in mathematics, how Kolmogorov's axiomatization of probabilities succeeded.
- Prove theorems.

Thank you