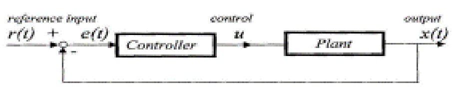
Sliding Mode
Control of Power Converters

SEMINAR, INSTITUTE OF CONTROL SCIENCES
LAB.№7,
OCTOBER 27, 2020

What is Sliding Mode?

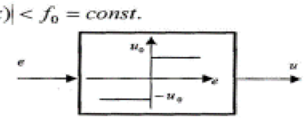
Primitive Examples

First-Order Tracking System



$$\dot{x} = f(x) + u, \quad |f(x)| < f_0 = const.$$

$$e(t) = r(t) - x(t),$$

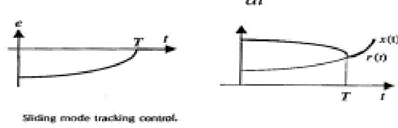


Relay control.

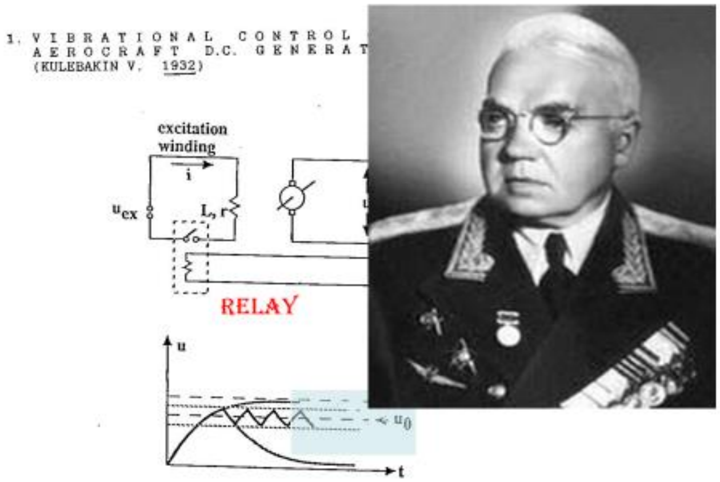
Control

$$u = \begin{cases} u_0 & \text{if} \quad e > 0 \\ -u_0 & \text{if} \quad e < 0 \end{cases}, \quad \text{or } u = u_0 sign(e) \quad u_0 = const.$$

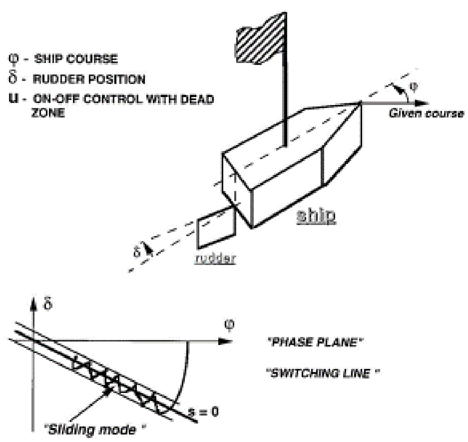
$$\frac{de}{dt} = \dot{r} - f(x) - u_0 sign(e), \quad e\frac{de}{dt} < 0, \quad u_0 > f_0 + |\dot{r}|.$$



Sliding mode tracking control.



2. ON AUTOMATIC STABILITY OF A SHIP ON A GIVEN COURSE
(NIKOLSKI G., 1934)

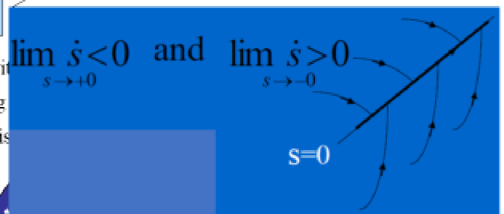


Introduction of Sliding Mode Control

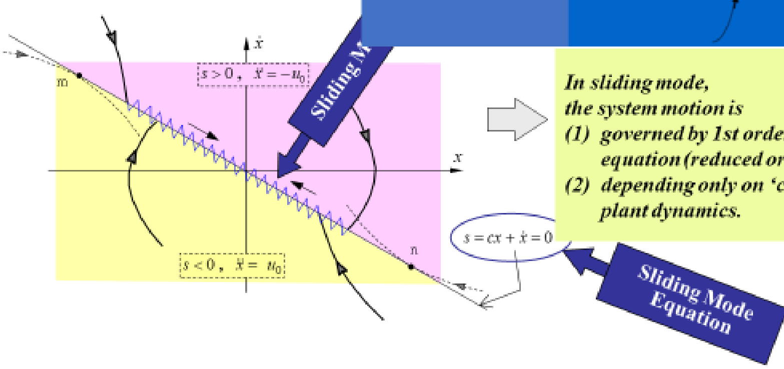
■ Concept of Sliding Mode (Second order relay system)

$\ddot{x} = u,$
$u = -u_0 \,\mathrm{sgn}(s), \quad s = cx + \dot{x}, \quad u_0, c : \text{const}$

Upper semi-plane : $s > 0 \;\rightarrow\; u = -u_0 \;\rightarrow\; \ddot{x} = -u_0$

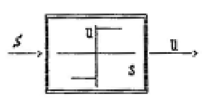


- State trajectories are towards the line swi
- State trajectories cannot leave and belong
- After sliding mode starts, further motion is



In sliding mode, the system motion is
(1) governed by 1st order equation (reduced order).
(2) depending only on 'c' not plant dynamics.

RELAY ON - OFF CONTROL BANG - BANG



$u = u_0 \,\mathrm{sign}\; s$

I. Flügge - Lotz
1953

Ya. Tsypkin
1955

DISCONTINUOUS
AUTOMATIC CONTROL

By IRMGARD FLÜGGE-LOTZ

1953
PRINCETON UNIVERSITY PRESS
PRINCETON, NEW JERSEY

Я. З. ЦЫПКИН

ТЕОРИЯ
РЕЛЕЙНЫХ СИСТЕМ
АВТОМАТИЧЕСКОГО
РЕГУЛИРОВАНИЯ

ГОСУДАРСТВЕННОЕ ИЗДАТЕЛЬСТВО
ТЕХНИКО-ТЕОРЕТИЧЕСКОЙ ЛИТЕРАТУРЫ
МОСКВА 1955

Canonical Space

$$\boxed{x^{(n)} + a_n x^{(n-1)} + a_{n-1} x^{(n-2)} + \ldots + a_1 x = b\,u + f(t)}$$

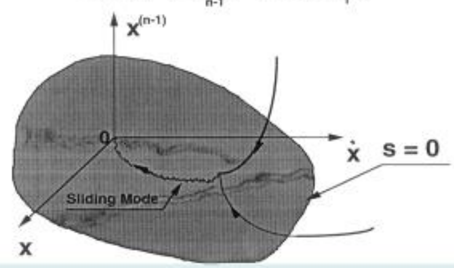
$a_i(t)$ and $b(t)$ are unknown parameters

$f(t)$ is disturbance

Control $u = \begin{cases} u^+(x,t) & \text{if } s > 0 \\ u^-(x,t) & \text{if } s < 0 \end{cases}$

$u^+(x,t) \neq u^-(x,t)$

$$s = x^{(n-1)} + c_{n-1} x^{(n-2)} + \ldots + c_1 x$$



Sliding Mode Equation **does not depend** on parameters and disturbances $\boxed{x^{(n-1)} + c_{n-1} x^{(n-2)} + \ldots + c_1 x = 0}$

1-st Stage:

Canonic Space Design

Disadvantages

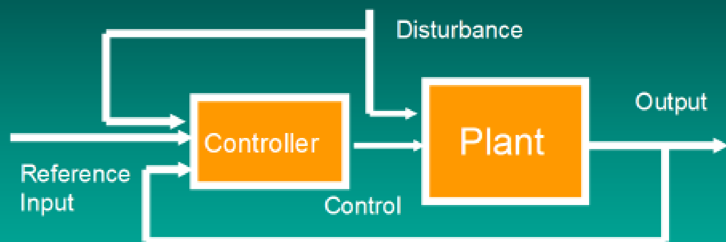
- High order time-derivatives
- Control and output may be vector-valued
- Accessible for measurement components of the system state are not utilized

2-nd Stage:
Multiinput - multioutput (MIMO) systems

****

SECOND STAGE

Finite Dimensional MIMO Systems



$$\dot{x} = f(x,t,u) \quad x \in R^n \qquad u \in R^m$$

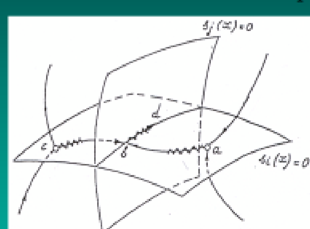
$$u_i = \begin{cases} u_i^+(x,t) & \text{if} \quad s_i(x) > 0 \\ u_i^-(x,t) & \text{if} \quad s_i(x) < 0 \end{cases}, \qquad i = 1,\dots,m$$

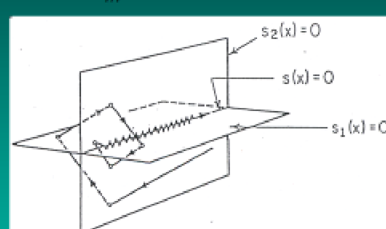
$$u_i^+(x,t) \neq u_i^-(x,t)$$

Multidimensional Sliding Modes

(Late Sixtieth)

$$s(x) = 0, \; s^T(x) = [s_1(x),\dots,s_m(x)], \; (s = 0) \in \Re^{n-m}$$





Two Dimensional Sliding Mode

Introduction of Sliding Mode Control

Multi-dimensional Sliding Mode

- **In general case,**

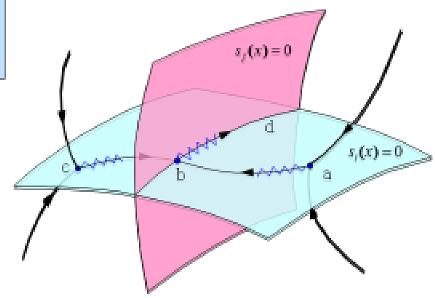
$$\dot{x} = f(x,t) + B(x,t)u, \qquad x \in R^n, u \in R^m$$

$$u_i = \begin{cases} u_i^+(x,t) & \text{if} \quad s_i(x) > 0 \\ u_i^-(x,t) & \text{if} \quad s_i(x) < 0 \end{cases} \qquad \mathrm{i} = 1,\ldots,m$$

- *Sliding mode occurs in the intersection of all m surfaces,*

  $s_i(x) = 0 \quad (i = 1,..,m)$

- *The order of motion equation is by 'm' less than the order of original system*



Sliding mode in discontinuity surfaces and their intersection

Introduction of Sliding Mode Control

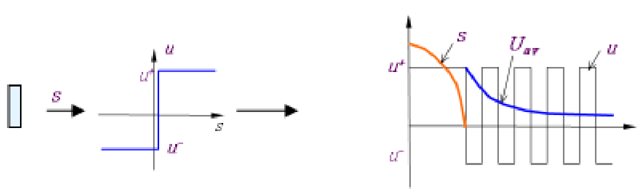
Outline of Sliding Mode Methodology

- **Order reduction of the system**

  ⇨ **Enables a designer to simplify and decouple the design procedure.**

- **Insensitivity with respect to parameter variations and disturbances**

  ⇨ **Attained by finite control actions, unlike continuous high gain controls in conventional system.**

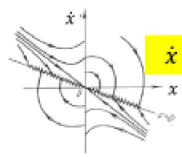


$$K_{eq} = \frac{U_{av}}{s} \approx \infty \qquad (\because s \to 0)$$

BASIC PROBLEMS OF SLIDING MODE CONTROL

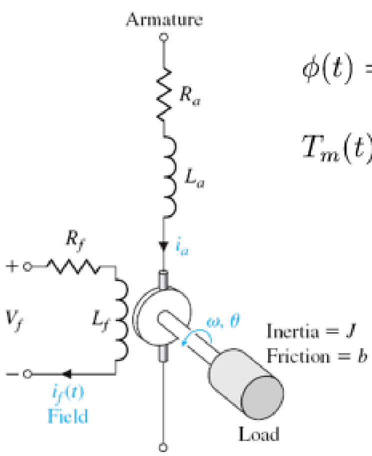
- ***Differential Equations of Sliding Mode***
- ***Conditions for Sliding Mode to Exist***
- ***Design Methods of Sliding Mode Control***
  - *How to select sliding surfaces with the desired motions*
  - *How to enforce sliding mode in their intersection*

Differential Equations of Sliding Mode



$\dot{x} + cx = 0$

DC MOTOR



$\phi(t) = K_f i_f(t)$ *Magnetic flux*

$T_m(t) = K_1\phi(t)i_a(t) = K_1 K_f i_f(t) i_a(t)$ *Torque*

$$L_a \frac{di_a}{dt} = -i_a R_a - K_2\omega + u$$

$$J\frac{d\omega}{dt} = K_1 K_f i_f i_a - T_L$$

$s = Ci_a + \omega, u = -u_0 sign(s)$ ⇨ Sliding mode, $s = 0$, $Ci_a = -\omega$???
sliding mode equation ???

Postulates

$$\dot{x} = f(x,t,u) \quad x \in R^n \qquad u \in R^m$$

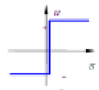
$$u_i = \begin{cases} u_i^+(x,t) & \text{if} \quad s_i(x) > 0 \\ u_i^-(x,t) & \text{if} \quad s_i(x) < 0 \end{cases}, \qquad i = 1,\dots,m$$

$$u_i^+(x,t) \neq u_i^-(x,t)$$

1. $\dot{x} = f(x), \; f(x) = \begin{cases} f^+ & \text{if} \;\; s(x) > 0 \\ f^- & \text{if} \;\; s(x) < 0 \end{cases}$ $\qquad \dot{x} = conv(f^+, f^-)$ ***Filippov***

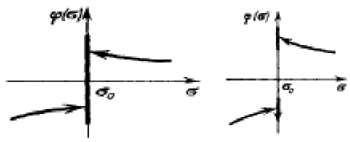
   Convex hull

2. $u = ks, \quad k \to \infty$

   Tsypkin

3. $\dot{x} = Ax + Bu, \; s = Cx,$ $\quad$ *Convolution equation* $\rightarrow$ $u\!: \; s = 0$ ***Neimark***

4. 

   $$\frac{dx}{dt} \in f(x, t)$$

   Yakubovich

5. $u \in U, \; v \in conv(U), \quad \dot{x} = conv f(x,t,v)$ ***Aizerman, Pyatnitskii***

THE QUESTION, WHICH IS CORRECT, IS NOT LEGITIMATE – ALL EQUATIONS ARE POSTULATED

Equivalent Control Method

Control system $\qquad \dot{x} = f(x,u,t)$

Discontinuous control u enforces sliding mode in manifold $s = (s_1, \dots, s_m) = 0.$ $\quad G(x) = \{\partial s / \partial x\}.$

Then solution to $\; \dot{s} = Gf(x,t,u) = 0 \Rightarrow u_{eq} = u_{eq}(x,t)$

is substituted to system equation

$$\dot{x} = f(x,t,u_{eq})$$

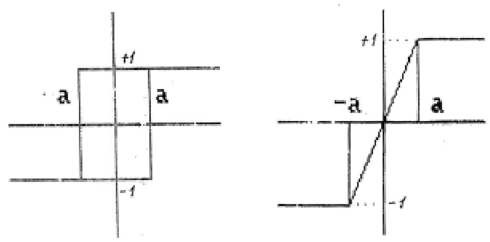
$$s(x) = 0 \longrightarrow x_2 = s_0(x_1) \quad x_2, s_0 \in \Re^m \quad x_1 \in \Re^{n-m}:$$

$$\dot{x}_1 = f_1[x_1, t, s_0(x_1)], \; f_1 \in \Re^{n-m}.$$

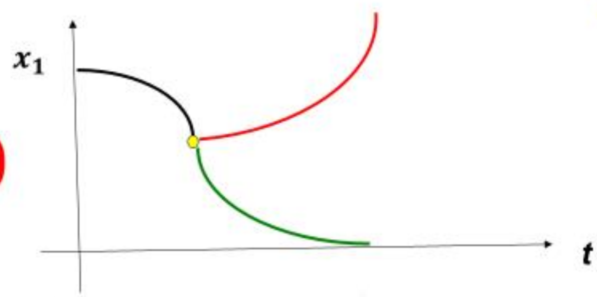
Uniqueness Problem ----

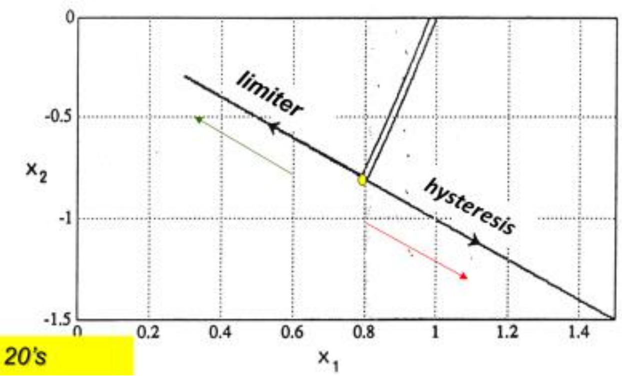
$$\dot{x}_1 = 0.3x_2 + ux_1$$
$$\dot{x}_2 = -0.7x_1 + 4u^3x_1,$$

$$u = -\mathrm{sign}(x_1 s),$$
$$s = x_1 + x_2.$$



$a \to 0$





Mandelshtam, in 20's

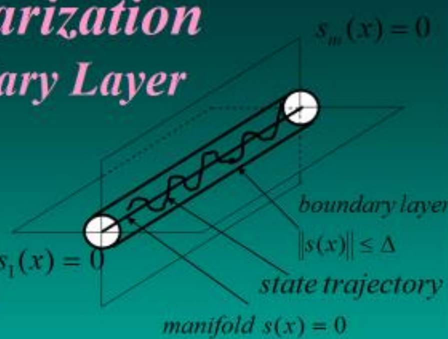
Regularization

Boundary Layer

$\dot{x} = f(x,u),$

$x, f \in \Re^n,\ u(x) \in \Re^m,$

$$u_i(x) = \begin{cases} u_i^+(x) & \text{if } s_i(x) > 0 \\ u_i^-(x) & \text{if } s_i(x) < 0 \end{cases}$$



$$\tilde{u}(x,\Delta) \Rightarrow \tilde{x}(t,\Delta),\ \|s(x)\| \le \Delta,\quad \|s\|^2 = (s^T, s).$$

$$\lim_{\Delta \to 0} \tilde{x}(t,\Delta) = x(t)$$

Equivalent Control Method

Affine system $\dot{x} = f(x,t) + B(x,t)u$

Discontinuous control u enforces sliding mode in manifold $s = (s_1, \dots, s_m) = 0$. $\det(GB) \neq 0,\ G(x) = \{\partial s/\partial x\}$.

Then solution to $\dot{s} = Gf + GBu = 0,\quad u_{eq} = -(GB)^{-1} Gf$.

is substituted to system equation

$$\dot{x} = f - B(GB)^{-1} Gf$$

$s(x) = 0 \longrightarrow x_2 = s_0(x_1)$ $x_2, s_0 \in \Re^m$ $x_1 \in \Re^{n-m}$:

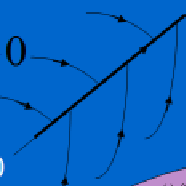
$$\dot{x}_1 = f_1[x_1, t, s_0(x_1)],\ f_1 \in \Re^{n-m}.$$

E.C.M. is substantiated by boundary layer regularization

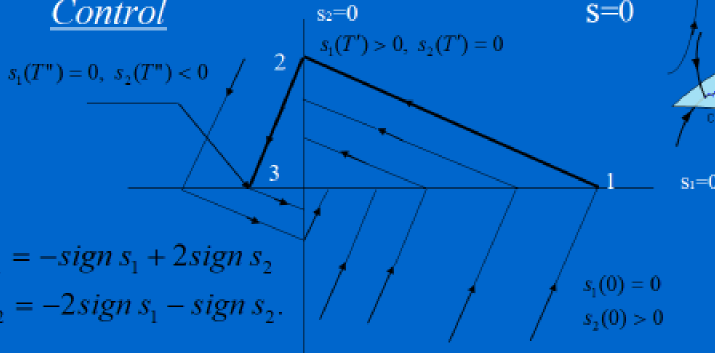
Mathematical Aspects II

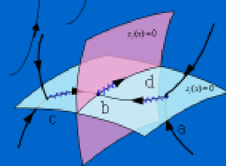
Sliding Mode Existence Conditions

Scalar Control: $\lim_{s \to +0} \dot{s} < 0$ and $\lim_{s \to -0} \dot{s} > 0$

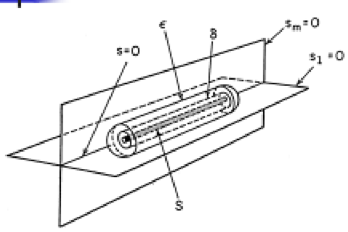


Vector Control





$$\dot{s}_1 = -sign\, s_1 + 2 sign\, s_2$$
$$\dot{s}_2 = -2 sign\, s_1 - sign\, s_2.$$



$$s^T(x) = [s_1(x), \ldots, s_m(x)]$$
$$G = \left\{ \frac{\partial s}{\partial x} \right\} \quad \text{is } m \times n \text{ matrix}$$

Stability of the origin in subspace S

Motion Equation:

$$\dot{s} = Gf + GBu$$

$$u = \begin{cases} u^+(x) & \text{if } s(x) > 0 \\ u^-(x) & \text{if } s(x) < 0 \end{cases} \quad \text{Component-wise}$$

$$s \in R^m, \; u \in R^m.$$

Piece-vise smooth Lyapunov functions I

Motion Equations:

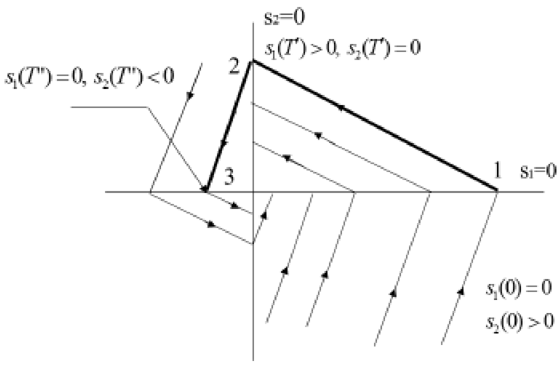
$$\dot{s}_1 = -sign(s_1) + 2sign(s_2),$$
$$\dot{s}_2 = -2sign(s_1) - sign(s_2)$$

Lyapunov Function:

$$V = s^T sign(s),\ s^T = (s_1, s_2),$$
$$sign(s) = \begin{pmatrix} sign(s_1) \\ sign(s_2) \end{pmatrix}$$

Time Derivative:

$$\dot{V} = \frac{\partial V}{\partial s_1}\dot{s}_1 + \frac{\partial V}{\partial s_2}\dot{s}_2 = -2$$



Sliding Mode Exists in s=0.

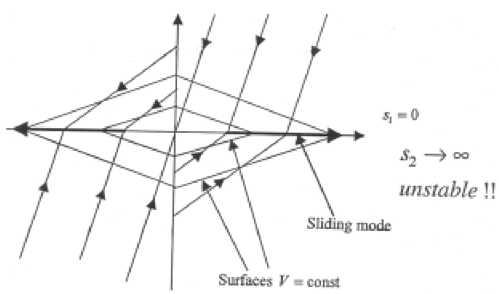
Piece-vise smooth Lyapunov functions II

Motion Equations:

$$\dot{s}_1 = -2sign(s_1) - sign(s_2),$$
$$\dot{s}_2 = -2sign(s_1) + sign(s_2)$$

Lyapunov Function:

$$V = s^T P sign(s),\ s^T = (s_1, s_2),$$
$$sign(s) = \begin{pmatrix} sign(s_1) \\ sign(s_2) \end{pmatrix},\ P = \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}.$$



Time Derivative:

$$\dot{V} = \frac{\partial V}{\partial s_1}\dot{s}_1 + \frac{\partial V}{\partial s_2}\dot{s}_2 = -7 - 6sign(s_1 s_2) < 0.$$

Sliding Mode Control Design

Sliding Mode Equation $\dot{x}_1 = f_1[x_1, t, s_0(x_1)],\ f_1 \in \Re^{n-m}$.

- is of reduced order
- does not depend on control
- depends on the equation of switching surfaces.

Design is decoupled into two independent subproblems:

- design of the desired dynamics for a system of the *(n-m)-th* order by a proper choice of a sliding manifold $s=0$;
- enforcing sliding motion in this manifold which is equivalent to stability problem of the *m-th* order system.

Design, Regular Form

$$\dot{x}_1 = f_1(x_1, x_2, t),\quad x_1 \in R^{n-m}$$

$$\dot{x}_2 = f_2(x_1, x_2, t) + B_2(x_1, x_2, t)u,\quad x_2 \in R^m,\ \det(B_2) \neq 0.$$

$x_2 = -s_0(x_1)$ - psuedo control

$s(x_1, x_2) = x_2 + s_0(x_1) = 0$ - sliding manifold

$\dot{x}_1 = f_1[x_1, -s_0(x_1), t]$ - sliding mode equation

depends neither $f_2(x_1, x_2, t)$ nor $B_2(x_1, x_2, t)$.

D C Motor:

$$J\frac{d\omega}{dt} = k_1 i - T_L,$$

$$L\frac{di}{dt} = -iR - k_2\omega + u,$$

Voltage is equal to u_0 or $-u_0$.

1.
$$i = K_p(\omega_{reference} - \omega) + K_i \int(\omega_{reference} - \omega)$$

2.
$$s = i - [K_p(\omega_{reference} - \omega) + K_i \int(\omega_{reference} - \omega)dt$$

3.
$$u = \begin{cases} -u_0 & \text{if } s > 0 \\ u_0 & \text{if } s < 0 \end{cases}.$$

$$\dot{s} = (1/L)u + |f(\omega, i, T_L)|. \qquad \boldsymbol{u = -u_0 sign(s)}$$

if $u_0 > L|f(\omega, i, T_L)|$ ***then*** $s\dot{s} < 0$

sliding mode exists, $s = 0$ ***and condition*** **1.** ***holds.***

DC-DC CONVERTERS

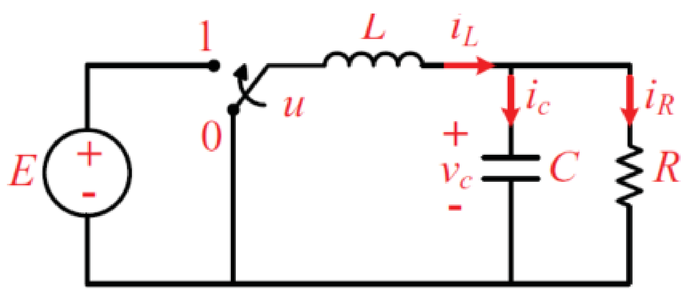


Figure 1. Buck-type DC/DC converter.

$$\dot{x}_1 = -\frac{1}{L}x_2 + u\frac{E}{L}$$

$$\dot{x}_2 = \frac{1}{C}x_1 - \frac{1}{RC}x_2$$

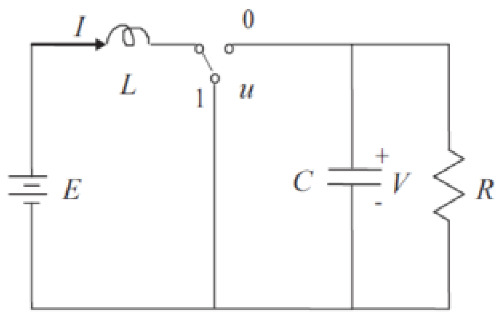
$x_1 = I$and $x_2 = V$.

$$u = \begin{cases} 1, & \text{switch is closed} \\ 0, & \text{switch is open} \end{cases}$$

$$u = \frac{1}{2}\left(1 - \text{sign}\,(s)\right) \qquad s = x_1 - x^*{}_1$$

$$\dot{s} = -\frac{E}{L}sign(s) + \cdots.$$

DC-DC CONVERTERS



Boost DC–DC converter.

$$\dot{x}_1 = -(1-u)\frac{1}{L}x_2 + \frac{E}{L},$$

$$\dot{x}_2 = (1-u)\frac{1}{C}x_1 - \frac{1}{RC}x_2,$$

with $x_1 = I$ and $x_2 = V$.

$$u = (1/2)(1 + \mathrm{sign}(s)),\ s = x_2 - x_2^*$$?!?

Unstable zero dynamics

$$u = \frac{1}{2}(1 - sign(s)) \quad s = x_1 - x_1^*$$

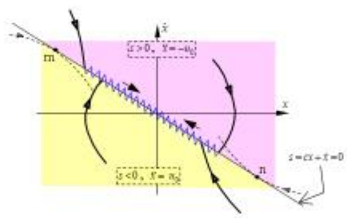
Stable zero dynamics

THE CHATTERING PROBLEM

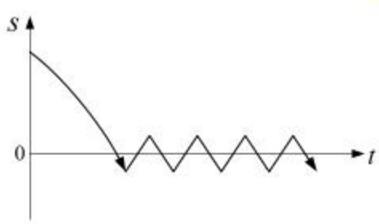
■ Causes of Chattering

1. Fast dynamics disregarded in an ideal model (unmodeled dynamics)

2. Switching frequency limitation

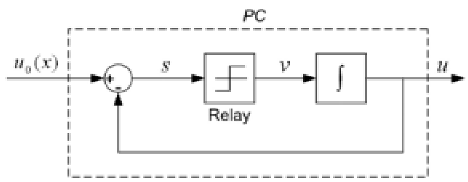


* Discrete time system (discretization chattering)



Sliding Mode Control

Power Converter



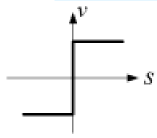
$u_0(t)$ **to be tracked.**

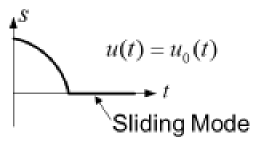
Error $s = u_0 - u$

$\dot{u} = v$

$\dot{s} = \dot{u}_0 - v$

$$v = Msign(s) \quad (M > |\dot{u}_0|)$$

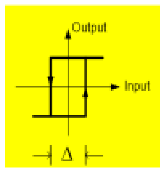


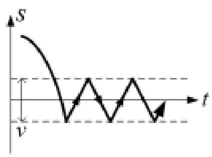


$$v = Msign_h(s)$$

Chattering (ripple) amplitude $A = \frac{\Delta}{2}$

Frequency $f = \frac{1}{\Delta}\frac{M^2 - (\dot{u}_0)^2}{2M}$



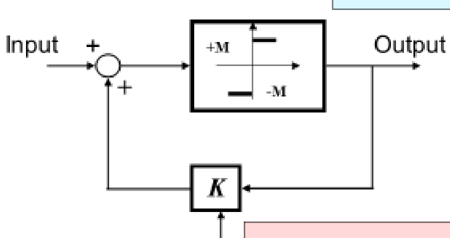


Frequency Control

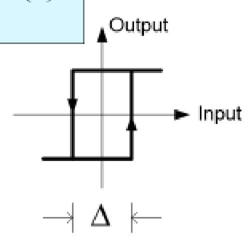
$f = \frac{1}{\Delta}\frac{M^2 - a^2}{2M}$ $\quad a = \dot{u}_0 \approx const.$

f^* : **Desired Frequency**

$$\Delta = \frac{1}{f^*}\frac{M^2 - a^2}{2M} \quad (*)$$



$=$



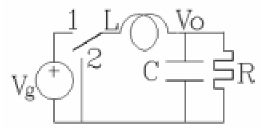
$$\Delta = 2KM \ (**)$$

(*), (**): $K = \frac{1}{f^*}\frac{M^2 - a^2}{4M^2}$

Multiplication? No

Chattering (Ripple) Problem

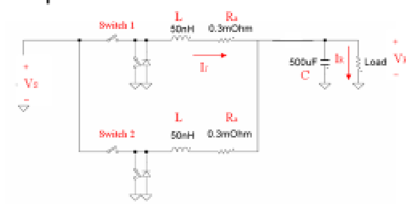
Power converter

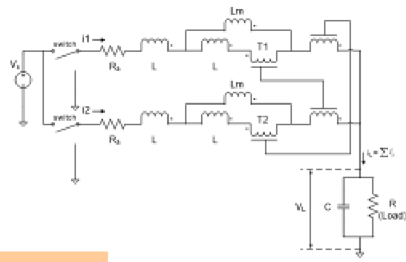


f^* : Switching Frequency

Amplitude of oscillation should be reduced.

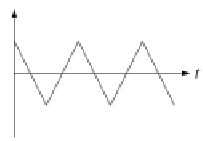
Multiphase converter





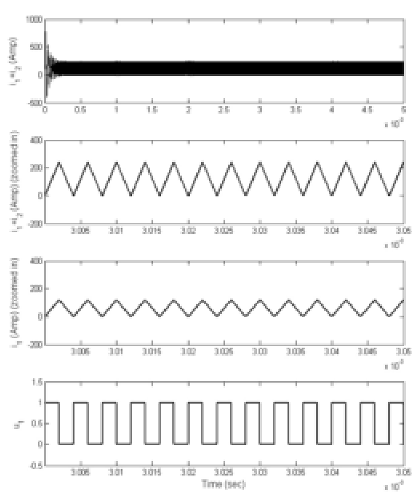
- Triangular wave with frequency f^* (offset)
- Timer (delay)
- Transformers
- Filter

Transformers, Filter → depends on frequency

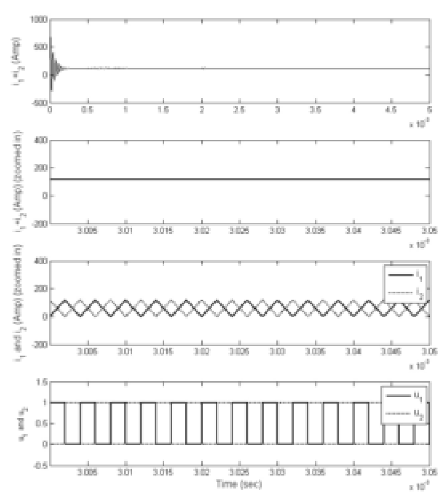


Chattering (Ripple) Problem

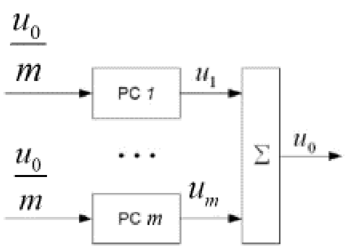
Two phases (without phase shift)



Two phases (with phase shift)



Implementation of Multiphase converter



$$s_i = \frac{u_0}{m} - u_i, \quad \dot{s}_i = \frac{\dot{u}_0}{m} - M sign_h(s_i)$$

- Phase in each PC depends on initial condition
- Phases in PC's are independent

$$\max A_{chattering} = mA_{i\ chattering}$$

Number of Phases I

m-phase converter

Any discontinuous signal v is a periodic function with **the same** frequency ω.

$$v_i = \sum_{k=1}^{\infty} g_i \sin(\omega kt + \varphi_k) \quad \text{(Fourier Series)}$$

The phase shift between each phase and preceding is equal to $T/m,\ T = 2\pi/\omega$

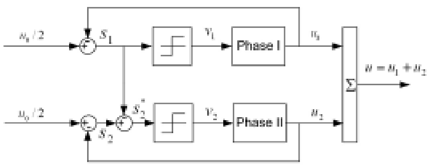
Then the k-th harmonic is the output signal (sum of the outputs).

$$\sum_{i=0}^{m-1} sin\left[\omega_k(t - \frac{2\pi}{\omega m}i)\right] = \sum_{i=0}^{m-1} Im\left[e^{j(\omega_k t - \frac{2\pi k}{m}i)}\right]$$
$$= Im(e^{j\omega_k t}Z), \quad Z = \sum_{i=0}^{m-1} e^{-j\frac{2\pi k}{m}i}$$

$$\boldsymbol{Ze^{-j\frac{2\pi k}{m}} = Z,\ e^{-j\frac{2\pi k}{m}} = 1\ only\ if\ \ k = m, 2m, \dots}$$

Then $Z=0$ for all harmonics except for $k=m, 2m, \dots$ and they cancel.

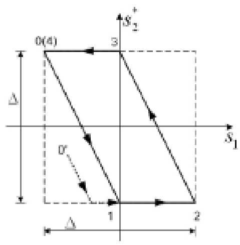
Design Principle (Two-phase PC)



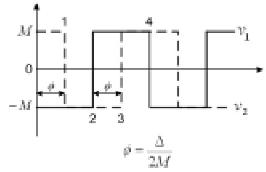
The relay function v_2 of the second phase depends on the two errors

$$s_2^* = s_2 - s_1$$

$$\dot{s}_1 = -M sign_h(s_1) + a$$
$$\dot{s}_2^* = M sign_h(s_1) - M sign_h(s_2^*) \quad \Rightarrow \quad \pm 2M \text{ or } 0$$



(a)



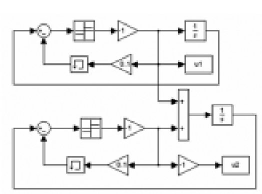
(b)

$a = 0$

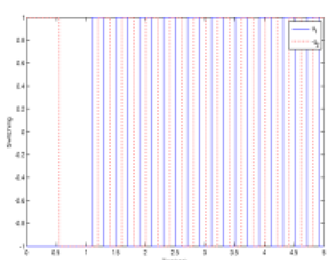
Period $T = \dfrac{2\Delta}{M}$

Phase Shift $\phi = \dfrac{\Delta}{2M} = \dfrac{T}{4}$

Simulation of Phase Shift

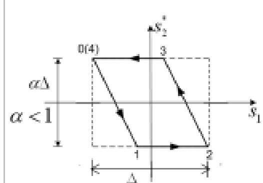


(a)

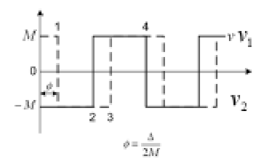


(b)

Phase shift can be controlled if $\Delta_2 = \alpha\Delta_1 \ (a = 0)$

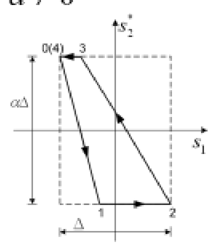


(a)

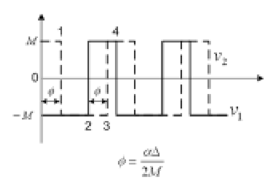


(b)

$a \neq 0$ **Phase shift** $\phi = \dfrac{\alpha\Delta}{2M} < \dfrac{T}{4}$

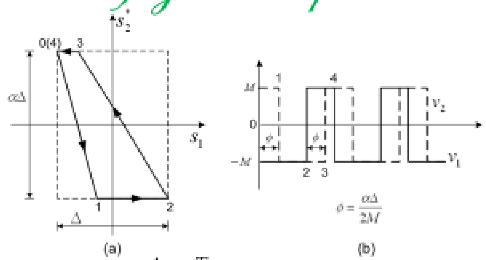


(a)



(b)

Selection of hysteresis loop width



Phase shift $\quad \phi = \dfrac{\alpha\Delta}{2M} = \dfrac{T}{m}$

$$\Delta = \frac{1}{f^*}\frac{M^2 - a^2}{2M}$$

Period $\quad T = \dfrac{2M\Delta}{M^2 - a^2}$

$$\alpha = \frac{4M^2}{m(M^2 - a^2)}$$

Simulation I

Power converter

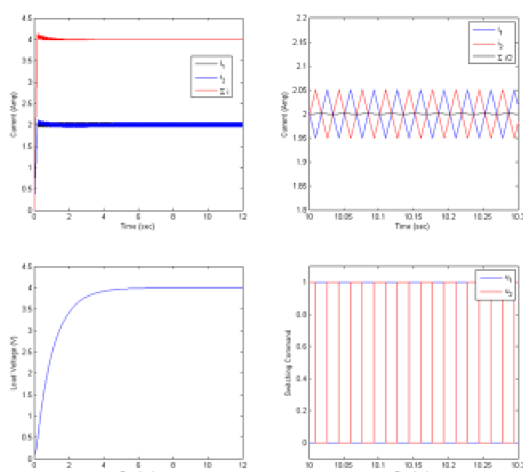
$$\dot{I}_k = \frac{1}{L}\left(-I_k R_a + u_k - V_L\right)$$

$$\dot{V}_L = \frac{1}{C}\left(\sum_{k=1}^{m} i_k - \frac{V_L}{R_L}\right).$$

$$(k = 1, 2, \cdots, m)$$

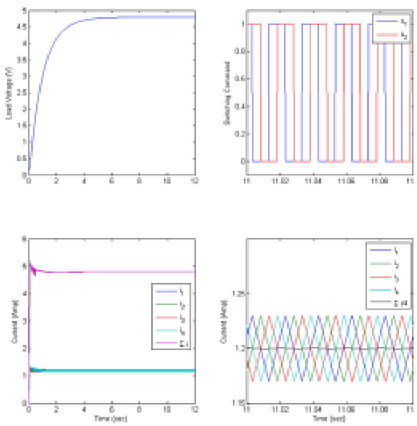
Parameter	Set I
L (H)	1
C (F)	1
R_a (Ω)	1
R_L (Ω)	1
V_s (V)	12

Two-phase Converter $(V_{ref} = 4V)$

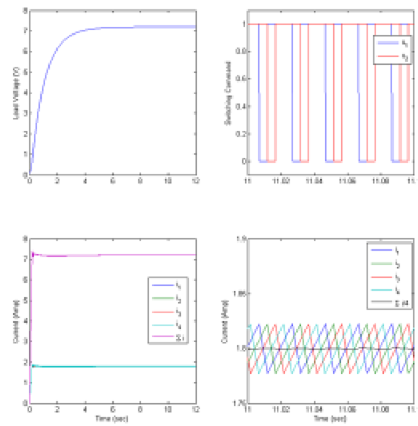


Simulation I

4-phase Converter $(V_{ref} = 4.8V)$



4-phase Converter $(V_{ref} = 7.2V)$

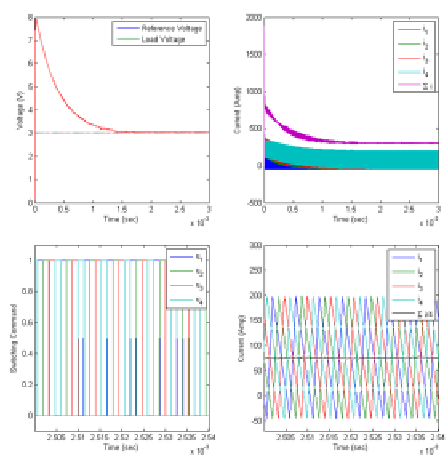


Simulation II

***IKOR* Power Converter**

Parameter	Set II
L (H)	5×10^{-8}
C (F)	1×10^{-3}
R_a (Ω)	3×10^{-4}
R_L (Ω)	1×10^{-2}
V_s (V)	12

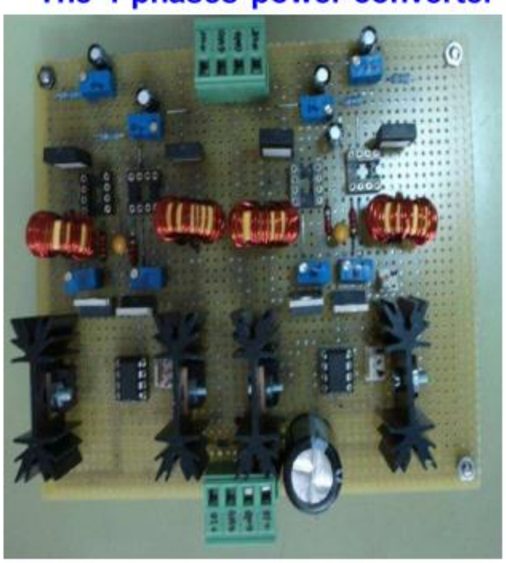
4-phase Converter $(V_{ref} = 3V)$



MULTIPHASE CONVERTER

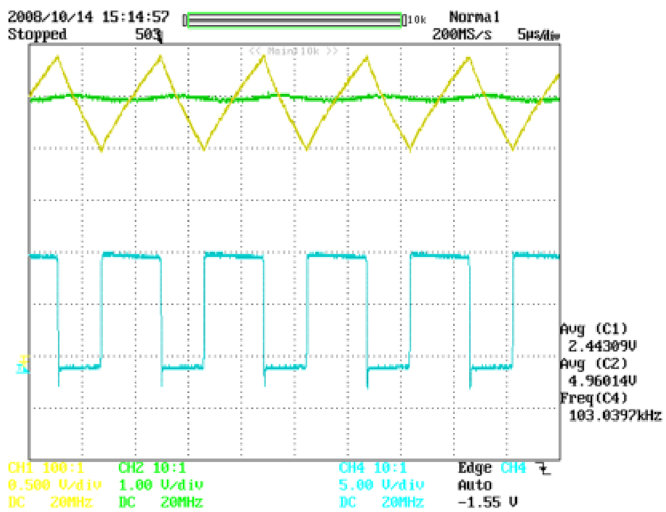
Resuts of Experiments

The 4-phases power converter

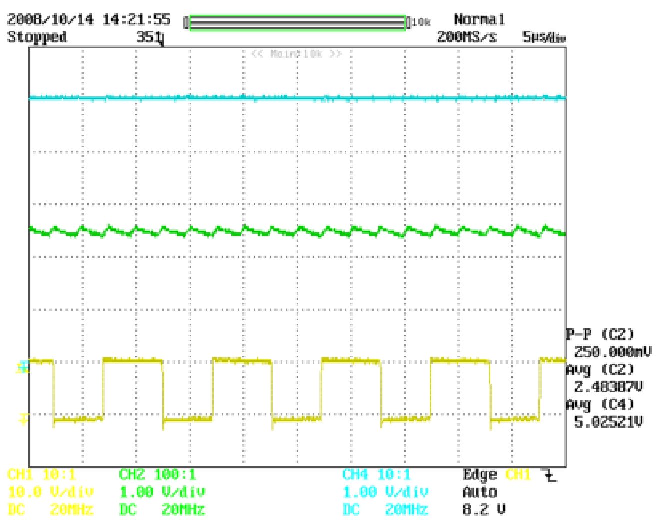


The converter parameter values are $E = 10$ V, $L = 22$ μH, $C = 10$ μF. The mosfet works at a frequency 100 kHertz.

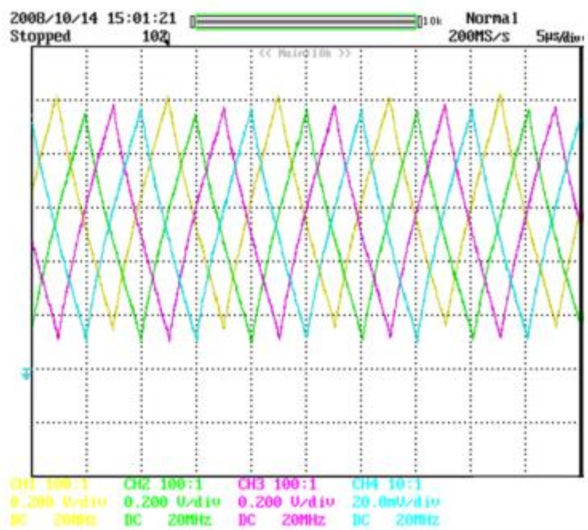
1 PHASE, 5 V, R = 2



4 PHASE, 5 V, R = 2



PHASE CURRENTS



PHASE CURRENTS

1 PHASE

i_L	$i_L$chattering
2.44 ± 0.47 A	± 0.47 A

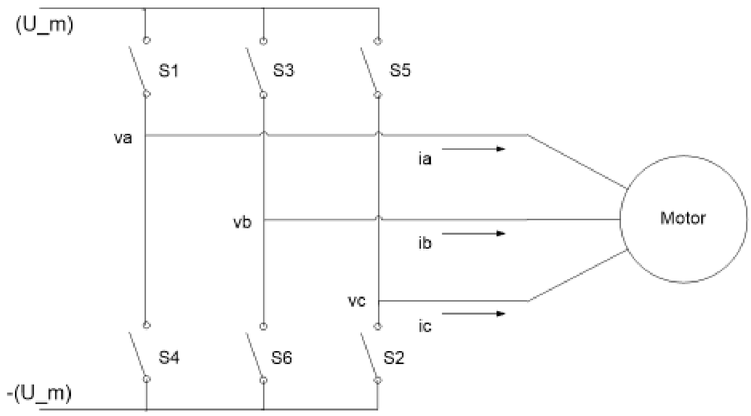
4 PHASES

i_L	$i_L -$chattering
2.48 ± 0.095 A	± 0.095 A

The current values of each phase are $i_{L_1} = 0.62 \pm 0.43$ A, $i_{L_2} = 0.6 \pm 0.42$ A, $i_{L_3} = 0.63 \pm 0.43$ A and $i_{L_4} = 0.66 \pm 0.42$ A.

Chattering reduction from 0.47 A to 0.095 A.

CONTROL OF INDUCTION MOTORS



Dynamic Model of Induction Motor

Mechanical equations

$$\frac{dn}{dt} = \frac{P}{J}(T - T_L)$$

$$T = \frac{3P}{2}\frac{x_H}{x_R}(i_\beta \phi_\alpha - i_\alpha \phi_\beta)$$

$$\begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix}\begin{bmatrix} u_R \\ u_S \\ u_T \end{bmatrix}$$

Rotor flux equations

$$\frac{d\phi_\alpha}{dt} = -\eta\phi_\alpha - n\phi_\beta + \eta x_H i_\alpha$$

$$\frac{d\phi_\beta}{dt} = -\eta\phi_\beta + n\phi_\alpha + \eta x_H i_\beta$$

Stator current equations

$$\frac{di_\alpha}{dt} = \beta\eta\phi_\alpha + \beta n\phi_\beta - \gamma i_\alpha + \frac{1}{\sigma x_S}u_\alpha$$

$$\frac{di_\beta}{dt} = \beta\eta\phi_\beta - \beta n\phi_\alpha - \gamma i_\beta + \frac{1}{\sigma x_S}u_\beta$$

Dynamic Model of Induction Motor

Mechanical equations

$$\frac{dn}{dt} = \frac{P}{J}\,(T - T_L)$$

$$T = \frac{3P}{2}\frac{x_H}{x_R}(i_\beta\phi_\alpha - i_\alpha\phi_\beta)$$

Rotor flux equations

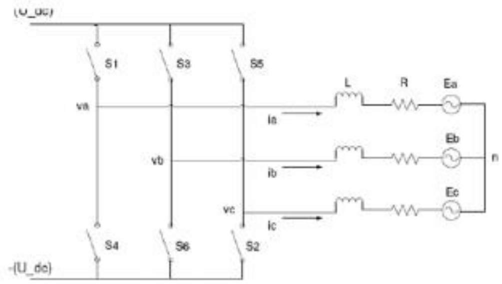
$$\frac{d\phi_\alpha}{dt} = -\eta\phi_\alpha - n\phi_\beta + \eta x_H i_\alpha$$

$$\frac{d\phi_\beta}{dt} = -\eta\phi_\beta + n\phi_\alpha + \eta x_H i_\beta$$

Stator current equations

$$\frac{di_\alpha}{dt} = \beta\eta\phi_\alpha + \beta n\phi_\beta - \gamma i_\alpha + \frac{1}{\sigma x_S}u_\alpha$$

$$\frac{di_\beta}{dt} = \beta\eta\phi_\beta - \beta n\phi_\alpha - \gamma i_\beta + \frac{1}{\sigma x_S}u_\beta$$



Based on circuit analysis, the system equations are:

$$\begin{aligned} L\dot{i}_a + Ri_a + E_a &= H_1U_{dc} - v_n \\ L\dot{i}_b + Ri_b + E_b &= H_3U_{dc} - v_n \\ L\dot{i}_c + Ri_c + E_c &= H_5U_{dc} - v_n \end{aligned} \tag{1}$$

where $H_1, H_3, H_5 \in \{\pm 1\}$

v_n is the voltage at the neutral point n.

E_a, E_b, E_c are voltage sources where $E_a + E_b + E_c = 0$

From kirchoff's current law: $i_a + i_b + i_c = 0$

$\Rightarrow \dot{i}_a + \dot{i}_b + \dot{i}_c = 0$

$$L(\dot{i}_a + \dot{i}_b + \dot{i}_c) + R(i_a + i_b + i_c) + (E_a + E_b + E_c) = (H_1 + H_3 + H_5)U_{dc} - 3v_n$$
$$\Rightarrow 0 = (H_1 + H_3 + H_5)U_{dc} - 3v_n$$
$$\Rightarrow v_n = \frac{U_{dc}}{3}(H_1 + H_3 + H_5) \tag{2}$$

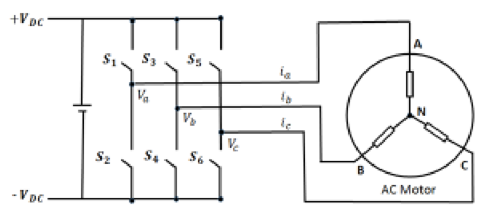
$$V_n = \frac{V_{dc}}{3}(H_1 + H_3 + H_5)$$

$$\dot{i}_a = K_a + \frac{V_{dc}}{3L}(2H_1 - H_3 - H_5)$$

$$\dot{i}_b = K_b + \frac{V_{dc}}{3L}(2H_3 - H_1 - H_5)$$

$$\dot{i}_c = K_c + \frac{V_{dc}}{3L}(2H_5 - H_1 - H_3)$$

$$H_i = \pm 1,\ i = 1, 3, 5$$



$$\begin{cases} K_a = -(R/L)i_a - E_a/L \\ K_b = -(R/L)i_b - E_b/L \\ K_c = -(R/L)i_c - E_c/L \end{cases}$$

Torque and Flux Control (cont.)

– Calculate

$$\dot{V} = f_3 + \frac{\boldsymbol{U_{dc}}}{\boldsymbol{3L}} \cdot (\Theta_1 s_R + \Theta_2 s_S + \Theta_3 s_T)$$

where f_3 does not depend on control and $\boldsymbol{s_{R,S,T} = \pm 1}$

$$\begin{cases} \Theta_1 &= -\psi_\beta S_T + \dfrac{\psi_\alpha}{\sqrt{\psi_\alpha^2 + \psi_\beta^2}} S_\psi \\ \Theta_2 &= (\dfrac{1}{2}\psi_\beta + \dfrac{\sqrt{3}}{2}\psi_\alpha) S_T + \dfrac{-\frac{1}{2}\psi_\alpha + \frac{\sqrt{3}}{2}\psi_\beta}{\sqrt{\psi_\alpha^2 + \psi_\beta^2}} S_\psi \\ \Theta_3 &= (\dfrac{1}{2}\psi_\beta - \dfrac{\sqrt{3}}{2}\psi_\alpha) S_T + \dfrac{-\frac{1}{2}\psi_\alpha - \frac{\sqrt{3}}{2}\psi_\beta}{\sqrt{\psi_\alpha^2 + \psi_\beta^2}} S_\psi \end{cases}$$

Torque and Flux Control (cont.)

– Select control logic such that $\dot{V} < 0$

$$\dot{V} = f_3 + \frac{\boldsymbol{U_{dc}}}{\boldsymbol{3L}} \cdot (\Theta_1 s_R + \Theta_2 s_S + \Theta_3 s_T)$$

Control Logic

$$\begin{cases} s_R &= -\mathrm{sign}(\Theta_1) \\ s_S &= -\mathrm{sign}(\Theta_2) \\ s_T &= -\mathrm{sign}(\Theta_3) \end{cases}$$

$$\left. \begin{array}{l} V = \frac{1}{2} S^T S \\ \dot{V} < 0 \end{array} \right\} \Rightarrow V \text{ tends to zero } \Rightarrow \begin{cases} \boxed{T = T^*} \\ S_\psi = 0 \Rightarrow \boxed{\|\psi\| \to \psi^*} \end{cases}$$

D C Motor:

$$J\frac{d\omega}{dt} = k_1 i - T_L,$$

$$L\frac{di}{dt} = -iR - k_2\omega + u,$$

Voltage is equal to u_0 or $-u_0$.

1.

$$i = K_p(\omega_{reference} - \omega) + K_i \int (\omega_{reference} - \omega)$$

2.

$$s = i - [K_p(\omega_{reference} - \omega) + K_i \int (\omega_{reference} - \omega)dt$$

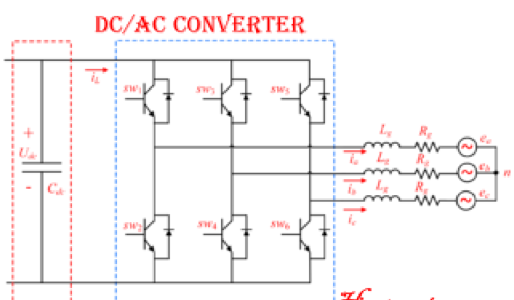
3.

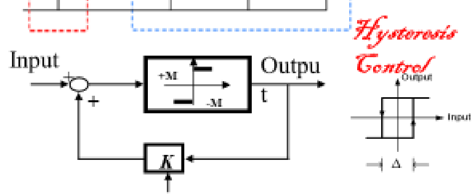
$$u = \begin{cases} -u_0 & \text{if } s > 0 \\ u_0 & \text{if } s < 0 \end{cases}.$$

$$\dot{s} = (1/L)u + |f(\omega, i, T_L)|. \qquad \boldsymbol{u = -u_0 sign(s)}$$

if $u_0 > L|f(\omega, i, T_L)|$ ***then*** $s\dot{s} < 0$

sliding mode exists, $s = 0$ ***and condition 1. holds.***





Three phase bridge

to control

Two phase current reference input

How to utilize

One extra degree of freedom?

DC/AC CONVERTER *(cont)*

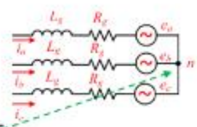
Accuracy depends on the width of hysteresis loop

High accuracy leads to high switching frequency and high heat losses.

Problem statement :

To utilize extra degree of freedom to minimize switching frequency for pre-selected width of hysteresis loop

$$\begin{cases} s_a = i_a^* - i_a \\ s_b = i_b^* - i_b \\ s_3 = \int (v_n^* - v_n)\, d\tau \end{cases}$$



Design ***Three switching surfaces***

v_n^* - reference input for the neutral point voltage

Lyapunov Approach

- ***Select control logic based on Lyapunov function such that sliding mode is enforced***
- Advantage: simple

$$V = \frac{1}{2} s^T s \qquad \begin{cases} s_a &= i_a^* - i_a \\ s_b &= i_b^* - i_b \\ s_3 &= \int (v_n^* - v_n) d\tau \end{cases}$$

$$v_n = \frac{V_{dc}}{3}(H_1 + H_3 + H_5) \qquad \dot{V} = s^T \dot{s} = f_1 - \frac{U_{dc}}{3L}[\alpha H_1 + \beta H_3 + \gamma H_5]$$

$$\begin{cases} \alpha = (2s_a - s_b + s_3 L) \\ \beta = (2s_b - s_a + s_3 L) \\ \gamma = (-s_a - s_b + s_3 L) \end{cases}$$

Control Logic

$$\begin{cases} H_1 = \mathrm{sign}(\alpha) \\ H_3 = \mathrm{sign}(\beta) \\ H_5 = \mathrm{sign}(\gamma) \end{cases}$$

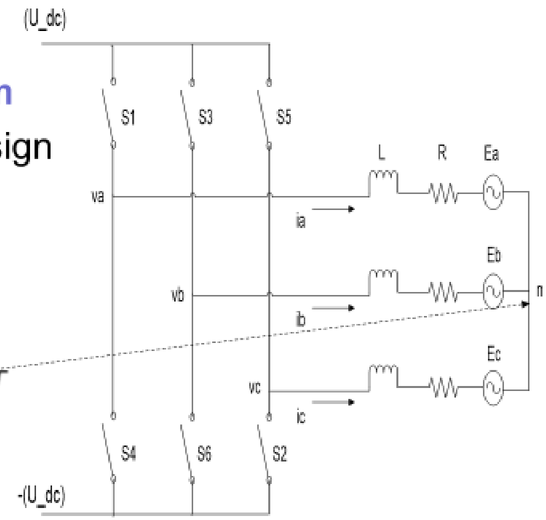
Cascade Sliding Mode Control

- **Idea: Utilize extra degree of freedom**
- Sliding surface design

$$\begin{cases} s_a &= i_a^* - i_a \\ s_b &= i_b^* - i_b \\ s_3 &= \int (v_n^* - v_n) d\tau \end{cases}$$

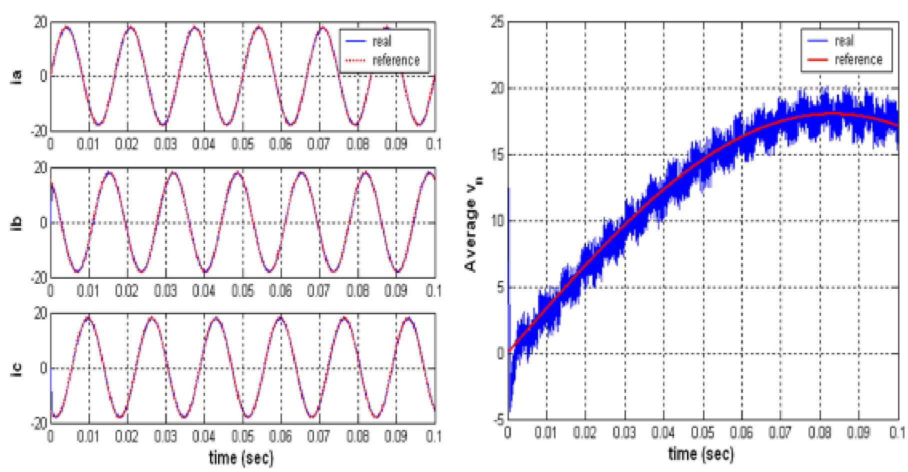
$$v_n = \frac{V_{dc}}{3}(S_1 + S_3 + S_5)$$

$$S_{1,3,5} = H_{1,3,5}$$

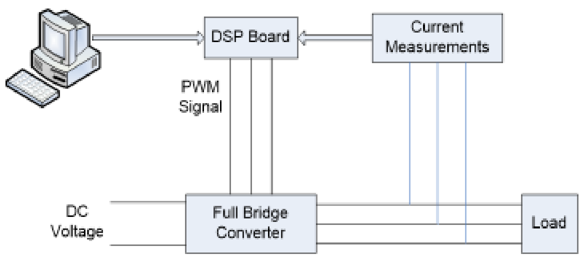


SMPWM Simulation Results

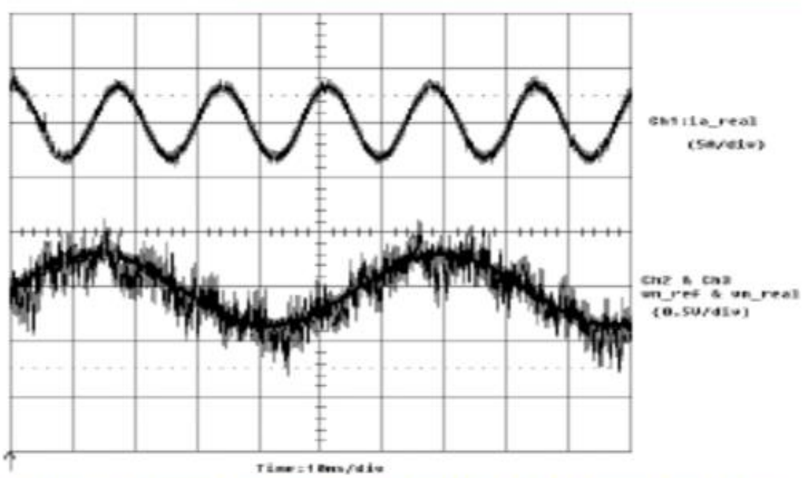
- Current tracking and v_n tracking



Experimental Setup



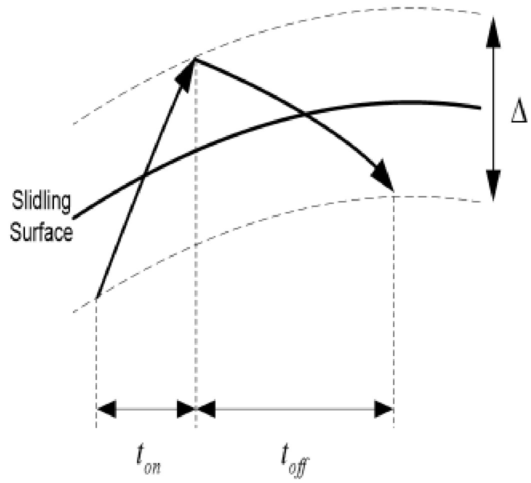
- Controller: TMS320F2812 DSP
- Switching devices: 2MBI100NC-12 IGBT
- 3 phase RL load
- 50 V DC supply



The first waveforms show the current tracking, the second ones show tracking a sinusoidal function by the average value of Vn (after filtering out a high frequency component of discontinuous function).

Vn Control

- *Objective: optimality by changing* v_n
- *Example: switching frequency reduction*

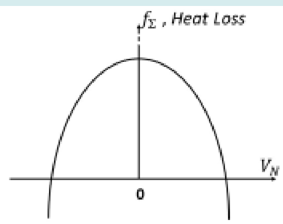


$$t_{\mathrm{on}\,i}(v_n) = \frac{\Delta}{\|f_{3i}(v_n) + \frac{U_{dc}}{3L}H_{2i-1}\|}$$

$$t_{\mathrm{off}\,i}(v_n) = \frac{-\Delta}{\|f_{3i}(v_n) - \frac{U_{dc}}{3L}H_{2i-1}\|}$$

$$f_{\mathrm{switch}}(v_n) = \frac{1}{t_{\mathrm{on}} + t_{\mathrm{off}}}$$

Minimization of frequency (heat loses)



$$a_1 = \underbrace{\frac{1}{3}i_a^* - \frac{1}{3}K_a}_{g_1} + \frac{1}{3L}V_N^*$$

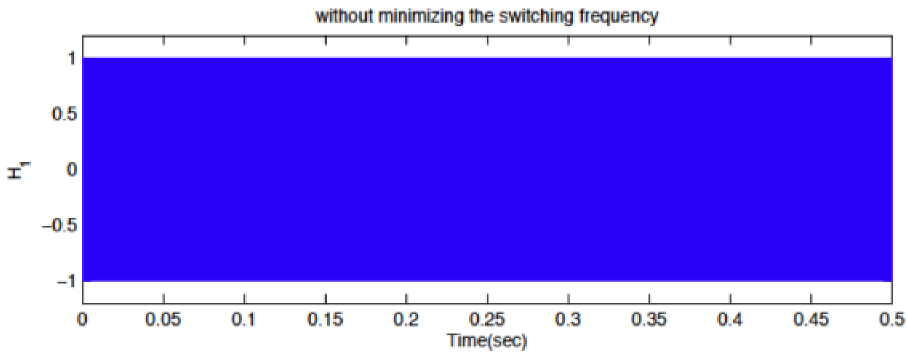
$$a_2 = \underbrace{\frac{1}{3}i_b^* - \frac{1}{3}K_b}_{g_2} + \frac{1}{3L}V_N^*$$

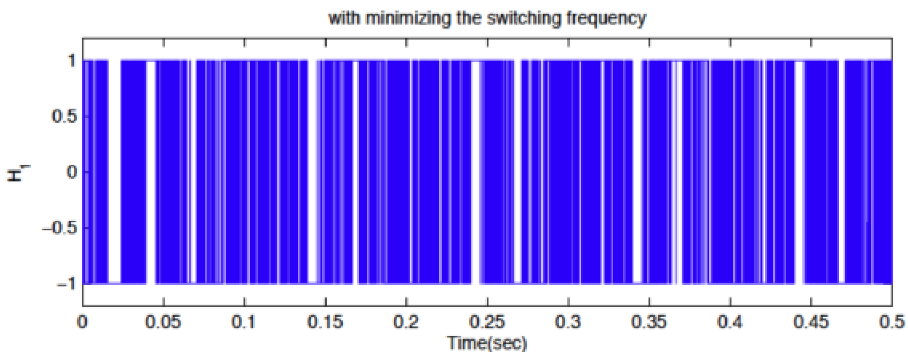
$$a_3 = \underbrace{-\frac{1}{3}(i_a^* + i_b^*) + \frac{1}{3}(K_a + K_b)}_{g_3} + \frac{1}{3L}V_N^*$$

$$f_\Sigma = \frac{3L}{2\Delta V_{dc}}\left[3\left(\frac{V_{dc}}{3L}\right)^2 - a_1^2 - a_2^2 - a_3^2\right] \qquad \frac{\partial f_\Sigma}{\partial V_N^*} = \frac{1}{\Delta V_{dc} L}V_N^*$$

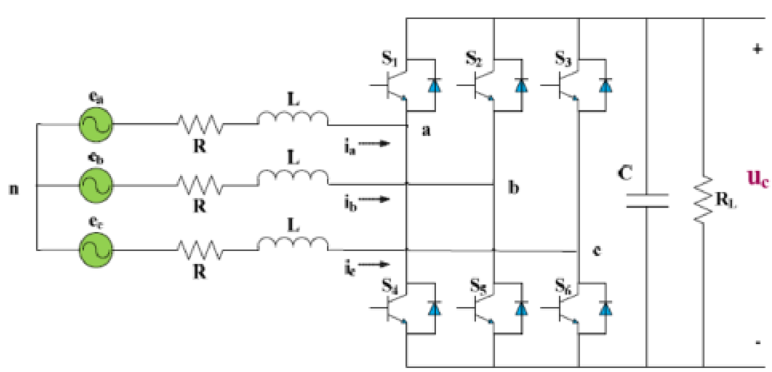
$$f_{switching}(V_N) = \sum_{i=1}^{3} f_{switching\_i}(V_N) \qquad f_{switching}(V_N) = \min \quad if \quad V_N = \max$$

$$\dot{s}_i = -\frac{V_{dc}}{3L}sign(s_i) + g_i + V_N, \quad \sum g_i = 0 \qquad \frac{\boldsymbol{V_{dc}}}{\boldsymbol{3L}} > |\boldsymbol{g_i} + \boldsymbol{V_N}|$$





- *Three-Phase, AC/DC Power Converter*



$$\begin{cases} L\dfrac{di_{ab}}{dt} = e_{ab} + u_c \Gamma S \\ \dfrac{du_c}{dt} = -\dfrac{u_c}{R} + (i_a, i_b, -i_a - i_b)S \end{cases}$$

$$\Gamma = \begin{bmatrix} \frac{-2}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{-2}{3} & \frac{1}{3} \end{bmatrix}$$

$$S_j = \begin{cases} 1, & S_j \text{ is close} \\ -1, & S_j \text{ is open} \end{cases} \qquad j = a, b, c$$

SLIDING MODE CURRENT TRACKING CONTROL

$\boldsymbol{i_{ab,ref}} = K * e_{ab}$ $\quad K \text{ is constant}$ - Power factor is equal to 1
- No high order harmonics

$\sigma_{ab} = L(i_{abref} - i_{ab})$

$$V = \frac{1}{2}\sigma_{ab}^T \sigma_{ab}$$

$$\dot{V} = \sigma_{ab}{}^T F(.) - u_c(\alpha, \beta, \gamma)S$$

where,

$$\alpha = \left(\frac{-2}{3}\sigma_a + \frac{1}{3}\sigma_b\right), \beta = \left(\frac{1}{3}\sigma_b + \frac{-2}{3}\sigma_b\right), \gamma = \left(\frac{1}{3}\sigma_a + \frac{-2}{3}\sigma_b\right)$$

If ${}^{u_c}/_{L}$ is large enough, $F(.)$ can be suppressed with conti given by:

$$S = \begin{cases} S_a = sign\ (\alpha) \\ S_b = sign\ (\beta) \\ S_c = sign\ (\gamma) \end{cases}$$

OUTPUT VOLTAGE CONTROL

$$\frac{d\sigma_{ab}}{dt} = L\frac{di_{abref}}{dt} - e_{ab} - u_c(\Gamma S)_{eq} = 0$$

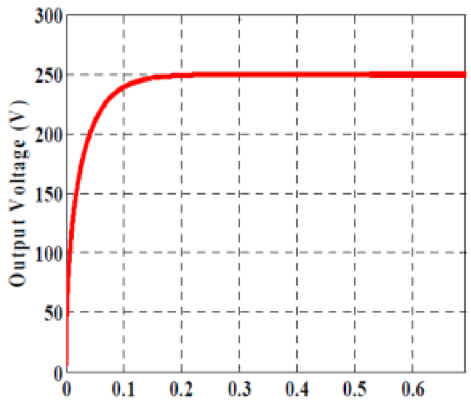
$$(\Gamma S)_{eq} = (L\frac{di_{abref}}{dt} - e_{ab})\frac{1}{u_c}$$

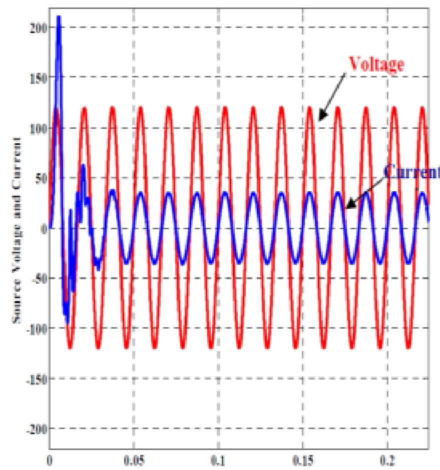
$$\frac{du_c}{dt} = -\frac{u_c}{R} + (i_a, i_b, -i_a - i_b)S \Longrightarrow C\frac{du_c}{dt} = -\frac{u_c}{R_L} + \frac{3}{2}KE_0{}^2\frac{1}{u_c}$$

Equilibrium point $\quad u_{css} = \sqrt{\frac{3}{2}KE_0{}^2R_L}$ $\quad$ *is asymptotically stable*

$$\boldsymbol{K = \frac{2}{3}\frac{u_{cref}^2}{E_0^2 R_L}}$$

SIMULATIONS

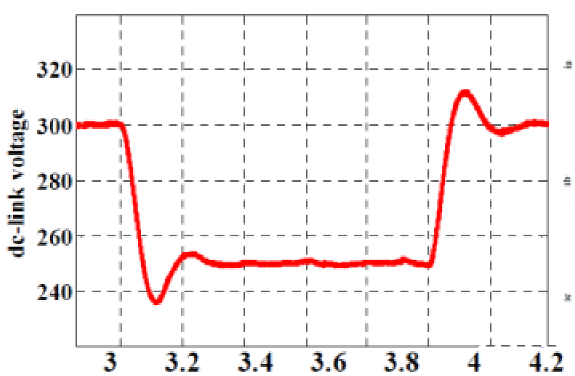


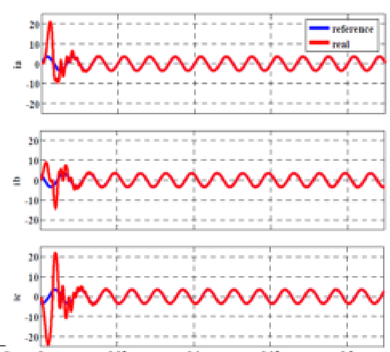


UNKNOWN PARAMETERS

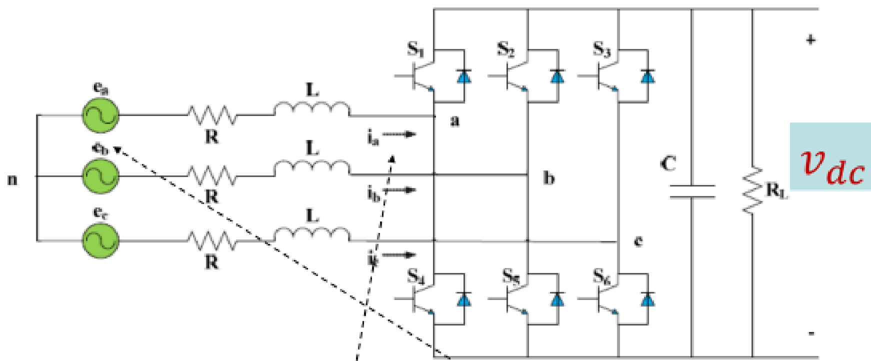
$$\dot{K} = \varepsilon\left(u_{cref} - u_c\right) \cdot$$

SIMULATIONS





DESIGN OF A CONTINUOUS SIGNAL GENERATOR BASED ON SLIDING MODE CONTROL OF THREE-PHASE AC-DC POWER CONVERTERS



$v_{dc,ref}$=*f(t)*, $i_{ab} = K(t)e_{ab}, K(t)$=???

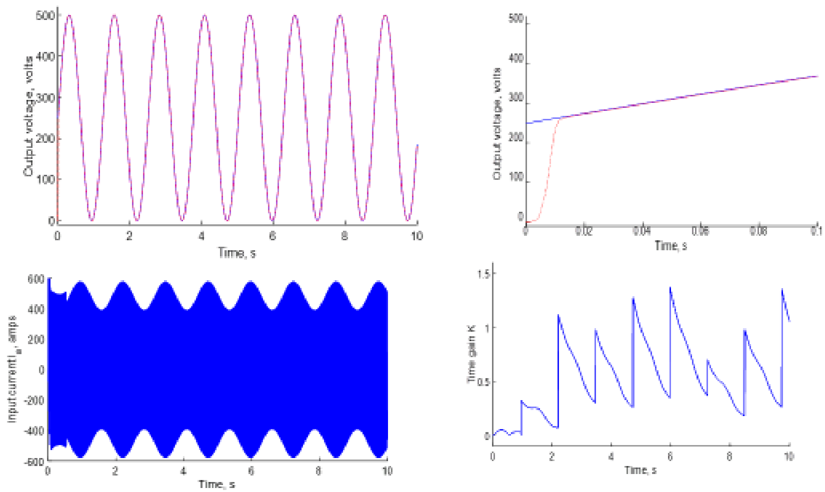
K(t) should satisfy differential equation

- $\left(\frac{1}{2}C\frac{d\boldsymbol{v_{dc}}}{dt} + \frac{1}{R_L}\boldsymbol{v_{dc}}\right)\frac{2}{3KE_0^2} = -L_g\frac{dK}{dt} + 1 - R_g K,$
- $\left(\frac{1}{2}C\frac{df(t)}{dt} + \frac{1}{R_L}\boldsymbol{f}(t)\right)\frac{2}{3KE_0^2} = -L_g\frac{dK}{dt} + 1 - R_g K$ **OBSERVER**
- $\left(\frac{1}{2}C\frac{d\Delta\boldsymbol{v}}{dt} + \frac{1}{R_L}\Delta\boldsymbol{v}\right)\frac{2}{3KE_0^2} = \mathbf{0},\ \Delta\boldsymbol{v} = \boldsymbol{f}(\boldsymbol{t}) - \boldsymbol{v_{dc}}$
- $\boldsymbol{\Delta v \to 0},$

$$\boldsymbol{v_{dc} \to f(t) = v_{dc,ref}}$$

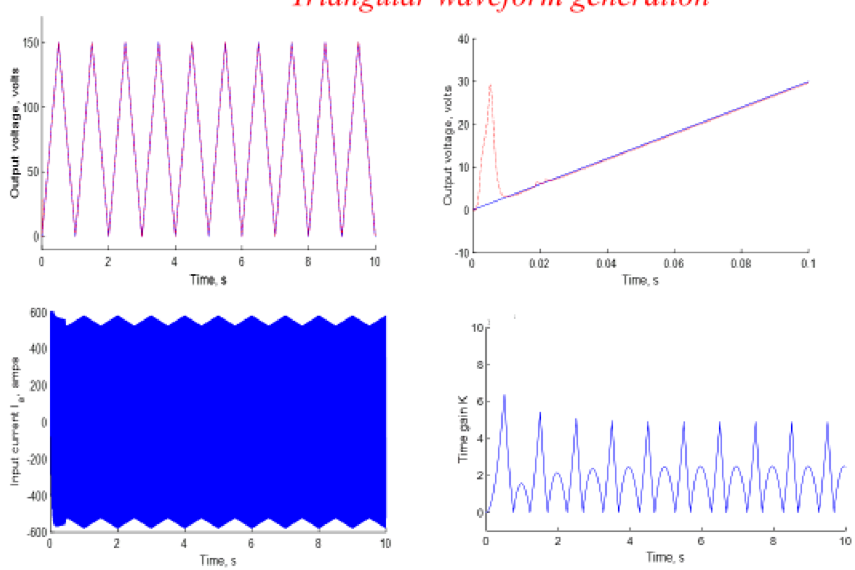
SIMULATION I

$$f'_1(t) = 250(\sin(15t) + 1)$$



SIMULATION II

Triangular waveform generation



• Problem statement

Control system: $\dot{x} = f(x, u),\ x \in R^n,$
$u \in R^m, \quad Feedback\ \ u = u(x, t).$
$Three - phase\ source\ is\ available\ only.$

Conventional approach –
a rectifier + DC/DC converter.

Problem: to get rid of a rectifier + DC/DC converter and have $u(x, t)$ as an output of three-phase converter.