### **Nonlocal Observables in Holography**

Irina Aref'eva
Steklov Mathematical Institute, Russian Academy of Sciences



International conference in memory of M. K. Polivanov «Polivanov–90»

16 - 17 December, 2020

Non-local observables in QFT

Non-local observables in QFT

Non-local observables in QFT

Why at this conference

Non-local observables in QFT

Why at this conference

Non-local observables in QFT

Why at this conference

List of non-local observables

Non-local observables in QFT

Why at this conference

List of non-local observables

Non-local observables in QFT

Why at this conference

List of non-local observables

Non-local observables in QFT

Why at this conference

List of non-local observables

Non-local observables in QFT

Why at this conference

List of non-local observables

Non-local observables in QFT

Why at this conference

List of non-local observables

Non-local observables in QFT

Why at this conference

List of non-local observables

Non-local observables in QFT

Why at this conference

List of non-local observables

Holography for non-local observables

Results

- Non-local observables in gauge theories
- D-dim YM or YM + quarks (QCD)

- Non-local observables in gauge theories
- D-dim YM or YM + quarks (QCD)
- Functional on a loop



Wilson loops

$$g(\Gamma) = P \exp \int_{\Gamma} A$$
 
$$W(\Gamma) = \langle \operatorname{Tr}(g(\Gamma)) \rangle$$

- Non-local observables in gauge theories
- D-dim YM or YM + quarks (QCD)

Functional on a loop



Wilson loops

$$g(\Gamma) = P \exp \int_{\Gamma} A$$
 
$$W(\Gamma) = \langle \operatorname{Tr}(g(\Gamma)) \rangle$$

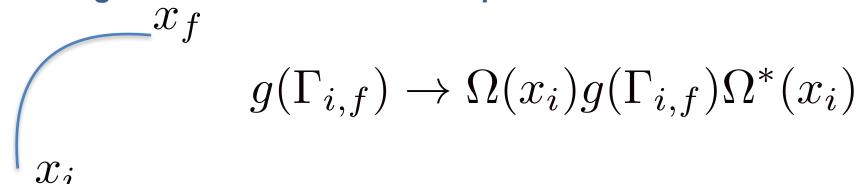
- Functional on a surface
  - B

Bags in QCD?

# Loop approach to YM equations

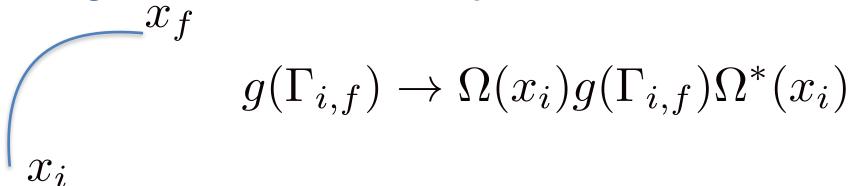
## Loop approach to YM equations

Gauge invariance of Wilson loops



## Loop approach to YM equations

Gauge invariance of Wilson loops



Dynamical equations in terms of Wilson loops

$$\mathcal{L}(\Gamma, x) \equiv g^*(\Gamma) \cdot \delta_{\Gamma, x} g(\Gamma)$$

$$\delta_{\Gamma,y} \mathcal{L}(\Gamma,x) - \delta_{\Gamma,x} \mathcal{L}(\Gamma,y) + [\mathcal{L}(\Gamma,x),\mathcal{L}(\Gamma,y)] = 0$$

$$\lim_{x \to y} \delta_{\Gamma,y} \mathcal{L}(\Gamma,x) = 0 \quad \longleftarrow \quad D_{\mu} F_{\mu\nu} = 0$$

# Why at this conference

## Why at this conference



N. N. Bogolyubov, B. V. Medvedev, and M. K. Polivanov,
 Problems of the Theory of Dispersion Relations), Fizmatgiz,
 1958

## Why at this conference



N. N. Bogolyubov, B. V. Medvedev, and M. K. Polivanov, Problems of the Theory of Dispersion Relations), Fizmatgiz, 1958

SOVIET PHYSICS JETP

APRIL, 1962

DEGREE OF GROWTH OF MATRIX ELEMENTS IN THE AXIOMATIC APPROACH

Mathematics Institute, Academy of Sciences, U.S.S.R. and Joint Institute for

Submitted to JETP editor March 22, 1961

It is shown that within the "axiomatic" approach for the construction of the scattering matrix, supplemented by the requirement that the theory be "renormalizable," some very strong restrictions arise on the possible degree of growth of the matrix elements.

 $I_N$  the last five years a lot of attention has been devoted to the study of the general structure of local quantum field theory.  $^{I_1=I_2}$  A central question in these investigations is the problem to what extent is the theory determined by only general requirements-relativistic invariance, unitarity and ompleteness of the system of positive energy

completenees of the system of positive energy completenees of the system of positive energy sumptions, the tare made when the theory is constituted to the basis of the fitself the system of the syst sents that must be satisfied by the matrix ele sary to introduce into the theory some local op-stors, since without them different points in s, since without them different points in time cannot be distinguished and the causal-jurement cannot be formulated. This can e (cf.<sup>[4]</sup>) by writing the S matrix as an sion in normal products of asymptotic fields:

fields  $\varphi(x)$ . A system of basic assumptions of this type has been formulated by Bogolyuboy  $\xi^{\ell}$  for a theory with adiabatic switching on and off of the interaction; this system, as was shown by Bogolyubov and

the authors <sup>(5,1)\*</sup> especially for derivation of dis-persion relations and spectral representations of the Killen-Lehmann type. We shall refer to this approach for the construction of quantum field theory, based on the set of fundamental assump-tions of BMP, Sec. 2, and resting on the methods of dispersion theory, as the dispersion approach The significance of the dispersion approach to quantum field theory is not restricted to the limited number of exact results, that have been ob tained with its help, but determines a new method

one term in the expansion after another. The ad- $S = \sum_{i=1}^{n} \left( dx_1 \dots dx_n \Phi^n(x_1, \dots, x_n) : \varphi(x_1) \dots \varphi(x_n) \right)$  to resort to the physically unsatisfactory adiabatic switching on and off procedure, and one will be and then extending to the nearry shell by only the photocolities of the nearry shell by only the photocolities of the nearry shell by the photocolities of the photocolities of the nearry shell by variational differentiation with respect to the following  $\mu$  of the photocolities of the nearry shell by variational differentiation with respect to the following  $\mu$  of the nearry shell by the photocolities of the nearry shell by variational differentiation with respect to the following  $\mu$  of the nearry shell  $\mu$  of the nearry shell B. V. Medvedev and M. K. Polivanov, JETP 41, 1130 (1961),

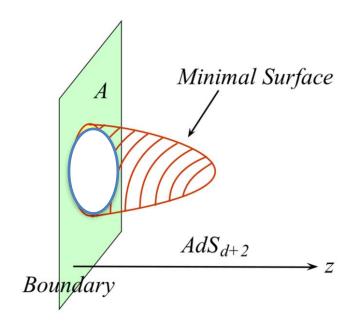
**Soviet Phys. JETP 14, 807 (1962)** 

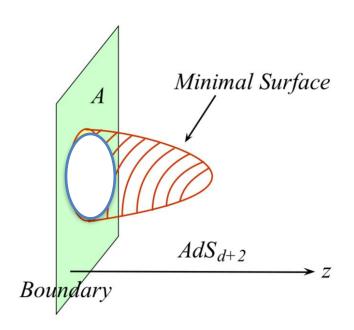
$$j(x) = i \frac{\delta S}{\delta \varphi(x)} S^+.$$
  $\mathcal{L}(\Gamma, x) \equiv g^*(\Gamma) \cdot \delta_{\Gamma, x} g(\Gamma)$ 

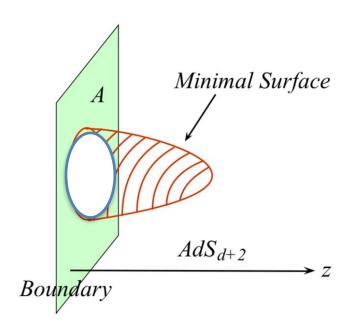
- **T-Dyson vs T-Wick**
- T-exp is just a Wilson line for time segment

\*To be referred to in the following as BMP.



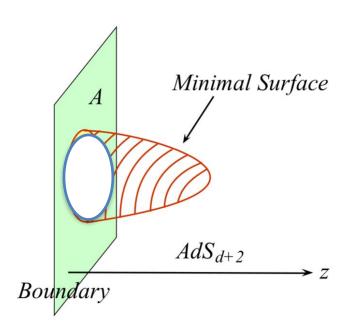






$$S = \frac{1}{2\pi\alpha'} \int d\sigma_1 d\sigma_2 \sqrt{-\det h_{\alpha\beta}}$$

$$h_{\alpha\beta} = g_{\mu\nu} \frac{\partial x^{\mu}}{\partial \sigma^{\alpha}} \frac{\partial x^{\nu}}{\partial \sigma^{\beta}}$$

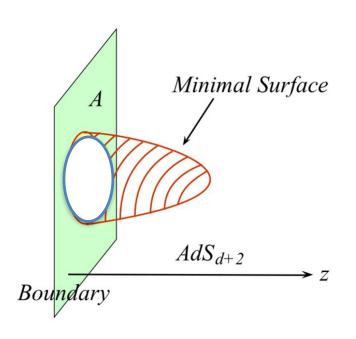


$$S = \frac{1}{2\pi\alpha'} \int d\sigma_1 d\sigma_2 \sqrt{-\det h_{\alpha\beta}}$$

$$h_{\alpha\beta} = g_{\mu\nu} \frac{\partial x^{\mu}}{\partial \sigma^{\alpha}} \frac{\partial x^{\nu}}{\partial \sigma^{\beta}}$$

$$\Gamma_{T,L} = T \times L$$

$$W(\Gamma_{T,L}) = e^{-TV(L)}$$

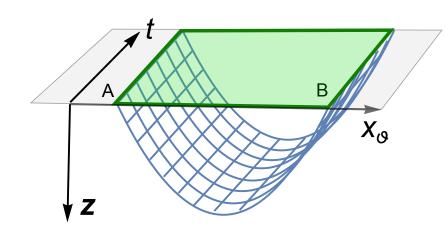


$$\Gamma_{T,L} = T \times L$$

$$W(\Gamma_{T,L}) = e^{-TV(L)}$$

$$S = \frac{1}{2\pi\alpha'} \int d\sigma_1 d\sigma_2 \sqrt{-\det h_{\alpha\beta}}$$

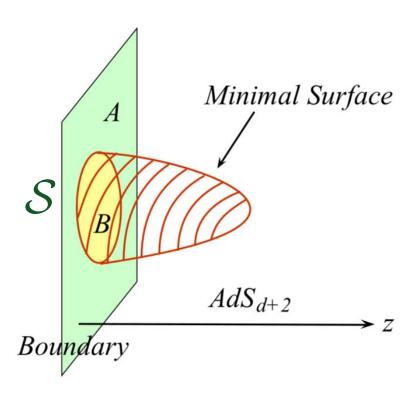
$$h_{\alpha\beta} = g_{\mu\nu} \frac{\partial x^{\mu}}{\partial \sigma^{\alpha}} \frac{\partial x^{\nu}}{\partial \sigma^{\beta}}$$



**=Entanglement Entropy** 

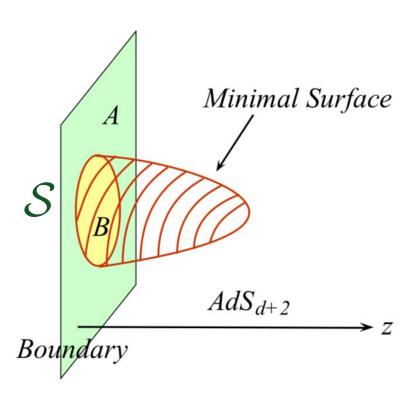


# **=Entanglement Entropy**



The holographic calculation of entanglement entropy via AdS/CFT.

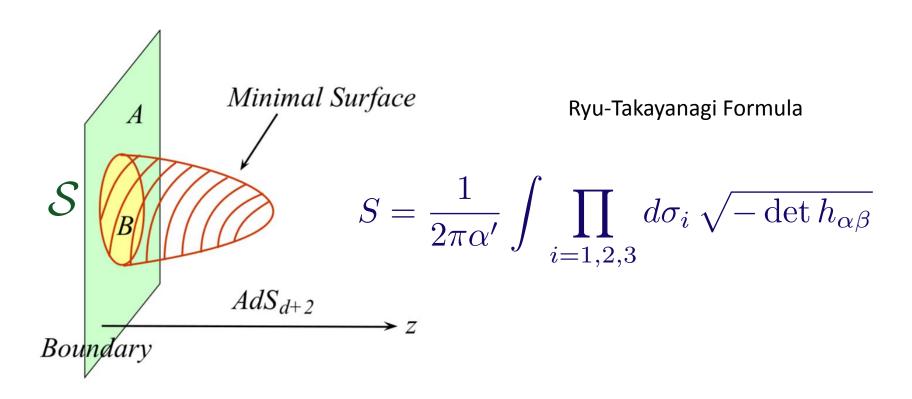
# **=Entanglement Entropy**



Ryu-Takayanagi Formula

The holographic calculation of entanglement entropy via AdS/CFT.

# **=Entanglement Entropy**

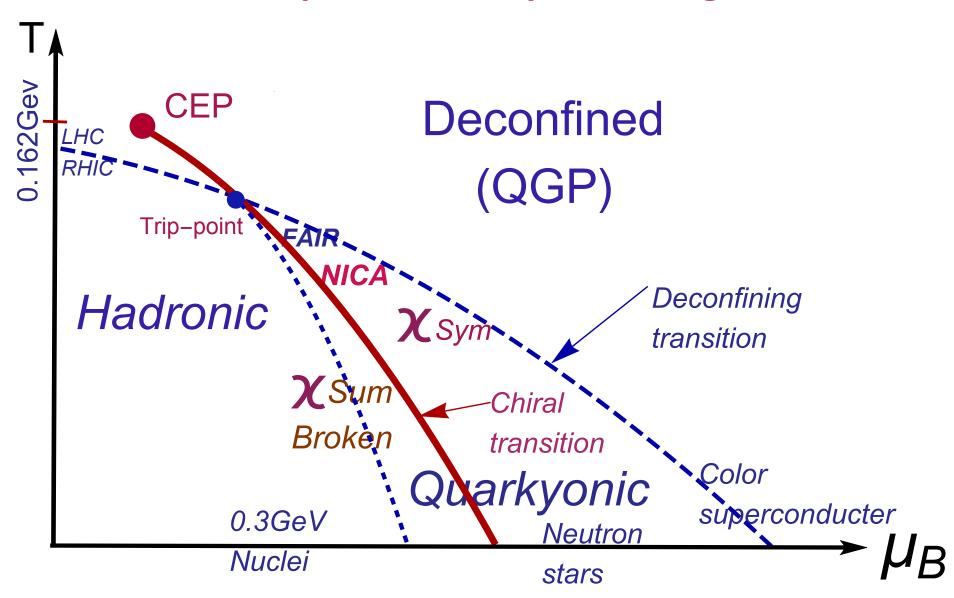


The holographic calculation of entanglement entropy via AdS/CFT.

## Applications to study phase transition in QGP

# The expected QCD phase diagram

## The expected QCD phase diagram





## Few remarks on holography (AdS/CFT)

Holography is nowadays one of the most effective tools to study quantum non-equilibrium physics of strongly interacting many body systems.

## Few remarks on holography (AdS/CFT)

Holography is nowadays one of the most effective tools to study quantum non-equilibrium physics of strongly interacting many body systems.

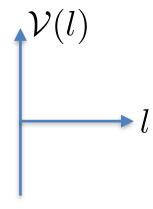
These systems include ultrarelativistic heavy-ion collisions, cold atom systems, quantum simulators, "ultrafast" techniques in condensed matter physics, etc.

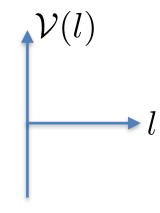
## Few remarks on holography (AdS/CFT)

Holography is nowadays one of the most effective tools to study quantum non-equilibrium physics of strongly interacting many body systems.

These systems include ultrarelativistic heavy-ion collisions, cold atom systems, quantum simulators, "ultrafast" techniques in condensed matter physics, etc.

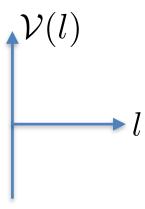
Holography translates the physics of quantum many body systems into a dual classical gravitational problem in a space-time with an extra dimension.





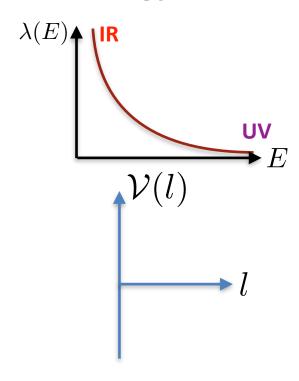
#### Starting point - 5-dim background. 5-th coordinate - energy scale

1) We have QCD and renormgroup flow (beta-function) (in 4D).



#### Starting point - 5-dim background. 5-th coordinate - energy scale

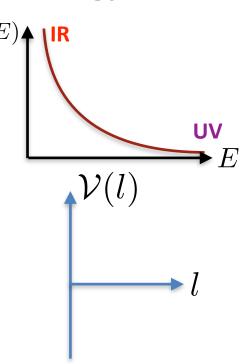
1) We have QCD and renormgroup flow (beta-function) (in 4D).



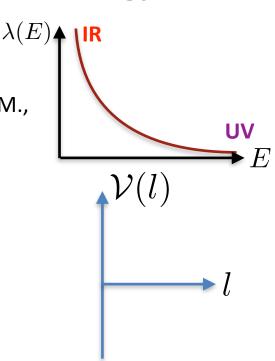
#### Starting point - 5-dim background. 5-th coordinate - energy scale

1) We have QCD and renormgroup flow (beta-function) (in 4D).

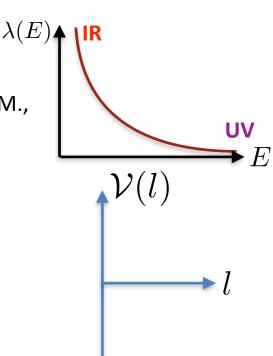
2) It's turn out that there are solutions to 5D classical Einstein E.O.M., that reproduce the QCD renormgroup flow (Holographic renormgroup flow)



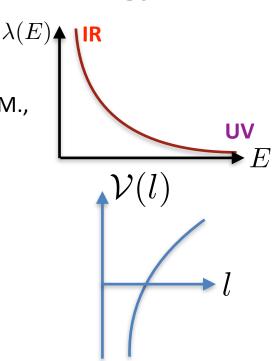
- 1) We have QCD and renormgroup flow (beta-function) (in 4D).
- 2) It's turn out that there are solutions to 5D classical Einstein E.O.M., that reproduce the QCD renormgroup flow (Holographic renormgroup flow)
- 3) These solution to classical Einstein eqs (with matter) that have the form of deformed AdS



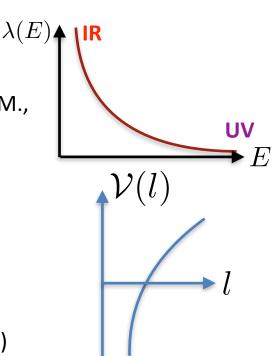
- 1) We have QCD and renormgroup flow (beta-function) (in 4D).
- 2) It's turn out that there are solutions to 5D classical Einstein E.O.M., that reproduce the QCD renormgroup flow (Holographic renormgroup flow)
- 3) These solution to classical Einstein eqs (with matter) that have the form of deformed AdS
- 4) QCD diagram as well as the Cornell potential at zero chemical potential has to be reproduced lattice data



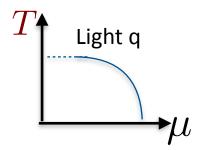
- 1) We have QCD and renormgroup flow (beta-function) (in 4D).
- 2) It's turn out that there are solutions to 5D classical Einstein E.O.M., that reproduce the QCD renormgroup flow (Holographic renormgroup flow)
- 3) These solution to classical Einstein eqs (with matter) that have the form of deformed AdS
- 4) QCD diagram as well as the Cornell potential at zero chemical potential has to be reproduced lattice data

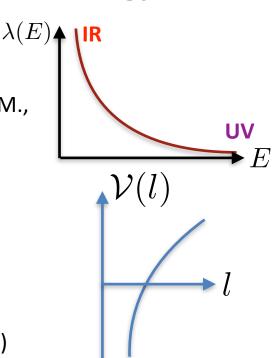


- 1) We have QCD and renormgroup flow (beta-function) (in 4D).
- 2) It's turn out that there are solutions to 5D classical Einstein E.O.M., that reproduce the QCD renormgroup flow (Holographic renormgroup flow)
- 3) These solution to classical Einstein eqs (with matter) that have the form of deformed AdS
- 4) QCD diagram as well as the Cornell potential at zero chemical potential has to be reproduced lattice data
- 5) Dependence on quark mass (RG coeff. depend on quark masses)

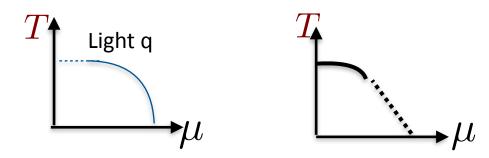


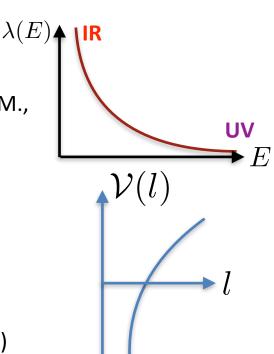
- 1) We have QCD and renormgroup flow (beta-function) (in 4D).
- 2) It's turn out that there are solutions to 5D classical Einstein E.O.M., that reproduce the QCD renormgroup flow (Holographic renormgroup flow)
- 3) These solution to classical Einstein eqs (with matter) that have the form of deformed AdS
- 4) QCD diagram as well as the Cornell potential at zero chemical potential has to be reproduced lattice data
- 5) Dependence on quark mass (RG coeff. depend on quark masses)



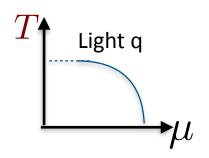


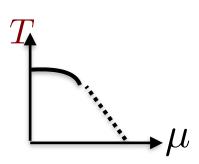
- 1) We have QCD and renormgroup flow (beta-function) (in 4D).
- 2) It's turn out that there are solutions to 5D classical Einstein E.O.M., that reproduce the QCD renormgroup flow (Holographic renormgroup flow)
- 3) These solution to classical Einstein eqs (with matter) that have the form of deformed AdS
- 4) QCD diagram as well as the Cornell potential at zero chemical potential has to be reproduced lattice data
- 5) Dependence on quark mass (RG coeff. depend on quark masses)

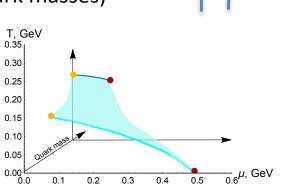


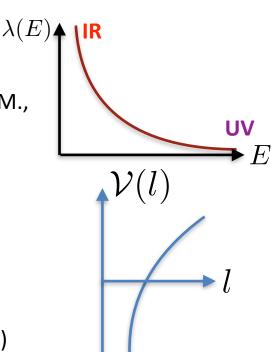


- 1) We have QCD and renormgroup flow (beta-function) (in 4D).
- 2) It's turn out that there are solutions to 5D classical Einstein E.O.M., that reproduce the QCD renormgroup flow (Holographic renormgroup flow)
- 3) These solution to classical Einstein eqs (with matter) that have the form of deformed AdS
- 4) QCD diagram as well as the Cornell potential at zero chemical potential has to be reproduced lattice data
- 5) Dependence on quark mass (RG coeff. depend on quark masses)



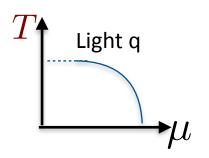


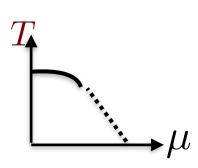


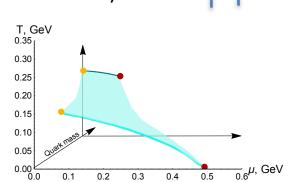


#### Starting point - 5-dim background. 5-th coordinate - energy scale

- 1) We have QCD and renormgroup flow (beta-function) (in 4D).
- 2) It's turn out that there are solutions to 5D classical Einstein E.O.M., that reproduce the QCD renormgroup flow (Holographic renormgroup flow)
- 3) These solution to classical Einstein eqs (with matter) that have the form of deformed AdS
- 4) QCD diagram as well as the Cornell potential at zero chemical potential has to be reproduced lattice data
- 5) Dependence on quark mass (RG coeff. depend on quark masses)



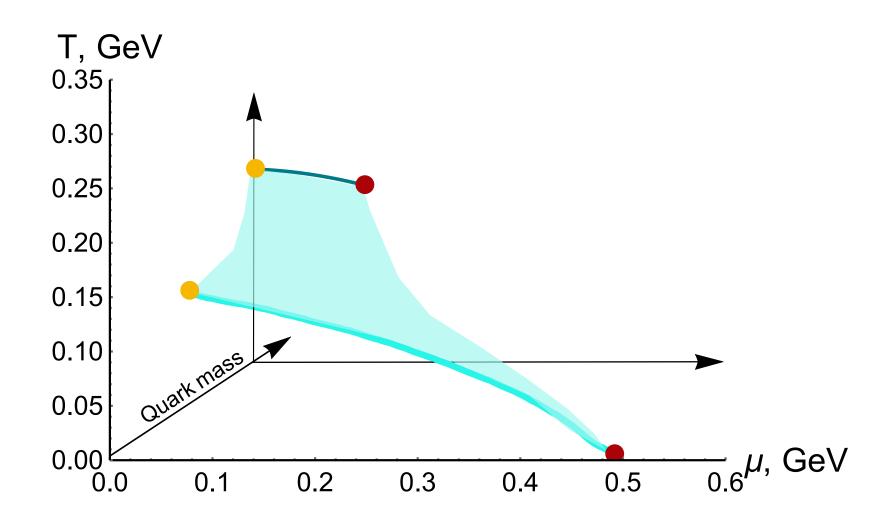




 $\lambda(E)$ 

The action has to be found by «trial and error method». What is a «best» 5-dim background?

## **QCD** diagram



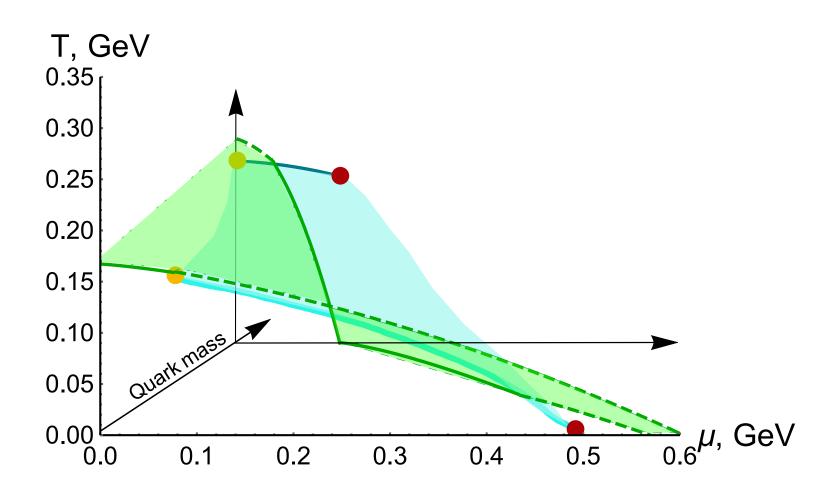
1-st order phase transition

# **QCD** diagrams

## **QCD** diagrams

1-st order phase transition and Wilson loop phase transition

## **QCD** diagrams



1-st order phase transition and Wilson loop phase transition

Holographic anisotropic phenomenological models

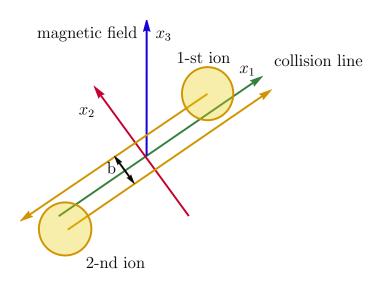
Origins of anisotropy

- Holographic anisotropic phenomenological models
  - Origins of anisotropy
    - Longitudinal and 2 transversal directions

- Holographic anisotropic phenomenological models
  - Origins of anisotropy
    - Longitudinal and 2 transversal directions
    - Full anisotropy (in strong magnetic) field)

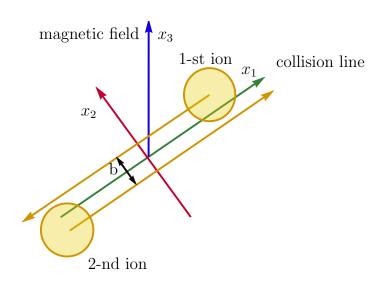
# Holographic anisotropic phenomenological models

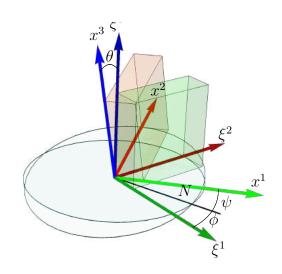
- Origins of anisotropy
  - Longitudinal and 2 transversal directions
  - Full anisotropy (in strong magnetic) field)



# Holographic anisotropic phenomenological models

- Origins of anisotropy
  - Longitudinal and 2 transversal directions
  - Full anisotropy (in strong magnetic) field)





Einstein-dilaton-two-Maxwell

I.A., K. Rannu, JHEP' 18

$$S = \int \frac{d^5x}{16\pi G_5} \sqrt{-\det(g_{\mu\nu})} \left[ R - \frac{f_1(\phi)}{4} F_{(1)}^2 - \frac{f_2(\phi)}{4} F_{(2)}^2 - \frac{1}{2} \partial_{\mu}\phi \partial^{\mu}\phi - V(\phi) \right]$$

Einstein-dilaton-two-Maxwell

I.A., K. Rannu, JHEP' 18

$$S = \int \frac{d^5x}{16\pi G_5} \sqrt{-\det(g_{\mu\nu})} \left[ R - \frac{f_1(\phi)}{4} F_{(1)}^2 - \frac{f_2(\phi)}{4} F_{(2)}^2 - \frac{1}{2} \partial_{\mu}\phi \partial^{\mu}\phi - V(\phi) \right]$$

$$ds^{2} = \frac{Lb(z)}{z^{2}} \left[ -g(z)dt^{2} + dx^{2} + R(z)(dy_{1}^{2} + dy_{2}^{2}) + \frac{1}{g(z)}dz^{2} \right]$$

$$\phi = \phi(z), \qquad A_{\mu}^{(1)} = A_t(z)\delta_{\mu}^0, \qquad F_{\mu\nu}^{(2)} = q \ dy^1 \wedge dy^2$$

Einstein-dilaton-two-Maxwell

I.A., K. Rannu, JHEP' 18

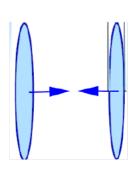
$$S = \int \frac{d^5x}{16\pi G_5} \sqrt{-\det(g_{\mu\nu})} \left[ R - \frac{f_1(\phi)}{4} F_{(1)}^2 - \frac{f_2(\phi)}{4} F_{(2)}^2 - \frac{1}{2} \partial_{\mu}\phi \partial^{\mu}\phi - V(\phi) \right]$$

$$ds^{2} = \frac{Lb(z)}{z^{2}} \left[ -g(z)dt^{2} + dx^{2} + R(z)(dy_{1}^{2} + dy_{2}^{2}) + \frac{1}{g(z)}dz^{2} \right]$$

$$\phi = \phi(z), \qquad A_{\mu}^{(1)} = A_t(z)\delta_{\mu}^0, \qquad F_{\mu\nu}^{(2)} = q \ dy^1 \wedge dy^2$$

$$F_{\mu\nu}^{(2)} = q \ dy^1 \wedge dy^2$$

**Schematic picture of** central HIC



$$y_1$$
  $\vec{y} = (y_1, y_2)$   $x - \text{beamer line}$ 

# **Full Anisotropic Background**

## **Full Anisotropic Background**

IA, Rannu, Slepov, 2011.07023

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left[ R - \frac{f_1(\phi)}{4} F_{(1)}^2 - \frac{f_2(\phi)}{4} F_{(2)}^2 - \frac{f_B(\phi)}{4} F_{(B)}^2 - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right],$$

$$ds^{2} = \frac{L^{2}}{z^{2}} e^{2A(z)} \left[ -g(z)dt^{2} + dx^{2} + \left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dy_{1}^{2} + e^{c_{B}z^{2}} \left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dy_{2}^{2} + \frac{dz^{2}}{g(z)} \right]$$

### **Full Anisotropic Background**

IA, Rannu, Slepov, 2011.07023

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left[ R - \frac{f_1(\phi)}{4} F_{(1)}^2 - \frac{f_2(\phi)}{4} F_{(2)}^2 - \frac{f_B(\phi)}{4} F_{(B)}^2 - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right],$$

$$ds^{2} = \frac{L^{2}}{z^{2}} e^{2A(z)} \left[ -g(z)dt^{2} + dx^{2} + \left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dy_{1}^{2} + e^{c_{B}z^{2}} \left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dy_{2}^{2} + \frac{dz^{2}}{g(z)} \right]$$

$$S = \frac{1}{2\kappa^2} \int d^5x \sqrt{-g} \left[ R + \mathcal{L}_M \right],$$

Gursoy, Jarvinen, Nijs, Pedraza, 2011.09474

$$\mathcal{L}_M = -\frac{1}{2}(\partial \phi)^2 + V(\phi) - \frac{1}{2}Z(\phi)(\partial \chi)^2$$

$$ds^{2} = e^{2A(r)} \left[ -f(r)dt^{2} + dx_{1}^{2} + e^{2U(r)}dx_{2}^{2} + e^{2W(r)}dx_{3}^{2} + \frac{dr^{2}}{f(r)} \right]$$

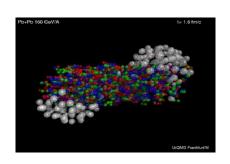
### **Thermalization via Holography**

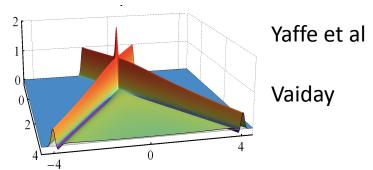
### Thermalization via Holography

### Thermalization in D <



### **BH formation in D+1**





Compare with W. Heisenberg, "Production of mesons as a shockwave problem," Zeit.Phys. 133 (1952) 65.

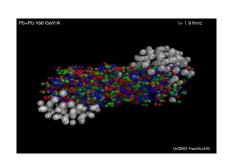
D=2 example

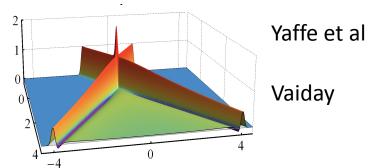
### Thermalization via Holography

#### **Thermalization in D**



#### BH formation in D+1

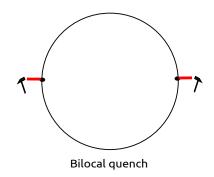


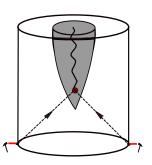


Compare with W. Heisenberg, "Production of mesons as a shockwave problem," Zeit.Phys. 133 (1952) 65.

#### D=2 example

Thermalization after holographic bilocal quench





I.A., Khramtsov, Tikhanovskaya JHEP09(2017)115 D.Ageev,I.A. JHEP 03 (2018) 103



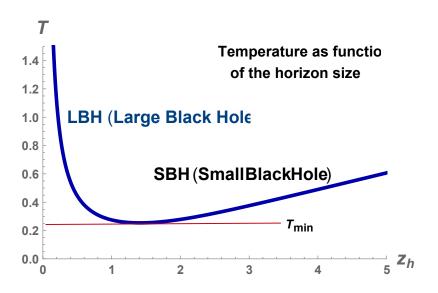


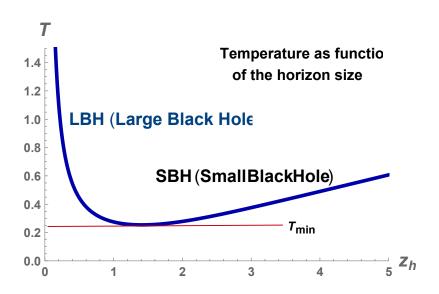
## Thermodynamics of the anisotropic background

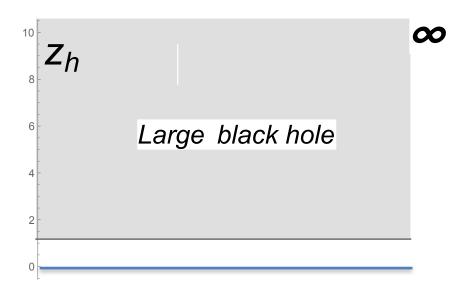


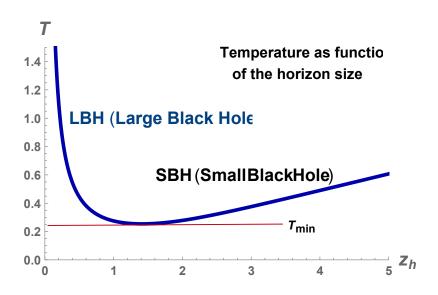
## Thermodynamics of the anisotropic background

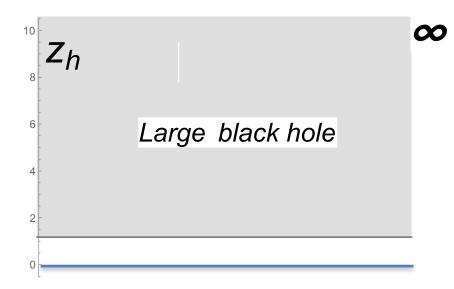
Thermodynamics for HIC (compare with Landau, 1952, Fermi, 1952)
Here thermodynamics for 5-dim BH

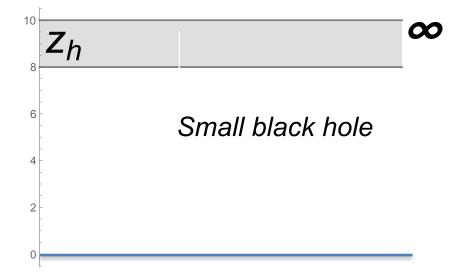


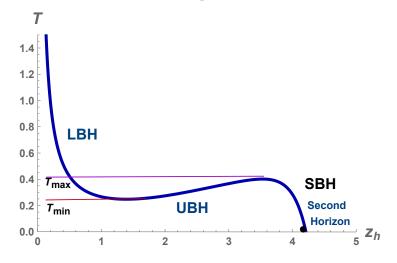


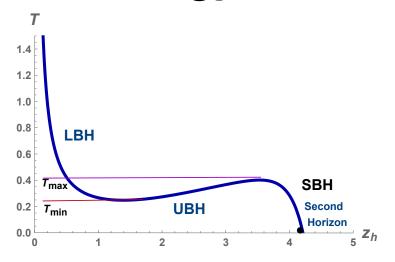


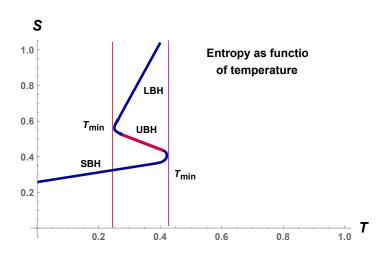


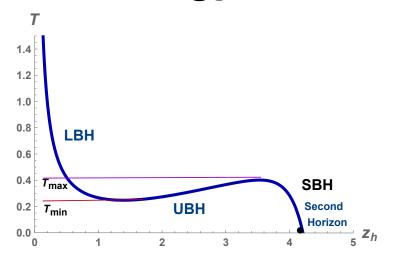


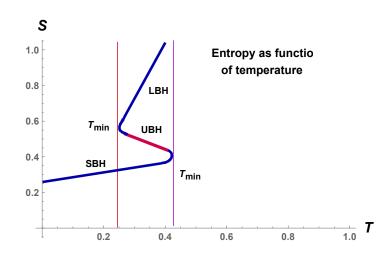




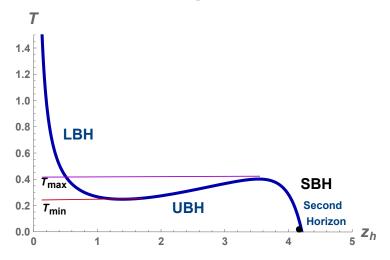




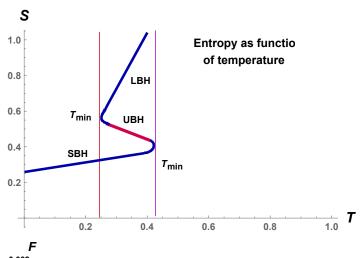


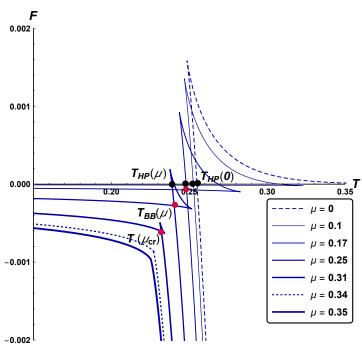


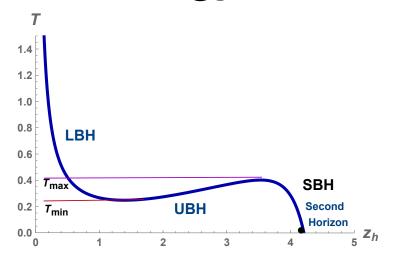
$$F(z_h, c, \nu) = \int_{z_h}^{\infty} s(z_h, c, \nu) T'(z_h, c, \nu) dz_h$$



$$F(z_h, c, \nu) = \int_{z_h}^{\infty} s(z_h, c, \nu) T'(z_h, c, \nu) dz_h$$

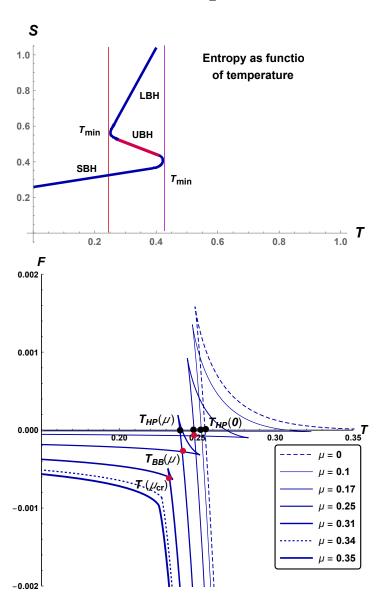


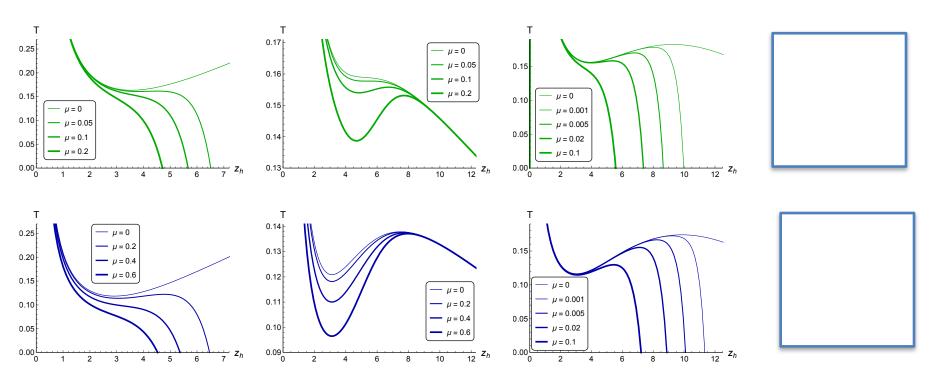


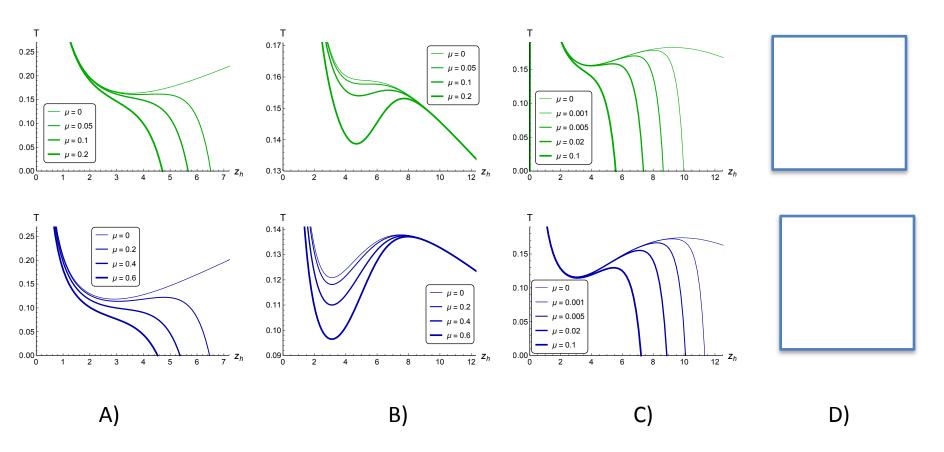


$$F(z_h, c, \nu) = \int_{z_h}^{\infty} s(z_h, c, \nu) T'(z_h, c, \nu) dz_h$$

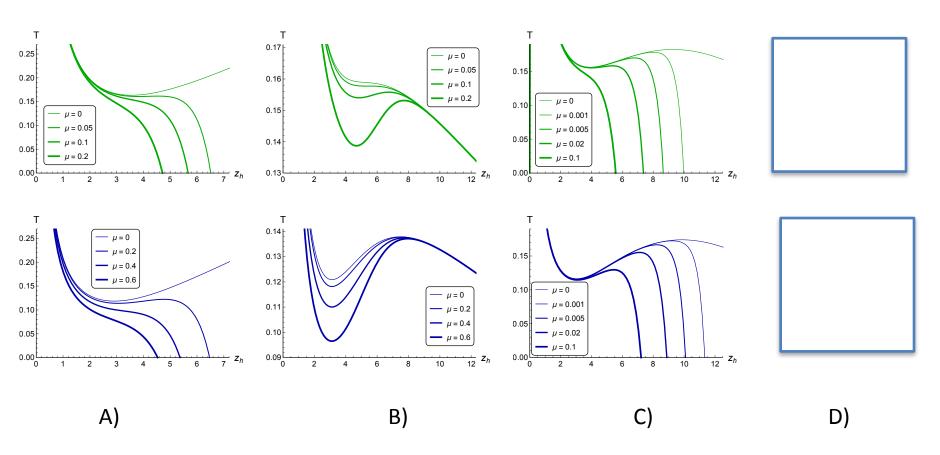
The swallow-tailed shape







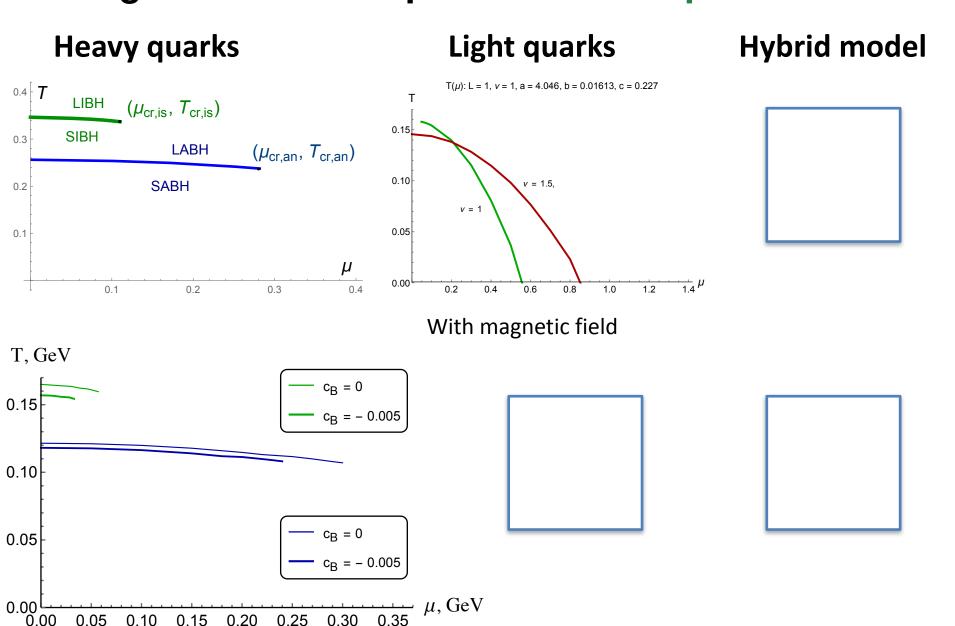
Temperature as function of horizon for different  $\mu$  in isotropic model for heavy A) and light B) quarks without magnetic field and with magnetic field C),  $c_B = -0.005$ , in the isotropic case,  $\nu = 1$  (top line) and anisotropic case,  $\nu = 4.5$  (bottom line).



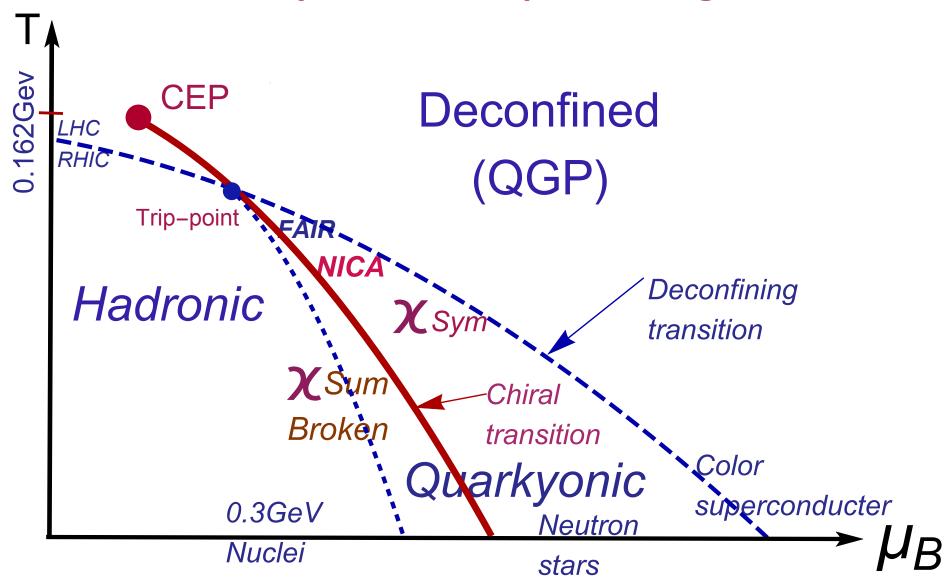
Temperature as function of horizon for different  $\mu$  in isotropic model for heavy A) and light B) quarks without magnetic field and with magnetic field C),  $c_B = -0.005$ , in the isotropic case,  $\nu = 1$  (top line) and anisotropic case,  $\nu = 4.5$  (bottom line).

D) Light quarks with magnetic field - work in progress

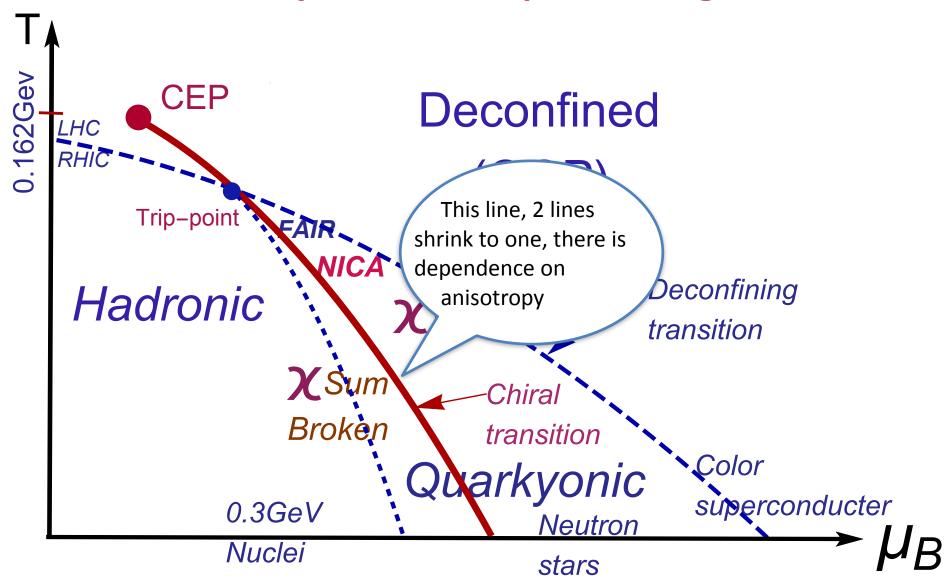
## Thermodynamics of the Anisotropic Backgrounds as compare with Isotropic ones



### The expected QCD phase diagram



### The expected QCD phase diagram





### Phase transitions for **non-local observables**

#### Wilson loop

temporal WL (confinement), spatial WL (drag forces, energy lost)

**Light-cone WL (jet quenching)** 

D.Ageev,IA, A.Golubtsova, E. Gourgoulhon, NuclPhysB,18' I.A,Golubtsova, Policastro, JHEP19'

IA,Rannu, JHEP18

IA, Rannu, Slepov, PhysLettB, 19';

2009.05562,2011.07023,2012.05758,

### Phase transitions for **non-local observables**

#### Wilson loop

temporal WL (confinement), spatial WL (drag forces, energy lost)

Light-cone WL (jet quenching)

D.Ageev,IA, A.Golubtsova, E. Gourgoulhon, NuclPhysB,18' I.A,Golubtsova, Policastro, JHEP19'

IA, Rannu, JHEP18

IA, Rannu, Slepov, PhysLettB, 19';

2009.05562,2011.07023,2012.05758,

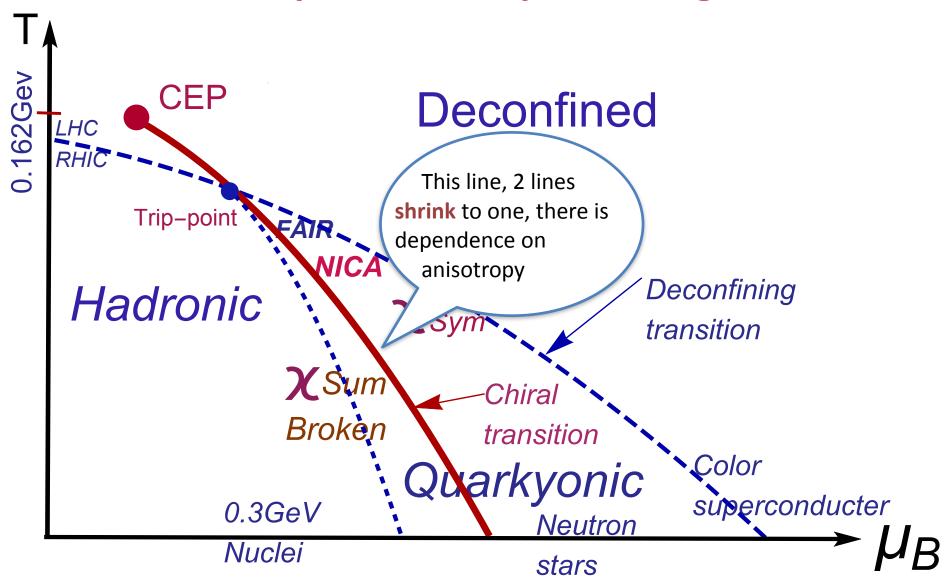
### **Entanglement Entropy**

How to measure in experiments?

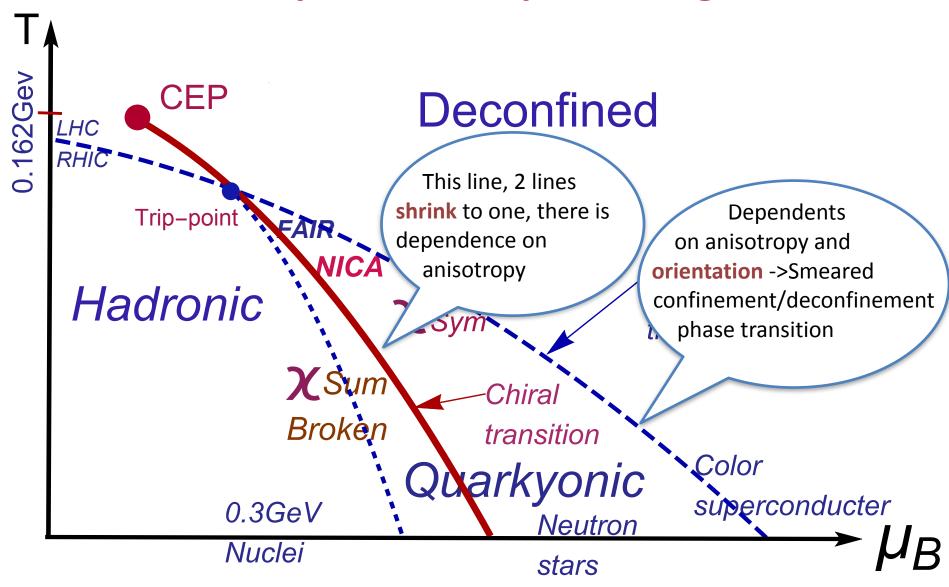
May be related with # of particles emitted from given volume

1. I.A., Patrushev, Slepov, JHEP20'

### The expected QCD phase diagram



### The expected QCD phase diagram



Holography is very suitable to study nonlocal observables in QFT

Holography is very suitable to study nonlocal observables in QFT

Holographic models are some kind of phenomenological models with few number of parameters

Holography is very suitable to study nonlocal observables in QFT

Holographic models are some kind of phenomenological models with few number of parameters

We have considered the anizotropic HQCD models that describe:

multiplicity, quark confinement;

#### predict:

smothering of transition lines between confinement/deconfinement phases

anisotropy parameter dependence of locations of phase transitions in (T,chemical potential, magnetic field)-plane inverse magnetic catalyse

## Thank you!