A mathematical model of primordial cosmology

"History … is mysterious and hardly is lending itself to any logical evaluation"

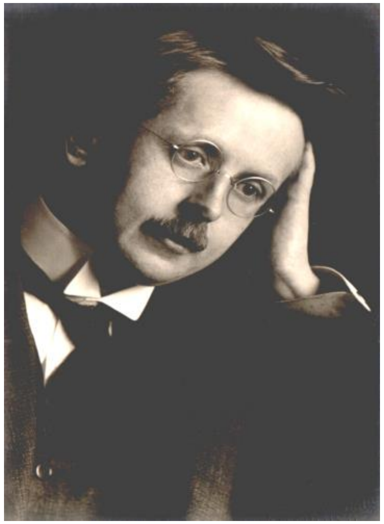
M.K.Polivanov

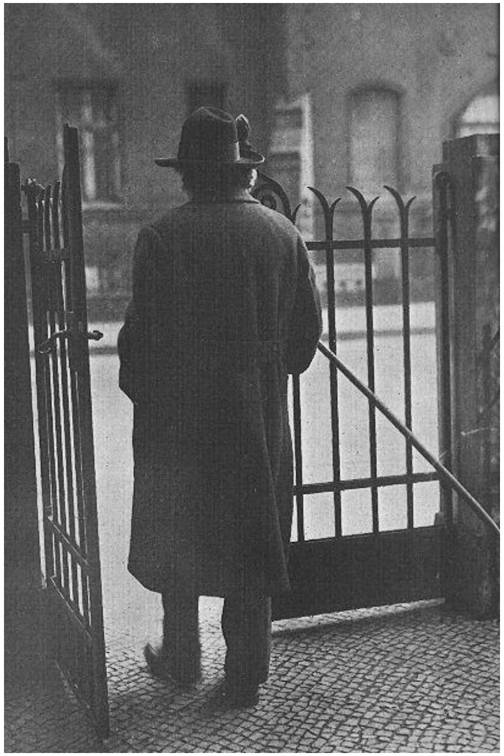
INTRODUCTION

Three faces of cosmology: 1. observational, 2. (astro)physical, 3. mathematical

What is mathematical cosmology?

At present we know the age (14 billion years) and dimension of the observable universe; homogeneous and isotropic distribution of different types of matter, approx. 5% of which is baryonic matter. Also, effective \La = 1,1 10^{-56} *cm*^{-2}, and *ca* 70% is DE. Then, *ca* 20% is DE. These data – for observable universe. BiBa is considered to be established. There is no reliable physical picture for the origin of the universe, and no initial conditions for the next scenario – inflationary universe, that is constructed by *trial and error* approach to get a reasonable exit into BB. With this aim, a special scalar field was "invented", the "potential" of which can be obtained only by trial end error approach. This goal was more or less achieved, but this is not a well defined mathematical model... (at least, to me?)





Ideas of Weyl, Eddington and Einstein

on **generalizing** *the* **geometry** *of* **General Relativity** *to* **incorporate photon**

(forgotten discovery)

1919--1923

Those who think of metaphysics as the most unconstrained are misinformed; compared with cosmology metaphysics is pedestrian and unimaginative.

S. Toulmin, British philosopher, 1922 -2009

Interesting reviews

The Cosmological Constant and Dark Energy. P.J.E. Peebles, Bharat Ratra -- **0207347**

Modern cosmology. G.F.R. Ellis, J-P Uzan – **0307347**

The big-bang theory: construction, evolution and status -- J-P Uzan – **1606.1112**

Primordial Cosmology-lectures, David Baumann --**1807.03098**

Nonsingular Ekpyrotic Cosmology with a Nearly Scale-Invariant

Spectrum of Cosmological Perturbations and Gravitational Waves

Robert Brandenberger, ∗ and Ziwei Wang **-- 2001.00638 (Ph.R.)**

Constructing and solving cosmologies of early universes with dark energy and dark matter

Alexandre T.Filippov, JINR.RU

New interpretation of Einstein's 3 papers of 1923: arXiv: **0812.2616**

[formulation of a **new unified theory** of **Dark Energy** and **vecton Dark Matter] .**

Also: **1003.0782, 1008.2333, 1011.2445, (vecton in cosmology as a relativistic particle);**

112.3023, 1302.6372 , (**general formulation in any dim**., **applied to cosmologies**);

1403.6815, 1506.01664, 1605.03948, **1905.0330,** (**general and special solutions**).

Older: **0612.258,** **0801.1312**, **0811.4501, 0902.4445** **(3 with Vittorio de Alfaro)**

in these publ. we proposed and studied **2-D Liouville - Toda models** describing

static, cosmological and **wave solutions** of ***dilaton gravity coupled to matter*.**

Almost Riemannian connection, the geodesics of which coincide with those of Riemannian, *up to parametrizations*

$$\gamma^i_{jk} = \Gamma^i_{jk}(g_{mn}) + \alpha\,(\delta^t_j\, a_k + \delta^i_k\, a_j)\,; \qquad r \equiv g^{ik}\, r_{ik} = R - 3\,\alpha^2 a_i\, a^i$$

$$2\kappa\,\mathcal{L}^{(4)}_{\rm geo} \;=\; \sqrt{-g}\,\{R - 3\,\alpha^2 a_i a^i - 2\Lambda\,[\,\det(\delta^i_j + \lambda f^i_j)\,]^{\frac{1}{2}}\,\}$$

$$f_{ij} \;\equiv\; \partial_i\, a_j \;-\; \partial_j\, a_i\;, \;\; f_{01} \;=\; \dot{a}_1 \;-\; a'_0 \qquad R \;\equiv\; g^{ik}\, R_{ik}$$

Only 2 free parameters will remain for vecton: here A_i replace a_i

$$2\kappa\,\mathcal{L}^{(4)}_{gAs} \;=\; \sqrt{-g}\left[R - 2\Lambda - \kappa\left(\frac{1}{2}F_{ij}F^{ij} + m^2 A_i\, A^i + \partial_i\psi\,\partial^i\psi + v(\psi)\right)\right]$$

This can be derived from the nonlinear Lagrangian for small a_i

$$2\kappa\,\mathcal{L}^{(4)}_{gas} \;=\; \sqrt{-g}\,\{R - 2\Lambda\,[\,\det(\delta^i_j + \lambda f^i_j)\,]^{\frac{1}{2}} \;-\kappa\,[m^2\, a_i\, a^i + \partial_i\psi\,\partial^i\psi + v(\psi)]\,\}$$

Preceding Lagrangian can be considered as \`linearization’ of the nonlinear one (below)

Reminding: The generalized Einstein (Eddington-Weyl) model in dimension D

>NB: *Higher dimensions are in fact unnecessary and, possibly,* ***misleading*****!**

The last two terms – pure geometry. The first – a generalization of Einstein's Lagrangian.
When D>4, the additional vector components produce scalar fields by dim. red.

$$\mathcal{L}_{eff} = \sqrt{-g}\left[-2\Lambda\left[\det(\delta_i^j + \lambda f_i^j)\right]^{1/(D-2)} + R(g) + c_a\, g^{ij} a_i a_j\right]$$

After dimensional reduction to D=4 and expanding the root term
up to the second order in the vector and scalar fields:

$$\mathcal{L}_{eff} \cong \sqrt{-g}\left[R[g] - 2\Lambda - \kappa\left(\frac{1}{2}F_{ij}F^{ij} + \mu^2 A_i A^i + g^{ij}\partial_i\psi\,\partial_j\psi + m^2\psi^2\right)\right]$$

$$\hbar = c = 1 \qquad A_i \sim a_i,\; F_{ij} \sim f_{ij},\; \kappa \equiv G/c^4$$

The **original Einstein square-root Lagrangian** is equivalent to so-called **DBI** one.

DBI either did not read E-1923 paper or forgot it. Anyway, its author declared it wrong!

`**Spherical**' metric: $ds_4^2 = e^{2\alpha}dr^2 + e^{2\beta}d\Omega^2(\theta,\phi) - e^{2\gamma}dt^2 + 2e^{2\delta}drdt$

Reduction to **cosmological**, **static** (or wave) solutions has the **main constraint**

$$-\dot{\beta}' - \dot{\beta}\beta' + \dot{\alpha}\beta' + \dot{\beta}\gamma' \;=\; \frac{1}{2}[\dot{\psi}\psi' + A_0 A_1] \propto T_{01}^{(m)} = \partial\mathcal{L}^{(m)}/\partial g^{01} \quad \text{for} \quad g^{01} \to 0$$

This is one of Einstein's equations corresponding to delta-variations

Separation of variables (dim. red. to static & cosmological states)

$$\alpha = \alpha_0(t) + \alpha_1(r), \quad \beta = \beta_0(t) + \beta_1(r),$$

Altogether 7 types of solutions: 3 cosmologies + 3 `dual' + 1 `self dual'

General anisotropic: $\beta' = \gamma' = 0$ FLRW cosmology: $\dot{\alpha} = \dot{\beta}\,, \quad \gamma' = 0$

We call the '*special' anisotropic* the cosmology with $\beta' = \gamma'$ and $\dot{\alpha} = 0$, which is dual to FRLW. The *flat isotropic* cosmology is obtained from the general anisotropic one if in addition $\bar{k} = k = 0$ and $\dot{\alpha} = \dot{\beta}$.

Mind the difference with FLRW !

2-D gravity: $$2\kappa\,\mathcal{L}_g^{(2)} \equiv e^{2\beta+\alpha+\gamma}\left[e^{-2\alpha}(2\beta'^2 + 4\beta'\gamma') - e^{-2\gamma}(2\dot\beta^2 + 4\dot\beta\dot\alpha) + 2\bar{k}\,e^{-2\beta}\right]$$

$$2\kappa\,\mathcal{L}_a^{(2)} = -\,e^{2\beta+\alpha+\gamma}\left\{2\,\Lambda\left[1 - \lambda^2 e^{-2(\alpha+\gamma)}(\dot a_1 - a_0')^2\right]^{\frac{1}{2}} + \kappa\, m^2(e^{-2\gamma}a_0^2 - e^{-2\alpha}a_1^2)\right\}$$

Linear appr.: $$2\kappa\,\mathcal{L}_A^{(2)} = -\,e^{2\beta+\alpha+\gamma}\left\{2\,\Lambda - \kappa\left[e^{-2(\alpha+\gamma)}F_{01}^2 + m^2(e^{-2\gamma}A_0^2 - e^{-2\alpha}A_1^2)\right]\right\}$$

1-D reduction: $$L_c = L_g + L_s + L_v\,, \quad \text{where} \quad L_g \equiv -e^{2\beta+\alpha-\gamma}(2\dot\beta^2 + 4\dot\beta\dot\alpha) - 6k\,e^{\alpha+\gamma}$$

$$L_s = \kappa\, e^{2\beta+\alpha+\gamma}\left[e^{-2\gamma}\dot\psi^2 - v(\psi)\right] \qquad L_v = e^{2\beta+\alpha+\gamma}\left\{-2\Lambda + \kappa\, e^{-2\alpha}\left[e^{-2\gamma}\dot A^2 - m^2A^2\right]\right\}$$

NB: $2\kappa\,\mathcal{L}_g^{(1)} \equiv L_g$, plus similarly defined $L_v\,,\ L_s$ $\qquad \lambda^2 = \kappa/\Lambda$

1-D exact vecton model, vecton field is effectively relativistic particle!

$$L_g + L_a \equiv L_g - e^{2\beta+\alpha+\gamma}\left[2\,\Lambda\sqrt{1 - \lambda^2\,\dot a^2\,e^{-2(\alpha+\gamma)}} + \kappa\, m^2 a^2 e^{-2\alpha}\right]$$

A `toy' scalar model for DME: $$L_{sa} \equiv -\,e^{2\beta+\alpha+\gamma}\left[2\,\Lambda\sqrt{1 - \lambda^2\,\dot\varphi^2\,e^{-2\gamma}} + \kappa\, m^2\varphi^2\right]$$

Why the nonlinear vecton model properties is most interesting for cosmology?

$$L = -2e^{2\beta}\left[e^{\alpha-\gamma}(\dot{\beta}^2 + 2\dot{\beta}\dot{\alpha}) + \Lambda\sqrt{e^{2(\alpha+\gamma)} - \lambda^2\dot{A}^2} + \frac{1}{2}\mu^2 A^2 e^{-\alpha+\gamma}\right]$$

In the gauge $\gamma = -\alpha$ and with notation $\alpha = \rho - 2\sigma$ and $\beta = \rho + \sigma$

$$\mathcal{L}_c = -2e^{2\beta}\left[3e^{2\alpha}(\dot{\rho}^2 - \dot{\sigma}^2) + \Lambda\sqrt{1 - \lambda^2\dot{A}^2} + \frac{1}{2}\mu^2 A^2 e^{-2\alpha}\right]$$

$$\mathcal{H} = \bar{c}\sqrt{p_A^2 + M_A^2\,\bar{c}^2} + \mu^2 A^2 e^{2(\beta-\alpha)} + \frac{1}{24}\,e^{2(\beta+\alpha)}\,(p_\sigma^2 - p_\rho^2)$$

Which is zero if there are no other fields. $M_A \equiv 2\lambda^2\Lambda e^{2\beta}$ $\lambda^{-1} \equiv \bar{c}$

In a sense, the vecton looks like a massive particle in a **gravitational accelerator** is it a source of BIG BANG? Can be!

Anisotropic variables and vecton contribution, *notation*

$$3\,\rho \equiv (\alpha + 2\beta)\,, \quad 3\,\sigma \equiv (\beta - \alpha)\,, \quad \alpha = \rho - 2\sigma$$

$$3\,A_{\pm} = e^{-2\rho+4\sigma}(\dot{A}^2 \pm m^2 e^{2\gamma} A^2)$$

$$\mathcal{L}_c = e^{3\rho-\gamma}(-6\dot{\rho}^2 \,+\, 6\dot{\sigma}^2 \,+\, \dot{\psi}^2) - \,6k\,e^{\rho-2\sigma+\gamma} \,-$$

$$-\,e^{3\rho+\gamma}\,v(\psi) \,+\, e^{3\rho\,-\,\gamma}\,3A_{-}$$

Anisotropic vecton plus scalaron Hamiltonian constraint

$$\mathcal{H}_c \equiv -6\dot{\rho}^2 + 6\dot{\sigma}^2 + \dot{\psi}^2 + 6k\,e^{2\gamma-2(\rho+\sigma)} \,+$$

$$+\,e^{2\gamma}\,v(\psi) + 3\,A_{+} \,=\, 0$$

1605.03948 hep-th

see also: 1506.01664

A fresh view of cosmological models describing very early Universe: general solution of the dynamical equations.

$$(\dot\rho,\, \dot\psi,\, \dot\sigma) \equiv [\,\xi(\rho)\,,\eta(\rho)\,,\zeta(\rho)\,] = [\,\xi(\rho),\, \xi\,\psi'(\rho),\, \xi\,\sigma'(\rho)\,] \equiv \xi(\rho)[\,1,\, \chi(\rho),\, \omega(\rho)]$$

Characteristic functions of cosmology depending also on anisotropy and parameter *k*

$\chi(\rho) \equiv \eta/\xi = \psi'(\rho)$ and $\omega(\rho) \equiv \zeta/\xi = \sigma'(\rho)$ are gauge invariant

$$v(\psi) = \bar v[\rho(\psi)] \qquad \text{for arbitrary } \bar v(\rho)$$

$$v'(\psi) = \frac{dv}{d\psi} = \frac{dv}{d\rho}\frac{d\rho}{d\psi} = \bar v'(\rho)\frac{\xi}{\eta} = \bar v'(\rho)/\chi(\rho)$$

Gauge invariant Anzatz for solving all equations for vanishing anisotropy

$$[\,x(\rho),\, y(\rho),\, z(\rho)\,] \equiv \exp\,(6\rho - 2\gamma)\,[\,\xi^2(\rho)\,,\eta^2(\rho),\, \zeta^2(\rho)\,]$$

Elementary ***integrating*** and ***approximating***

some `*non-integrable'* models of very early Universes

with **vector** (scalar) **DM** is our program of **formal mathematical cosmology**).

1. The dynamics of any spherical cosmology with a scalar field **(`scalaron')** coupling to gravity is described by **3** nonlinear second-order differential equations for depending on `time' **2** metric functions and the scalaron. The equations depend on the scalaron potential and arbitrary gauge function but can be reduced to gauge invariant ones.

2. Replacing `time' by `metric' allows to **explicitly integrate general isotropic flat model** in **any gauge,** with **arbitrary potentials depending on metric. Anisotropic corrections** are **asymptotically small** in a rather general scalaron theory.

3. Restrictions on the potentials arise from our **positivity criterion** of the exact solutions that are in fact ***canonical momenta squared*** and on conditions controlling scenario (contracting, bouncing), which must be imposed on **characteristic** functions.

4. An inverse problem – finding, with a given scenario, proper expressions for $V(\alpha)$ that in fact proved to be **approximately** (**only!**) constant in the inflationary domain.

5. The approach is presently being applied to *anisotropic models* with a neutral massive vector field **(`vecton' -- *Dark Matter candidate theory*),** based on our remake of Einstein's attempts to construct a simple affine gravity, which also **predicts** a dynamical effective **Dark Energy** (not constant!).

E.O.M. for the **anisotropic scalaron *plus* linear vecton**

$$\ddot{\rho} + (3\,\dot{\rho} - \dot{\gamma})\,\dot{\rho} - e^{2\gamma}\,v(\psi)/2 \;=$$

$$=\; 2k\,e^{2\gamma-2(\rho+\sigma)} \;+\; (3A_+ \;-\; A_-)/4\,,$$

$$\ddot{\sigma} \;+\; (3\,\dot{\rho} \;-\; \dot{\gamma})\,\dot{\sigma} \;=\; k\,e^{2\gamma-2(\rho+\sigma)} \;+\; A_-\,,$$

$$\ddot{\psi} + (3\,\dot{\rho} - \dot{\gamma})\,\dot{\psi} \;+\; e^{2\gamma}\,v'(\psi)/2 = 0\,,$$

$$\ddot{A} + (\dot{\rho} + 4\dot{\sigma} - \dot{\gamma})\dot{A} + e^{2\gamma}\,m^2 A = 0\,.$$

Note **simplicity** of scalaron and vecton equations and

$(3\,\dot{\rho} - \dot{\gamma})$ dependence on the gauge parameter

Important *equation of motion* **independent of scalar potential**

$$\ddot{\rho} - \dot{\rho}\dot{\gamma} + 3\,\dot{\sigma}^2 + \dot{\psi}^2/2 =$$

Vecton part is positive

$$= -k\,e^{2\gamma-2(\rho+\sigma)} - (3A_+ + A_-)/4$$

Gauge dependent **generalized** Hubble function[6] $H(t) \equiv \dot{\rho}$

Strong restrictions on the generalized Hubble function

$$\dot{H}(t) \equiv \ddot{\rho}(t) \geq 0, \quad \text{if} \quad \gamma'(\rho) \geq 3,\ v \geq 0,\ k \geq 0;$$

$$\dot{H}(t) \leq 0, \quad \text{if} \quad \gamma = 0,\ k \geq 0.$$

Canonical momenta for *graviton, scalaron and vecton*

$$(p_\rho,\, p_\psi,\, p_\sigma) = 2\,e^{3\rho-\gamma}(-6\dot{\rho},\, \dot{\psi},\, 6\,\dot{\sigma})\,, \quad p_A = 2\,e^{\rho+4\sigma-\gamma}\dot{A}$$

Define all momenta

Equations of motion for the **positive square-momentum type variables**

$$y'(\rho) + V'(\rho) - 6V(\rho) = 0\,, \qquad V \equiv e^{6\rho}\,\bar{v}(\rho)\,.$$

Independent of *k, \sigma*

$$x'(\rho) - V(\rho) = 4k\,e^{4\rho-2\sigma}, \qquad z'(\rho) = 2k\,e^{4\rho-2\sigma}\,\sigma'(\rho).$$

The constraint eqn.: $$6\,x(\rho) \;=\; y(\rho) + V(\rho) \;+\; 6z(\rho) \;+\; 6k\,e^{4\rho-2\sigma}\,.$$

$$y(\rho) = 6\left(C_y + \int V(\rho)\right) - V(\rho)$$

The solution of the *psi* equation

Solution of the *X* - eq.

$$x(\rho) = \left(C_x + \int V(\rho)\right) + 4k\int e^{4\rho-2\sigma(\rho)}$$

Sigma
equation

$$x(\rho)\,\sigma'^2(\rho) \;\equiv\; C_x - C_y + 2k\int \sigma'(\rho)\,e^{4\rho-2\sigma(\rho)}\,.$$

In general, our solutions may become negative even when the potential is positive

One has to require positivity at least in the classical domain \alfa > 0

Construction of **positive** general solutions

$$\bar{v}(\rho) > 0 \qquad y(\rho) \equiv \lambda \int_{\rho_0}^{\rho} e^{\lambda\rho}\, \bar{v}(\rho) \,-\, e^{\lambda\,\rho}\, \bar{v}(\rho) + e^{\lambda\,\rho_0} \bar{v}(\rho_0)$$

$$y(\rho) \,=\, - \int_{\rho_0}^{\rho} e^{\lambda\rho}\, \bar{v}'(\rho)\,, \quad y'(\rho) \,=\, -e^{\lambda\rho}\, \bar{v}'(\rho) > 0$$

$$\text{positive if } \bar{v}'\,(\rho) < 0$$

The fundamental expressions for the solution with vanishing anisotropy and exact relation between the fundamental cosmolo0gical functions

$$\hat{r}(\rho) \equiv \dot{\psi}^2 e^{-2\gamma}/v(\psi) = \chi^2(1+6k\,e^{-2\rho}/\bar{v})\,[6(1-\omega^2) - \chi^2]^{-1}$$

$$\hat{r}(\rho) = \frac{6\,C_y}{V(\rho)} + \frac{6}{V}\int V(\rho) - 1 = \frac{6\,C_y}{V(\rho)} + \sum_1^\infty (-1)^n \frac{\bar{v}^{(n)}(\rho)}{6^n\,\bar{v}(\rho)}$$

$$\chi^2 \equiv \frac{y}{x} \equiv \frac{6I_y(\rho) - V(\rho)}{I_x(\rho) + i_x(\rho)} \overset{\sigma\to 0}{\equiv} 6 - \frac{V(\alpha) + 6ke^{4\alpha}}{I(\alpha) + ke^{4\alpha}} \overset{k\to 0}{\equiv} 6 - \frac{V}{I}$$

$$I \equiv C_0 + \int V(\alpha)$$

$$\chi^2 = 6(1-\omega^2)\left[\sum_1^\infty (-1)^n \frac{\bar{v}^{(n)}(\rho)}{6^n\,\bar{v}(\rho)} + \frac{6\,C_y}{V}\right] \text{ x}$$

$$\left[1 + \sum_1^\infty (-1)^n \frac{\bar{v}^{(n)}(\rho)}{6^n\,\bar{v}(\rho)} + \frac{6}{V}\left(k\,e^{4\rho-2\sigma} + C_y\right)\right]^{-1}$$

All these formulas are exact, **the second is independent** on \sig and *k*

When $k = \sig = 0$, the exact equation for $\mathbf{r} = \hat{\chi}^2(1-\hat{\chi}^2)^{-1}$. $\hat{\chi}^2(\rho) \equiv \chi^2/6$.

$$\mathbf{r}'(\rho) + (6 + \bar{l}')\,\mathbf{r}(\rho) = -\bar{l}'(\rho)$$

Is equivalent to the equation for χ^2 and we easily find the normalized potential

$$\bar{v}(\alpha) = 6\,\xi^2 - \eta^2 = 6\,[1 - \hat{\chi}^2(\alpha)]\,\exp\left[-6\int \hat{\chi}^2(\alpha)\right]$$

which is obviously slowly varying in the inflationary domain, i.e., for small $\hat{\chi}^2$

As in inflationary domain $\hat{\chi}^2(\alpha) \ll 1$ and $\mathbf{r}(\alpha) \ll 1$ but $[\chi^2(\alpha)]'$

and $\mathbf{r}'(\rho)$ must be positive, the potential cannot be really constant.

Asymptotic hierarchy:

flat isotropic $\subset$ isotropic $\subset$ anysotropic

$$\mathrm{O}(1) \subset \mathrm{O}(e^{-2\alpha}) \subset \mathrm{O}(e^{-4\alpha})$$

$\Theta(\alpha)$ **-- important POSITIVE FUNCTION**

$$[\chi^2(\alpha)]' = -(6-\chi^2)\,[\,\chi^2(\alpha) + \bar{l}\,'(\alpha)] \equiv (6-\chi^2)\,\Theta(\alpha)\,,$$

$$\Theta(\alpha) = -[\,\ln(6-\chi^2(\alpha)]\,' = [\,\ln(\mathbf{r}+1)]\,'$$

$$-\Theta'(\alpha) = \Theta^2 + (6+\bar{l}')\,\Theta + \bar{l}'' \equiv (\Theta-\Theta_+)(\Theta-\Theta_-)$$

$$(\chi^2)'' = (\Theta + 6 + \bar{l}')\,(\Theta' - \Theta^2) = -(\Theta + 6 + \bar{l}')\,[2\Theta^2 + (6+\bar{l}')\,\Theta + \bar{l}'']$$

Change sign of $(\chi^2)''$ is possible to gain only when $\bar{l}'' < 0$.

First terms of the exact expression for the transition function

$$\chi^2 = (1-\omega^2)\left[\left(-\bar{l}' + \mathrm{o}(\bar{l}')\right) + 36\, C_y \,\frac{e^{-6\rho}}{\bar{v}(\rho)}\right] \;\times$$

$$\left[\left(1 - \frac{1}{6}\,\bar{l}' + \mathrm{o}(\bar{l}')\right) + 6\,\frac{e^{-2\rho}}{\bar{v}(\rho)}\left(k\, e^{-2\sigma} + C_y\, e^{-4\rho}\right)\right]^{-1}$$

$$\chi^2 = -\bar{l}'(\rho) + \mathrm{o}(\bar{l}') = -\chi\, v'(\psi)/v(\psi) + \ldots$$

$$\chi = -v'(\psi)/v(\psi) + \ldots \equiv -l'(\psi) + \mathrm{o}(l')$$

In fact we constructed a perturbative algorithm relating \rho and \psi pictures but it is rather complex. Nevertheless it is important as a matter of principle.

This provides the relation to approximate standard formulas**.**

The small **anisotropic corrections** are derived in **asymptotic domain** of **large** ***\rho***

Approximate solution of the anisotropy equation

estimating $\sigma(\rho)$ in the weak anisotropy limit

by asymptotically solving** the **Sigma equation

$$\sigma(\rho) = -3k \int_{\rho}^{\infty} \frac{e^{-2\rho}[\bar{v}(\rho)]^{-1}}{1 + \Sigma_1(\rho) + O(e^{-2\rho})} + O(e^{-4\rho})$$

$$\Sigma_1(\rho) = \sum_{1}^{\infty} (-1)^n \frac{\bar{v}^{(n)}(\rho)}{6^n \, \bar{v}(\rho)} \, .$$

Final remarks on the vecton theory:

1. The structure of the linearized theory is similar to the scalaron case but anisotropy requires additional efforts.

2. With zero anisotropy, the equations of the linearized theory can be solved, otherwise they give a sort of very useful `sum rules'. But, asymptotically small anisotropy approximation can be as effective as in the scalaron case.

3. The most difficult 'large vecton momentum' case for nonlinear vecton may also be treated asymptotically. It is very interesting for transition from inflation to particle production processes, or else, for description of the so called bouncing phenomena.

Complete nonlinear vecton cosmology is worth efforts to solve!

Here, we consider special anisotropic cosmology with $\dot{\alpha} = \beta' = \gamma' = 0$:

$$\mathcal{L}_c = e^{2\beta}\big[e^{-\gamma}(\dot{a}^2 - 2\dot{\beta}^2) - e^{\gamma}(2\Lambda + m^2a^2)\big] - 6ke^{\gamma}. \tag{76}$$

Denoting $\xi \equiv \dot{\beta}$, $b \equiv \dot{a}$, $v_0 \equiv 2\Lambda$, $v(a) \equiv v_0 + m^2a^2$, we find the constraint

$$e^{2\beta}\big[e^{-\gamma}(b^2 - 2\xi^2) + e^{\gamma}v(a)\big] + 6ke^{\gamma} = 0, \tag{77}$$

which is quadratic in the momenta: $(p_a, p_\beta) \equiv 2e^{2\beta-\gamma}(b, -2\xi)$. Equations of motion are

$$2\dot{b} + 2(2\xi - \dot{\gamma})b + e^{2\gamma}v'(a) = 0, \tag{78}$$

$$2\dot{\xi} + 4\xi^2 - 2\xi\dot{\gamma} - 2[v(a) + 3ke^{-2\beta}]e^{2\gamma} = 0. \tag{79}$$

Defining the new dynamical variables by $y(\beta) \equiv b^2\, e^{4\beta-2\gamma}$, $x(\beta) \equiv \xi^2 e^{4\beta-2\gamma}$ we find the gauge invariant equations and constraint similar to Eqs.(37-39) but there is no σ-equation:

$$x' - 2e^{4\beta}\,\bar{v}(\beta) + 6ke^{2\beta} = 0, \quad (a); \qquad y' + \bar{v}'(\beta)\, e^{4\beta} = 0, \quad (b). \tag{83}$$

$$2x(\beta) = y(\beta) + V(\beta) + 6\,k\,e^{2\beta}, \quad \text{where} \quad V(\beta) \equiv e^{4\beta}\,\bar{v}(\beta). \tag{84}$$

Their exact solution is also similar (40) with $\sigma \equiv 0$

$$2x = 4I(\beta) + 6ke^{2\beta}, \quad y = 4I(\beta) - V(\beta); \qquad I(\beta) \equiv \int V(\beta) + C \tag{85}$$

anisotropic model. It is sufficient to define our main characteristic of cosmology

$$\tilde{\chi}^2(\beta) \equiv y(\beta)/2x(\beta) = 1 - V(\beta)/4I(\beta), \quad \text{for} \quad k = 0, \tag{86}$$

which satisfy equation similar to equation (70) for $\hat{\chi}^2(\alpha) \equiv \chi^2/6$:

$$(\tilde{\chi}^2)' = -(1 - \tilde{\chi}^2)(4\tilde{\chi}^2 + \bar{l}') \equiv (1 - \tilde{\chi}^2)\,\tilde{\Theta}(\beta), \qquad \tilde{\chi}^2 < 1. \tag{87}$$

THE

END

THANK YOU FOR ATTENTION!