MARINA: Faster Non-Convex Distributed Learning with Compression

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Federated
Learning
One
World
Seminar

IITP RAS

MIPT
MOSCOW INSTITUTE OF PHYSICS AND TECHNOLOGY

KAUST

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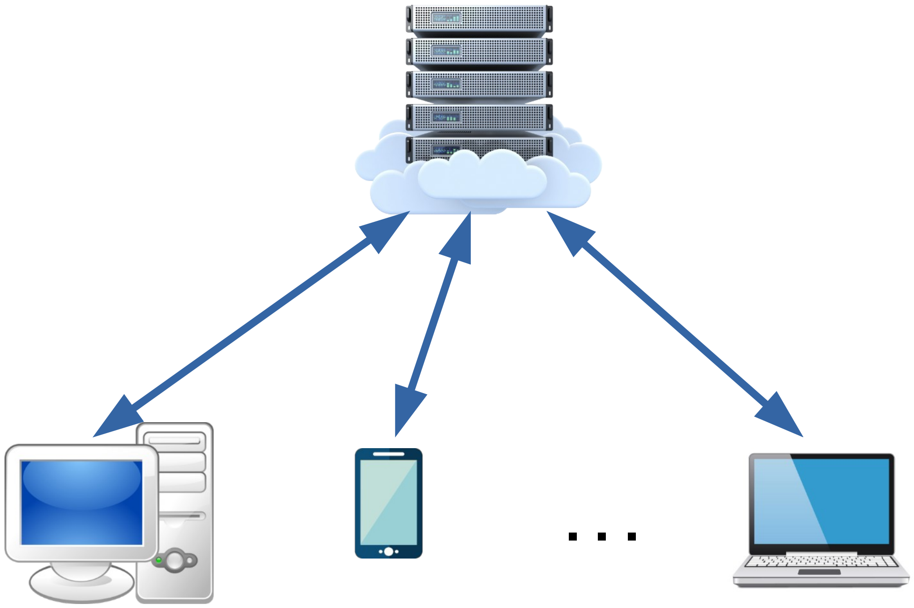
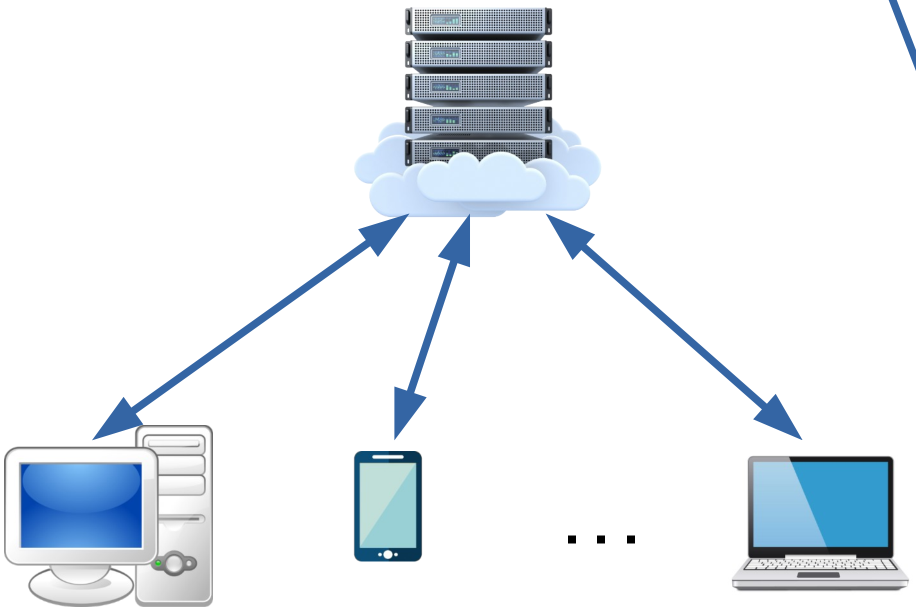
Outline

1. The Problem



$$\min_{x\in\mathbb{R}^d}\left\{f(x) := \frac{1}{n}\sum_{i=1}^{n} f_i(x)\right\}$$



parameters of the model

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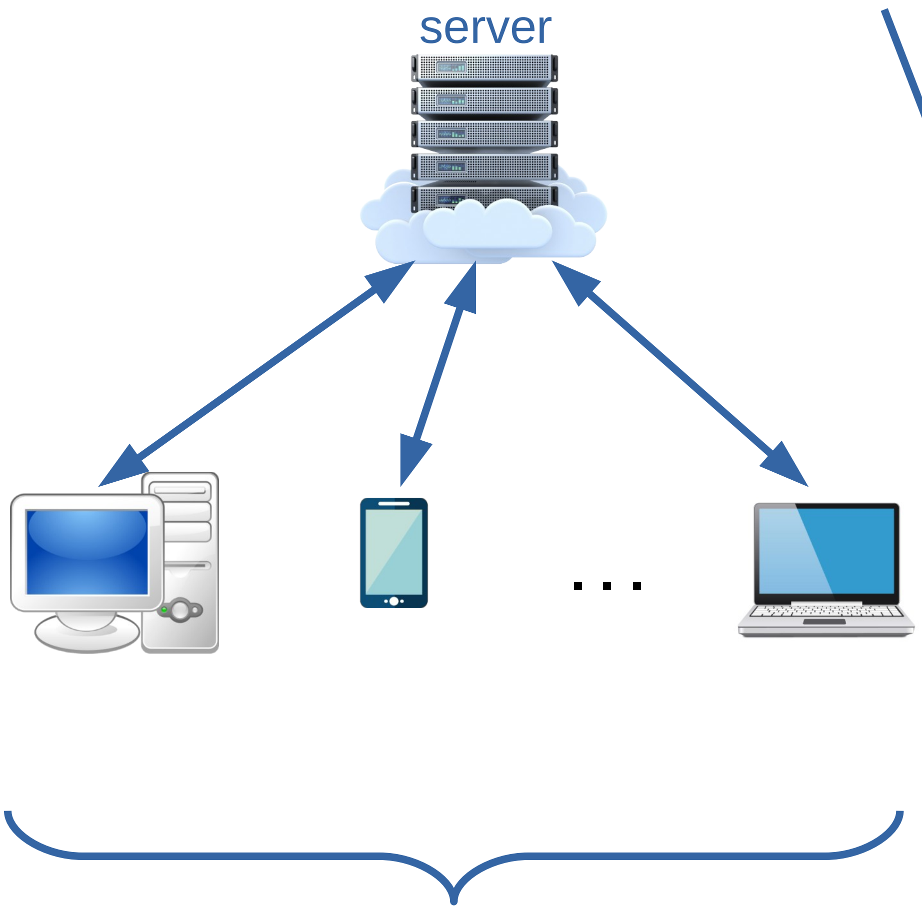
# of parameters



$\boxed{n}$ workers/devices

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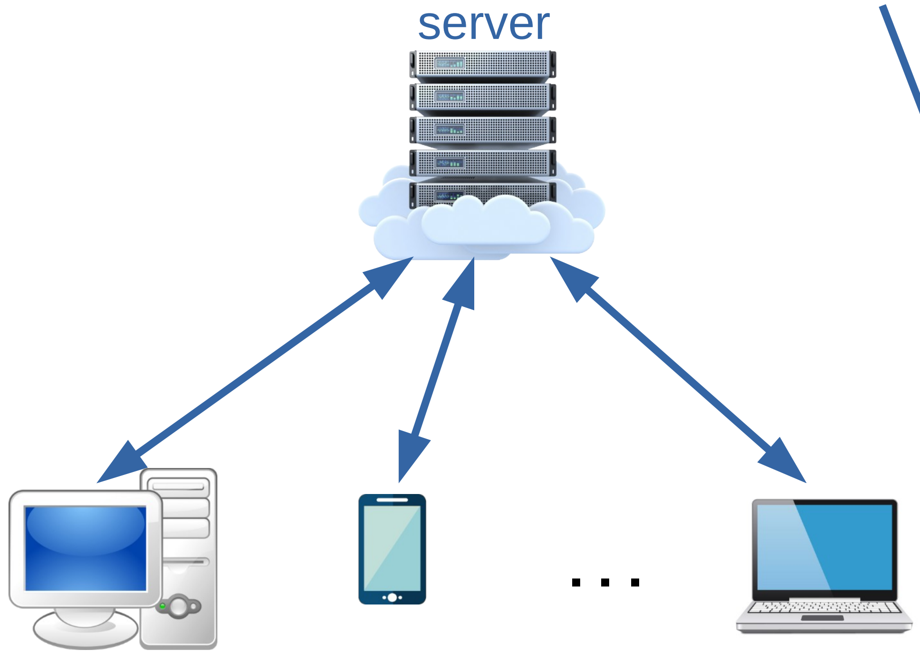

$$\underbrace{f_1(x) \qquad f_2(x) \qquad \cdots \qquad f_n(x)}_{\boxed{n}\text{ workers/devices}}$$

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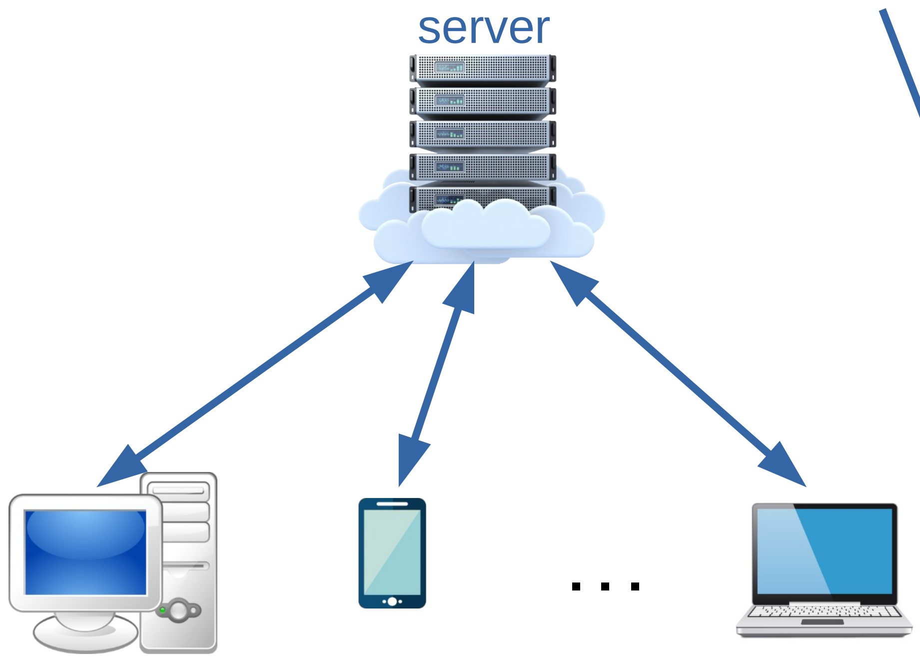
loss on the data accessible by worker *i*



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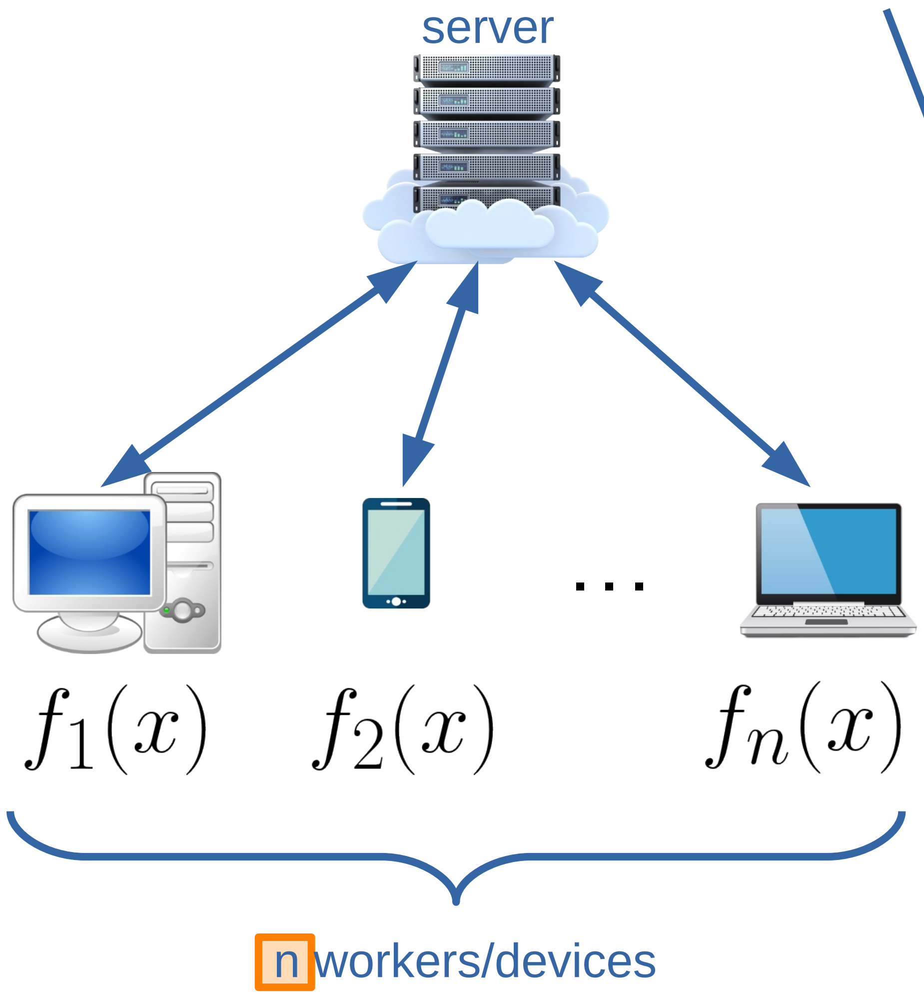

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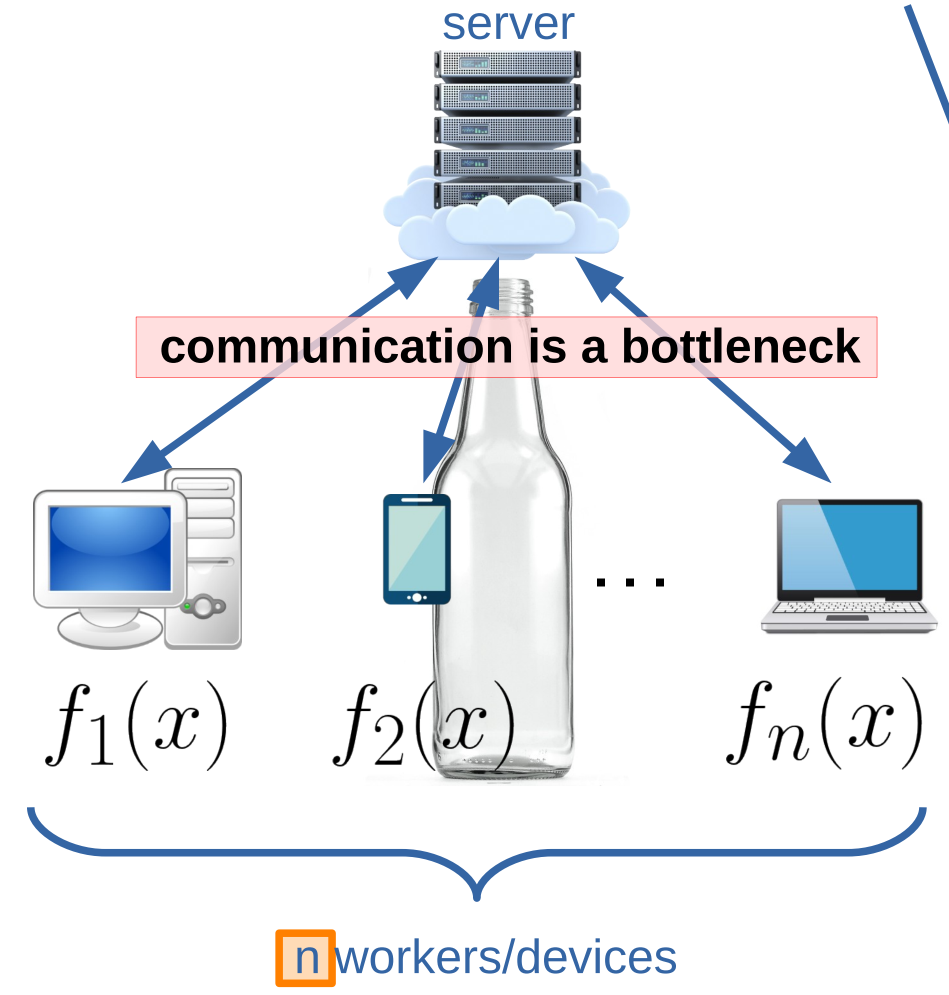
- $f_i(x) = \frac{1}{m}\sum_{j=1}^{m} f_{ij}(x)$



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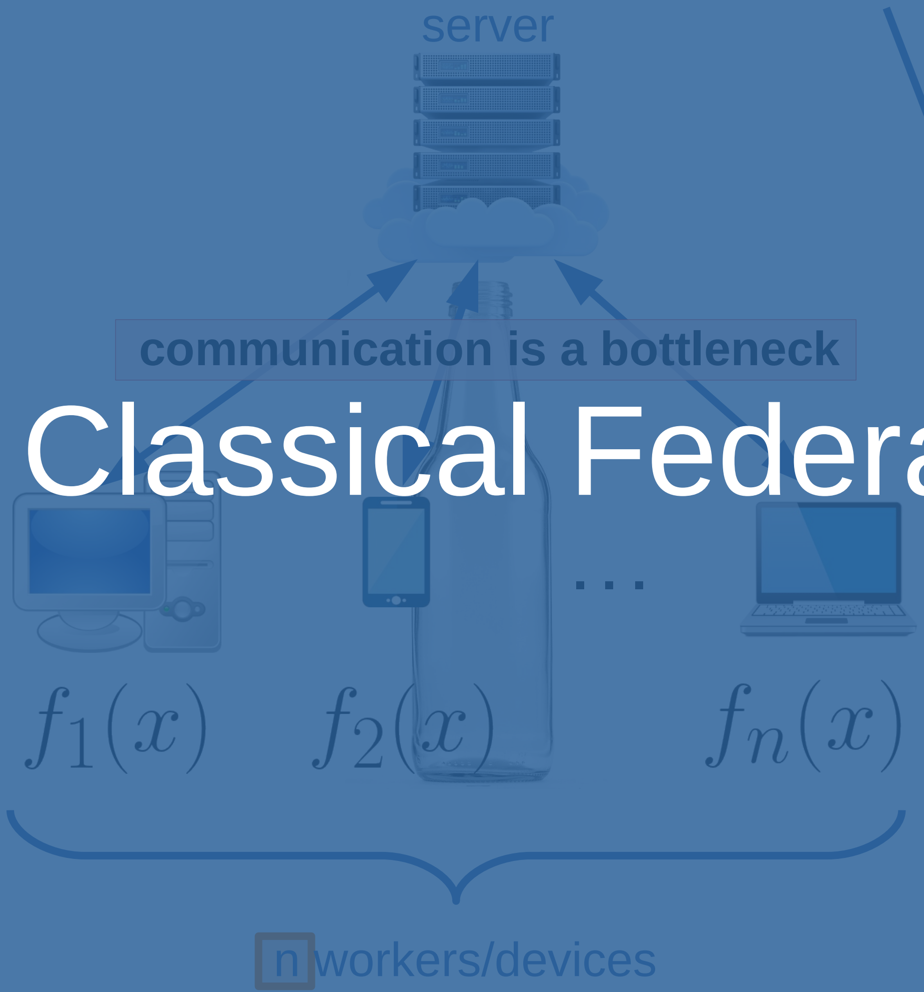
Classical Federated Learning Setup



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How to Handle Communication Bottleneck?

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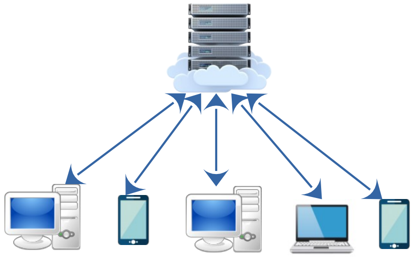
- Change the topology of the network



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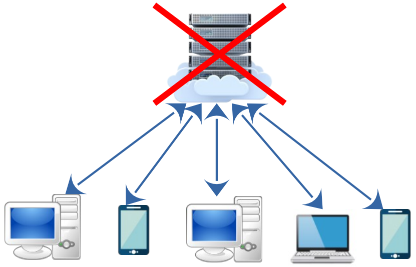

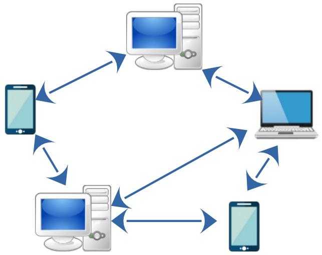

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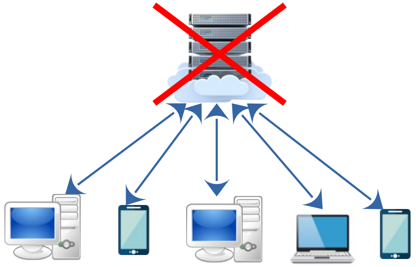

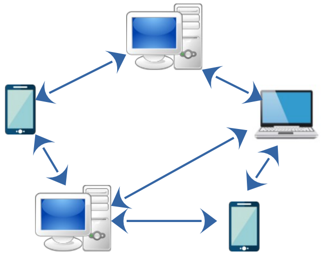

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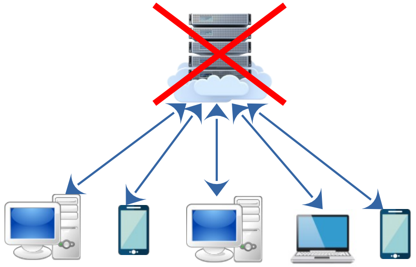

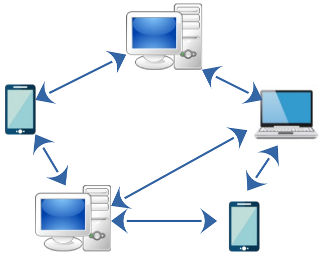

- Do more work on each worker in the hope of communicating less → Local-SGD/Federated Averaging

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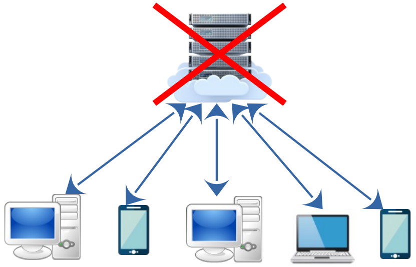

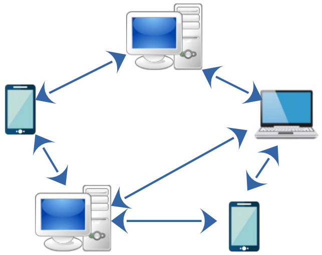

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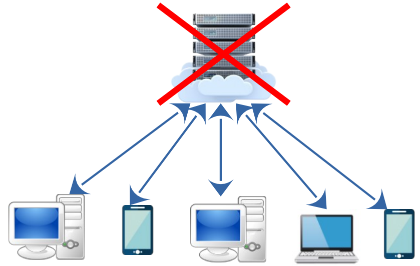
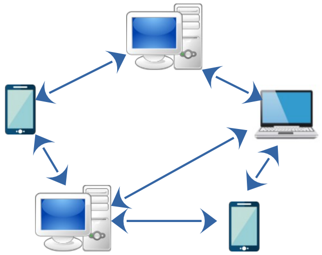


→



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Workers send dense vectors

$$g = \begin{pmatrix} 1 \\ -15 \\ 0.2 \\ -7 \\ 10 \end{pmatrix}$$

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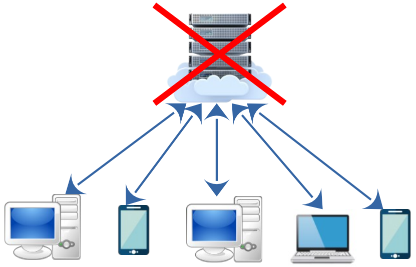

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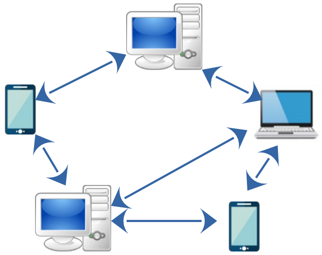

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Workers send dense vectors → Workers send compressed/sparse vectors

$$g = \begin{pmatrix} 1 \\ -15 \\ 0.2 \\ -7 \\ 10 \end{pmatrix} \quad \longrightarrow \quad \mathcal{Q}(g) = \frac{5}{2} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \\ -7 \\ 0 \end{pmatrix}$$

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We study this approach

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Example: RandK (for K = $\boxed{2}$)

$$\boxed{d = 5} \left\{ \begin{pmatrix} 1 \\ -15 \\ 0.2 \\ -7 \\ 10 \end{pmatrix} \right. \Rightarrow \frac{\boxed{5}}{\boxed{2}} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \\ -7 \\ 0 \end{pmatrix} \qquad \omega = \frac{\boxed{d}}{\boxed{K}} - 1$$

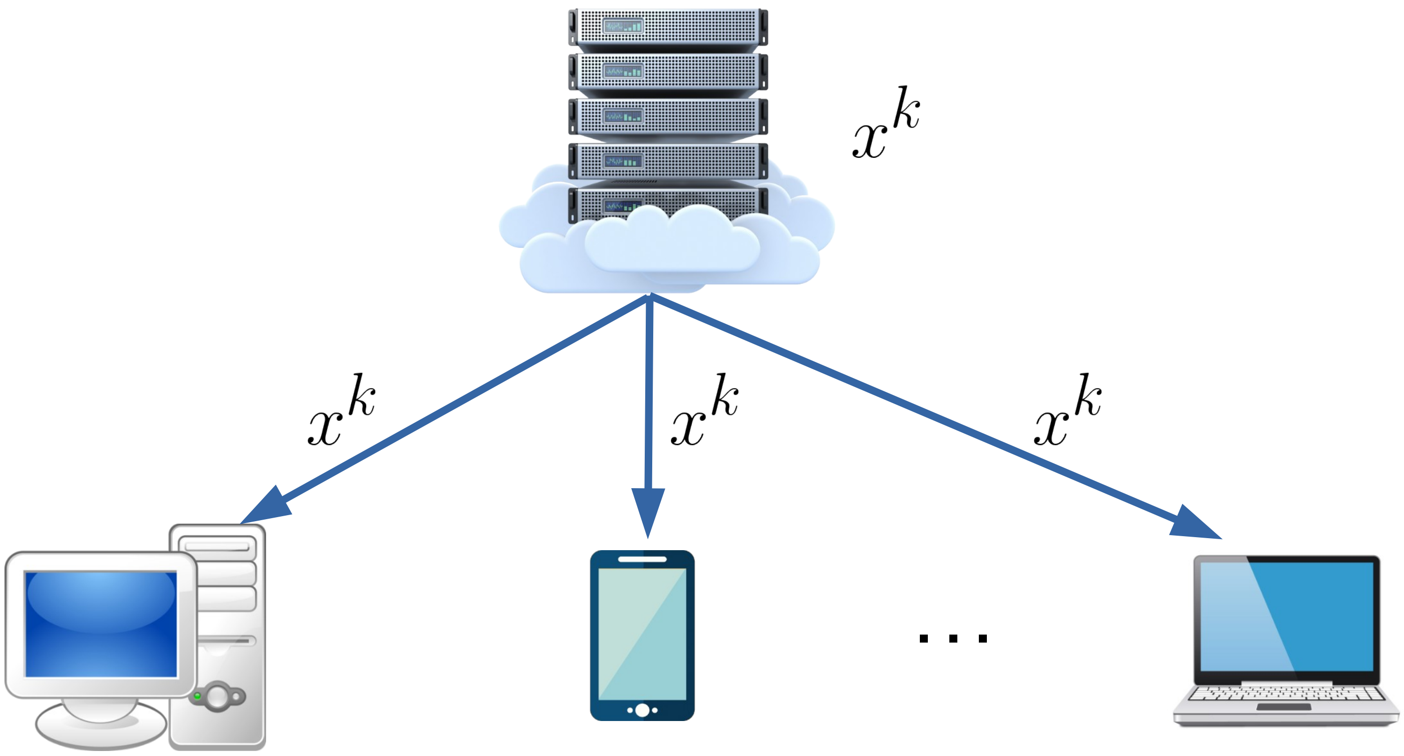
for unbiasedness

Pick K = $\boxed{2}$ components uniformly at random

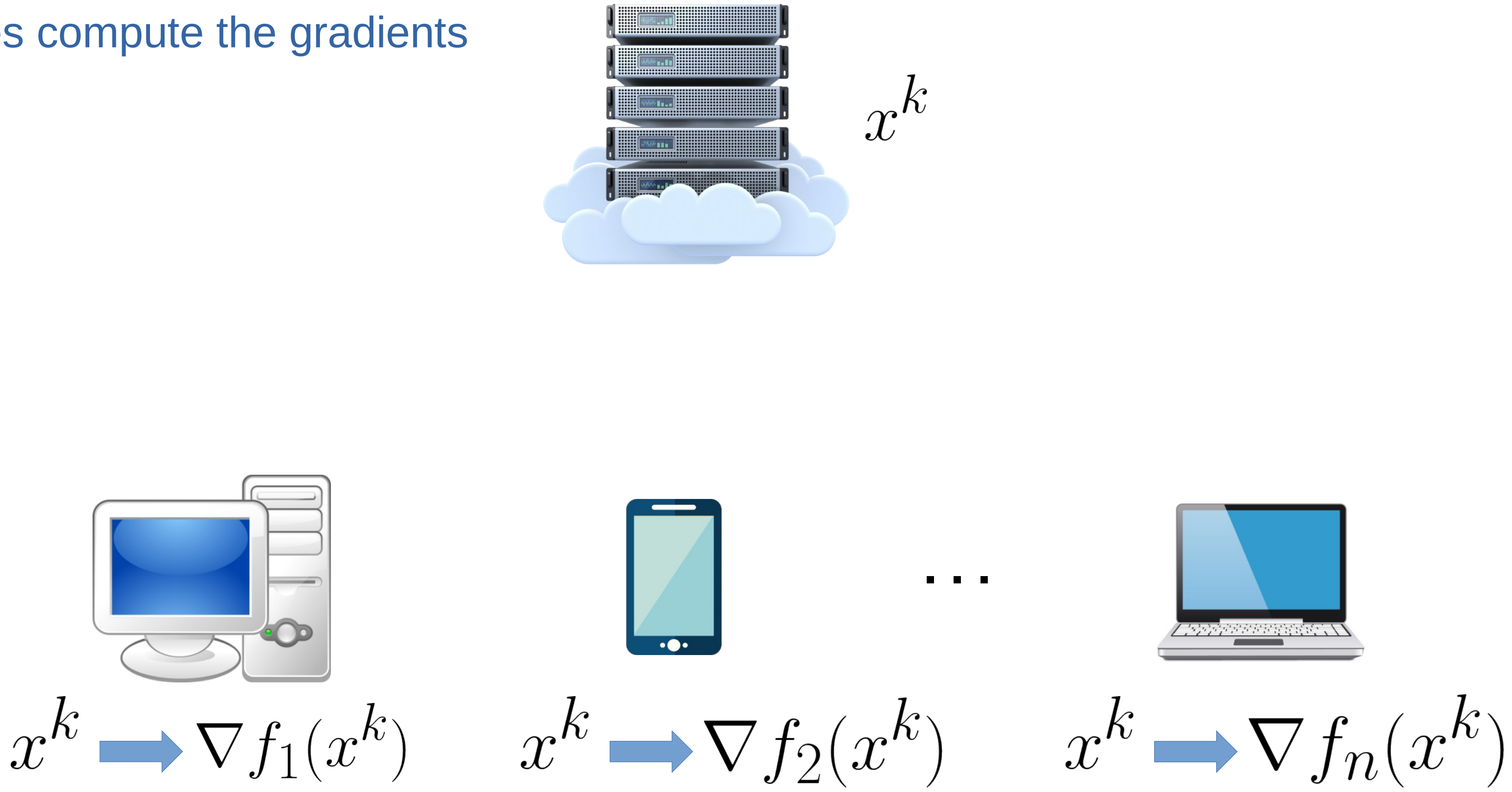
2. Quantized Gradient Descent (QGD)

Alistarh, Dan, Demjan Grubic, Jerry Li, Ryota Tomioka, and Milan Vojnovic. **"QSGD: Communication-efficient SGD via gradient quantization and encoding."** *In Advances in Neural Information Processing Systems*, pp. 1709-1720. 2017.

1 Server broadcasts the parameters

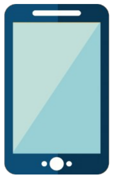


1. Server broadcasts the parameters
2. Devices compute the gradients



1. Server broadcasts the parameters
2. Devices compute the gradients
3. Devices quantize the gradients

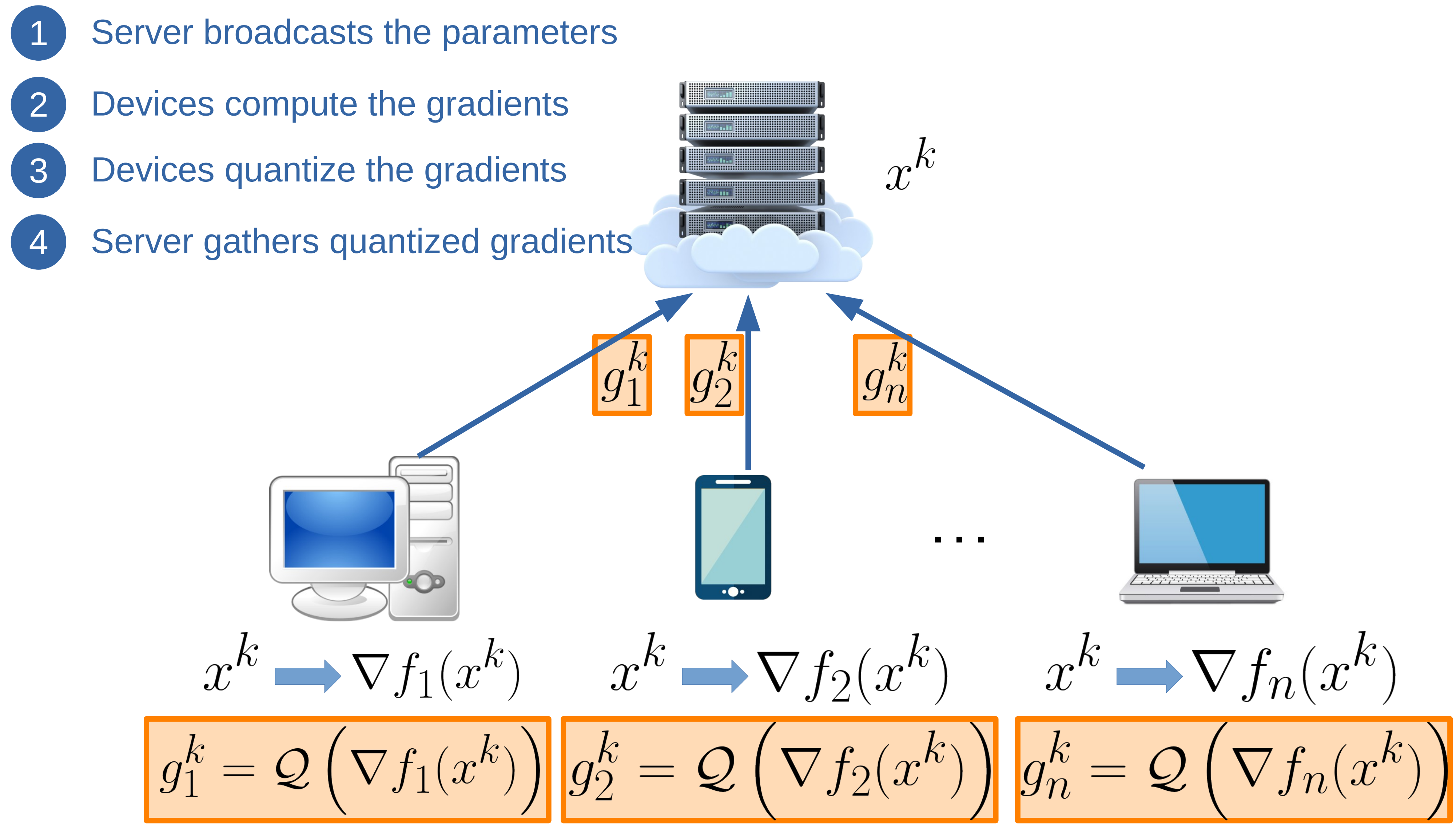
x^k



$\cdots$

$x^k \rightarrow \nabla f_1(x^k)$ $\quad$ $x^k \rightarrow \nabla f_2(x^k)$ $\quad$ $x^k \rightarrow \nabla f_n(x^k)$

$\boxed{g_1^k = \mathcal{Q}\left(\nabla f_1(x^k)\right)}$ $\quad$ $\boxed{g_2^k = \mathcal{Q}\left(\nabla f_2(x^k)\right)}$ $\quad$ $\boxed{g_n^k = \mathcal{Q}\left(\nabla f_n(x^k)\right)}$


1 Server broadcasts the parameters
2 Devices compute the gradients
3 Devices quantize the gradients
4 Server gathers quantized gradients
x^k
g_1^k
g_2^k
g_n^k
$\cdots$
$x^k \rightarrow \nabla f_1(x^k)$
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$x^k \rightarrow \nabla f_n(x^k)$
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1. Server broadcasts the parameters
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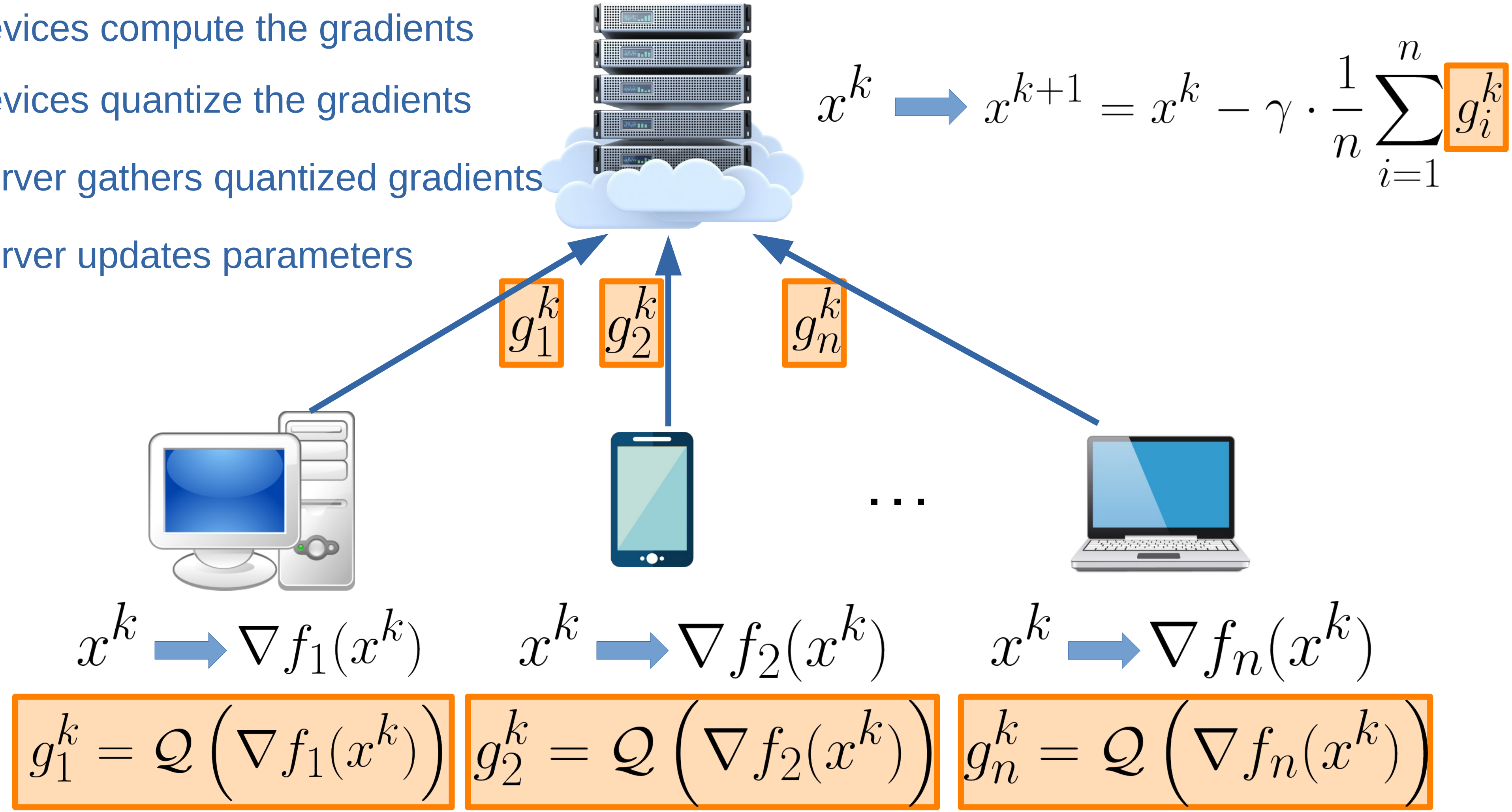
5. Server updates parameters



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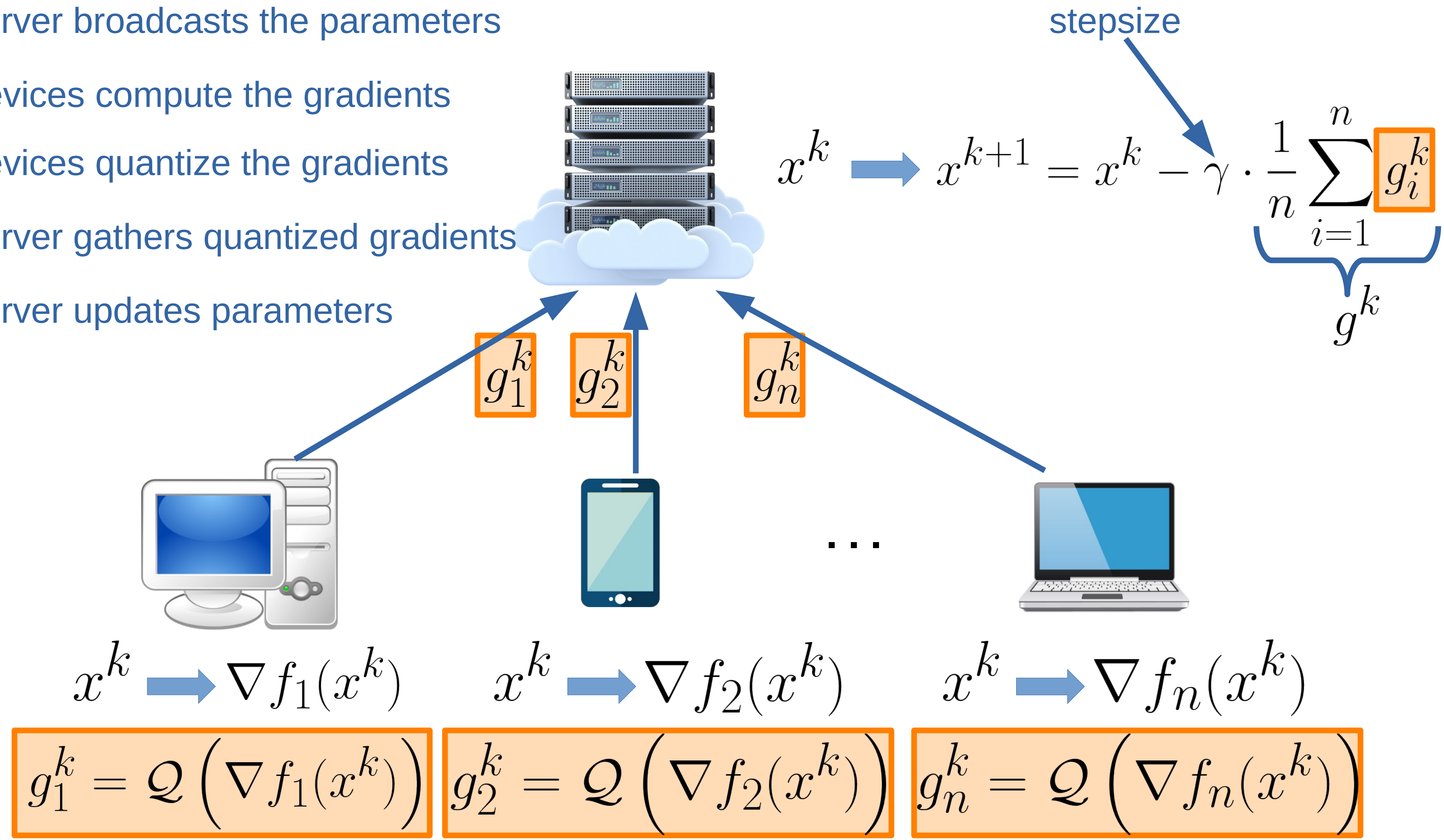
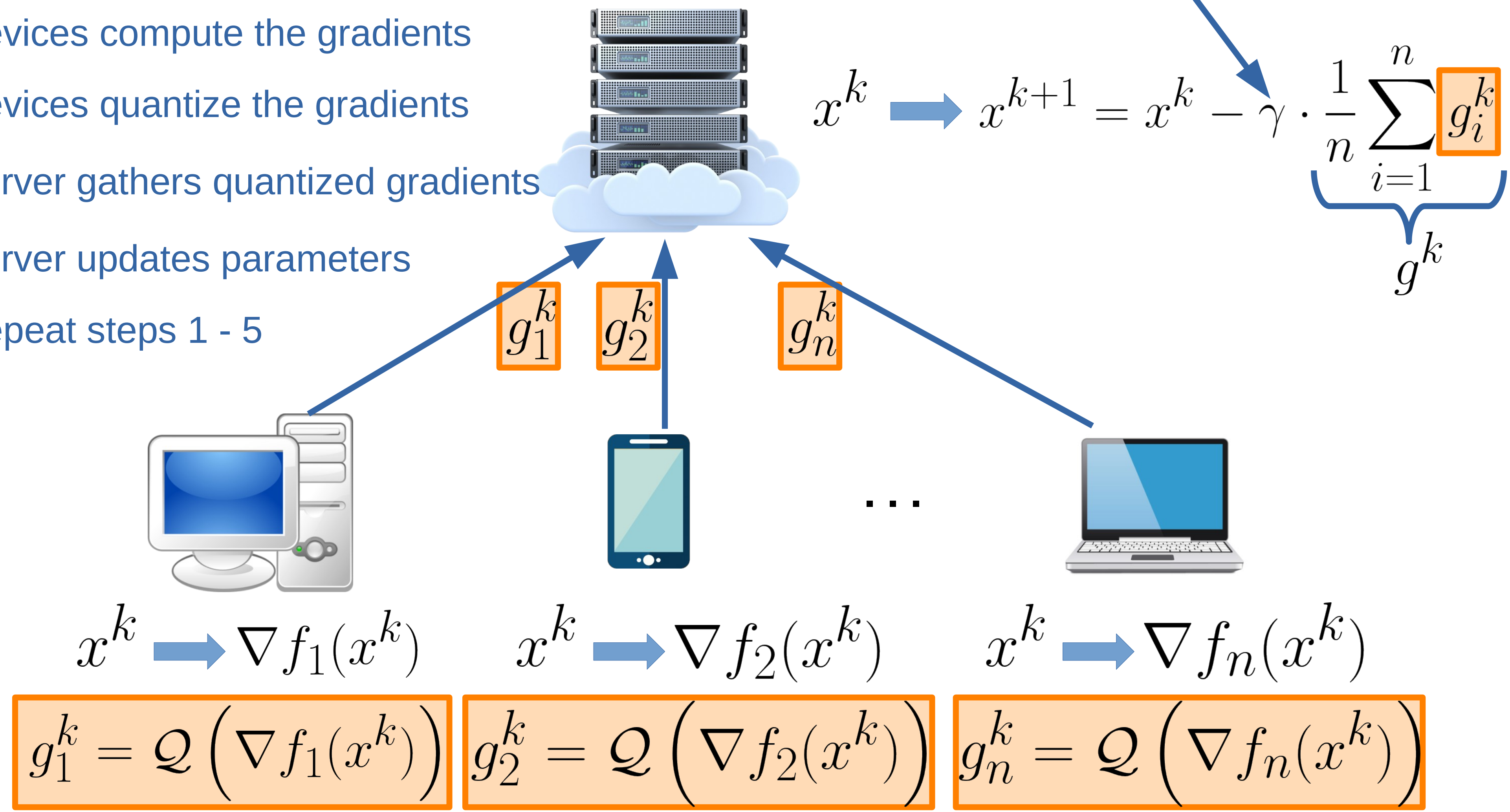


1. Server broadcasts the parameters
2. Devices compute the gradients
3. Devices quantize the gradients
4. Server gathers quantized gradients
5. Server updates parameters
6. Repeat steps 1 - 5



Assumptions

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2. Smoothness:

Assumptions

1 Uniform lower bound: $\exists f_* \in \mathbb{R}: \ \forall x \in \mathbb{R}^d \quad f(x) \geq f_*$

2 Smoothness: $\|\nabla f_i(x) - \nabla f_i(y)\| \leq L_i \|x - y\|$

Complexity Bound for QGD

Khaled, Ahmed, and Peter Richtárik. **"Better theory for SGD in the nonconvex world."** arXiv preprint arXiv:2002.03329 (2020).

QGD finds such $\hat{x}$ that $\mathbb{E}\left[\|\nabla f(\hat{x})\|^2\right] \leq \varepsilon^2$ after

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$$\mathcal{O}\left(\frac{\Delta_0}{\varepsilon^2} + \frac{(1+\omega)\Delta_0^2}{\varepsilon^4 n} + \frac{(1+\omega)\Delta_0\Delta_f^*}{\varepsilon^4 n}\right)$$

communication rounds

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Hides numerical factors and smoothness constants

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Hides numerical factors and smoothness constants → $\mathcal{O}\left(\frac{\boxed{\Delta_0}}{\boxed{\varepsilon^2}} + \frac{(1+\boxed{\omega})\boxed{\Delta_0^2}}{\boxed{\varepsilon^4} n} + \frac{(1+\boxed{\omega})\boxed{\Delta_0}\boxed{\Delta_f^*}}{\boxed{\varepsilon^4} n}\right)$ communication rounds

$$\boxed{\mathbb{E}\|Q(x)-x\|^2 \leq \omega\|x\|^2}$$

$$\boxed{\Delta_0 = f(x^0) - f_*}$$

$$\boxed{\Delta_f^* = f_* - \frac{1}{n}\sum_{i=1}^n f_{i,*}}$$

3. DIANA

Mishchenko, Konstantin, Eduard Gorbunov, Martin Takáč, and Peter Richtárik. **"Distributed learning with compressed gradient differences."** arXiv preprint arXiv:1901.09269 (2019).

Horváth, Samuel, Dmitry Kovalev, Konstantin Mishchenko, Sebastian Stich, and Peter Richtárik. **"Stochastic distributed learning with gradient quantization and variance reduction."** arXiv preprint arXiv:1904.05115 (2019).

$$x^{k+1} = x^k - \gamma \cdot \frac{1}{n} \sum_{i=1}^{n} g_i^k$$

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learnable local shifts

$$\boxed{h_i^{k+1} = h_i^k + \alpha \mathcal{Q}\left(\nabla f_i(x^k) - h_i^k\right)}$$

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communication rounds

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DIANA finds such $\hat{x}$ that $\mathbb{E}\left[\|\nabla f(\hat{x})\|^2\right] \leq \varepsilon^2$ after

Hides numerical factors and smoothness constants

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Hides numerical factors and smoothness constants

$$\boxed{\mathbb{E}\|\mathcal{Q}(x)-x\|^2 \leq \omega\|x\|^2}$$

$$\boxed{\Delta_0 = f(x^0) - f_*}$$

Complexity Bounds for DIANA and QGD

QGD: $$\mathcal{O}\left(\frac{\Delta_0}{\varepsilon^2} + \frac{(1+\omega)\Delta_0^2}{\varepsilon^4 n} + \frac{(1+\omega)\Delta_0\Delta_f^*}{\varepsilon^4 n}\right)$$

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Complexity Bound for DIANA

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Is it possible to get better rates?

DIANA: $\mathcal{O}\left(\frac{\Delta_0\left(1+(1+\omega)\sqrt{\omega/n}\right)}{\varepsilon^2}\right)$

4. MARINA

$$x^{k+1} = x^k - \gamma \cdot \frac{1}{n} \sum_{i=1}^{n} g_i^k$$

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$$x^{k+1} = x^k - \gamma \cdot \boxed{\frac{1}{n}\sum_{i=1}^{n} g_i^k} = x^k - \gamma \boxed{g^k}$$

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$$h_i^{k+1} = h_i^k + \alpha \mathcal{Q}\left(\nabla f_i(x^k) - h_i^k\right) \quad \mathbb{E}\left[\boxed{g^k} \mid x^k\right] = \nabla f(x^k)$$

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MARINA: $g_i^k = \begin{cases} \boxed{\nabla f_i\left(x^k\right)} & \text{w.p. } p \\ g^{k-1} + \boxed{\mathcal{Q}\left(\nabla f_i\left(x^k\right) - \boxed{\nabla f_i\left(x^{k-1}\right)}\right)} & \text{w.p. } 1-p \end{cases}$

$$\mathbb{E}\left[\boxed{g^k} \mid x^k\right] \neq \nabla f(x^k)$$

Complexity Bound for MARINA

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$$\boxed{\Delta_0 = f(x^0) - f_*}$$

$$p = \frac{1}{\omega+1} = \Theta\left(\frac{\boxed{\zeta_{\mathcal{Q}}}}{d}\right)$$

$$\boxed{\zeta_{\mathcal{Q}}} = \sup_{x\in\mathbb{R}^d} \mathbb{E}\left[\|\mathcal{Q}(x)\|_0\right]$$

assumption (holds for RandK, l2-quantization)

expected density

Complexity Bounds for MARINA and DIANA

DIANA: $$\mathcal{O}\left(\frac{\Delta_0\left(1+(1+\omega)\sqrt{\omega/n}\right)}{\varepsilon^2}\right)$$

MARINA: $$\mathcal{O}\left(\frac{\Delta_0\left(1+\omega/\sqrt{n}\right)}{\varepsilon^2}\right)$$

Complexity Bounds for MARINA and DIANA

DIANA: $$\mathcal{O}\left(\frac{\Delta_0\left(1+(1+\boxed{\omega})\sqrt{\boxed{\omega}/n}\right)}{\varepsilon^2}\right)$$

MARINA: $$\mathcal{O}\left(\frac{\Delta_0\,(1+\boxed{\omega}/\sqrt{n})}{\varepsilon^2}\right)$$

5. MARINA and Variance Reduction

$$f_i(x) = \frac{1}{m} \sum_{j=1}^{m} f_{ij}(x)$$

$$f_i(x) = \frac{1}{m}\sum_{j=1}^{m} f_{ij}(x) \qquad x^{k+1} = x^k - \gamma \cdot \frac{1}{n}\sum_{i=1}^{n} g_i^k$$

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MARINA: $g_i^k = \begin{cases} \nabla f_i\left(x^k\right) & \text{w.p. } p \\ g^{k-1} + \mathcal{Q}\left(\nabla f_i\left(x^k\right) - \nabla f_i\left(x^{k-1}\right)\right) & \text{w.p. } 1-p \end{cases}$

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VR-MARINA: $g_i^k = \begin{cases} \nabla f_i\left(x^k\right) & \text{w.p. } p \\ g^{k-1} + \mathcal{Q}\left(\nabla f_{ij}\left(x^k\right) - \nabla f_{ij}\left(x^{k-1}\right)\right) & \text{w.p. } 1-p \end{cases}$

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MARINA: $g_i^k = \begin{cases} \nabla f_i\left(x^k\right) & \text{w.p. } p \\ g^{k-1} + \mathcal{Q}\left(\nabla f_i\left(x^k\right) - \nabla f_i\left(x^{k-1}\right)\right) & \text{w.p. } 1-p \end{cases}$

VR-MARINA: $g_i^k = \begin{cases} \nabla f_i\left(x^k\right) & \text{w.p. } p \\ g^{k-1} + \mathcal{Q}\left(\nabla f_{i\boxed{j}}\left(x^k\right) - \nabla f_{i\boxed{j}}\left(x^{k-1}\right)\right) & \text{w.p. } 1-p \end{cases}$

$\boxed{j} \sim \{1, \ldots, m\}$ uniformly at random

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MARINA: $g_i^k = \begin{cases} \nabla f_i\left(x^k\right) & \text{w.p. } p \\ g^{k-1} + \mathcal{Q}\left(\nabla f_i\left(x^k\right) - \nabla f_i\left(x^{k-1}\right)\right) & \text{w.p. } 1-p \end{cases}$

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Complexity Bound for VR-MARINA

VR-MARINA finds such $\hat{x}$ that $\mathbb{E}\left[\|\nabla f(\hat{x})\|^2\right] \leq \varepsilon^2$ after

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communication rounds/oracle calls per node

Complexity Bound for VR-MARINA

VR-MARINA finds such $\hat{x}$ that $\mathbb{E}\left[\|\nabla f(\hat{x})\|^2\right] \leq \varepsilon^2$ after

Hides numerical factors and smoothness constants

$$\mathcal{O}\left(\frac{\Delta_0\left(1+\max\{\omega, \sqrt{(1+\omega)m}\}/\sqrt{n}\right)}{\varepsilon^2}\right)$$

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communication rounds/oracle calls per node

$$\boxed{\mathbb{E}\|\mathcal{Q}(x)-x\|^2 \leq \omega\|x\|^2}$$

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communication rounds/oracle calls per node

$$\boxed{\mathbb{E}\|\mathcal{Q}(x)-x\|^2 \leq \omega\|x\|^2}$$

$$\boxed{\Delta_0 = f(x^0) - f_*}$$

$$p = \min\left\{\frac{1}{\boxed{\omega}+1}, \frac{1}{\boxed{m}+1}\right\}$$

Complexity Bounds for MARINA and VR-MARINA

$$f_i(x) = \frac{1}{\boxed{m}} \sum_{j=1}^{\boxed{m}} f_{ij}(x)$$

	communication rounds	oracle calls per node
MARINA:	$\mathcal{O}\left(\frac{\Delta_0 (1+\boxed{\omega}/\sqrt{n})}{\varepsilon^2}\right)$	$\mathcal{O}\left(\frac{\boxed{m}\Delta_0 (1+\boxed{\omega}/\sqrt{n})}{\varepsilon^2}\right)$

VR-MARINA:

$$\mathcal{O}\left(\frac{\Delta_0 \left(1+\max\{\boxed{\omega}, \sqrt{(1+\boxed{\omega})\boxed{m}}\}/\sqrt{n}\right)}{\varepsilon^2}\right)$$

Complexity Bounds for VR-DIANA and VR-MARINA

$$f_i(x) = \frac{1}{\boxed{m}} \sum_{j=1}^{\boxed{m}} f_{ij}(x)$$

communication rounds oracle calls per node

VR-DIANA: $$\mathcal{O}\left(\frac{\Delta_0 \left(\boxed{m^{2/3}} + \boxed{\omega}\right) \sqrt{1 + \boxed{\omega}/n}}{\varepsilon^2}\right)$$

VR-MARINA: $$\mathcal{O}\left(\frac{\Delta_0 \left(1 + \max\{\boxed{\omega}, \sqrt{(1+\boxed{\omega})\boxed{m}}\}/\sqrt{n}\right)}{\varepsilon^2}\right)$$

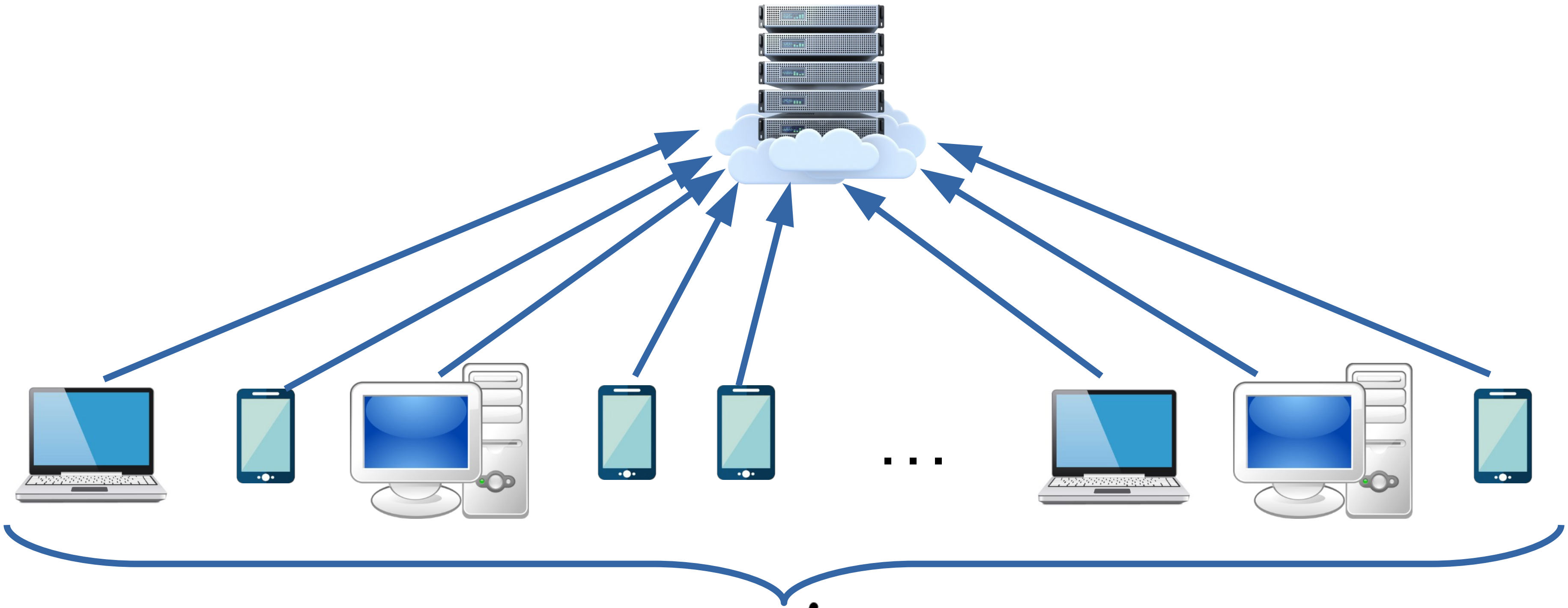
6. MARINA and Partial Participation

PP-MARINA

PP-MARINA

I_k' - the set of clients participating in communication on step k

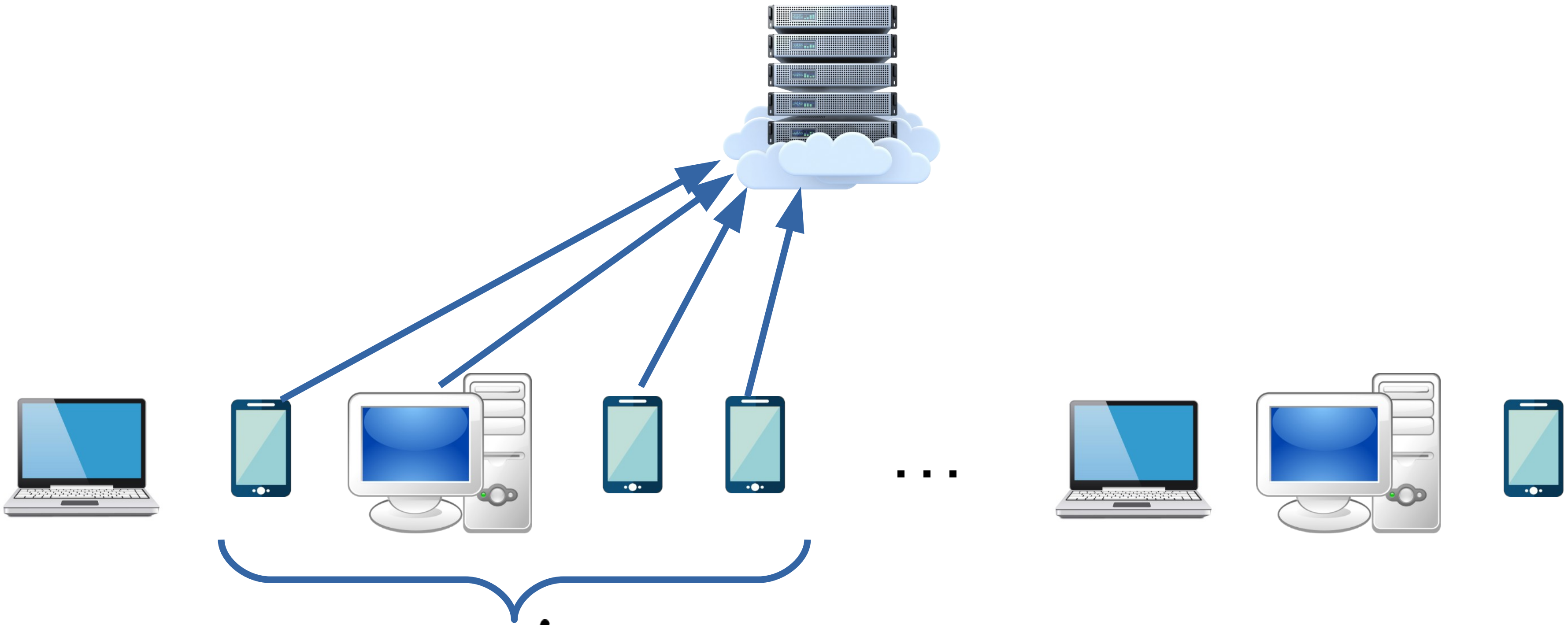
PP-MARINA



I'_k with small probability p

I'_k - the set of clients participating in communication on step k

PP-MARINA



I'_k with large probability 1 - p

I'_k - the set of clients participating in communication on step k

PP-MARINA

$$x^{k+1} = x^k - \gamma g^k$$

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$$g^k = \begin{cases} \nabla f\left(x^k\right) & \text{if } c_k = 1 \\ g^{k-1} + \frac{1}{r} \sum\limits_{i_k \in I'_k} \mathcal{Q}\left(\nabla f_{i_k}\left(x^k\right) - \nabla f_{i_k}\left(x^{k-1}\right)\right) & \text{if } c_k = 0 \end{cases}$$

PP-MARINA

$$x^{k+1} = x^k - \gamma g^k$$

$$\boxed{c_k \sim \mathrm{Be}(p)}$$

$$g^k = \begin{cases} \nabla f\left(x^k\right) & \text{if } \boxed{c_k} = 1 \\ g^{k-1} + \frac{1}{r} \sum\limits_{i_k \in I'_k} \mathcal{Q}\left(\nabla f_{i_k}\left(x^k\right) - \nabla f_{i_k}\left(x^{k-1}\right)\right) & \text{if } \boxed{c_k} = 0 \end{cases}$$

PP-MARINA

$$x^{k+1} = x^k - \gamma g^k$$

full participation & uncompressed communication

$$\boxed{c_k \sim \mathrm{Be}(p)}$$

$$g^k = \begin{cases} \boxed{\nabla f\left(x^k\right)} & \text{if } \boxed{c_k} = 1 \\ g^{k-1} + \frac{1}{r} \sum\limits_{i_k \in I'_k} \mathcal{Q}\left(\nabla f_{i_k}\left(x^k\right) - \nabla f_{i_k}\left(x^{k-1}\right)\right) & \text{if } \boxed{c_k} = 0 \end{cases}$$

PP-MARINA

$$x^{k+1} = x^k - \gamma g^k$$

full participation & uncompressed communication

$$\boxed{c_k \sim \mathrm{Be}(p)}$$

$$g^k = \begin{cases} \boxed{\nabla f\left(x^k\right)} & \text{if } \boxed{c_k} = 1 \\ g^{k-1} + \frac{1}{\boxed{r}} \sum\limits_{i_k \in \boxed{I'_k}} \boxed{\mathcal{Q}\left(\nabla f_{i_k}\left(x^k\right) - \nabla f_{i_k}\left(x^{k-1}\right)\right)} & \text{if } \boxed{c_k} = 0 \end{cases}$$

partial participation & compressed communication

$$\boxed{|I'_k| = r}$$

Complexity Bound for PP-MARINA

PP-MARINA finds such $\hat{x}$ that $\mathbb{E}\left[\|\nabla f(\hat{x})\|^2\right] \leq \varepsilon^2$ after

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communication rounds

Complexity Bound for PP-MARINA

PP-MARINA finds such $\hat{x}$ that $\mathbb{E}\left[\|\nabla f(\hat{x})\|^2\right] \leq \boxed{\varepsilon^2}$ after

Hides numerical factors and smoothness constants

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$$p = \frac{\boxed{r}}{n(\omega+1)} = \Theta\left(\frac{\boxed{r}\zeta_{\mathcal{Q}}}{nd}\right)$$

$$\zeta_{\mathcal{Q}} = \sup_{x \in \mathbb{R}^d} \mathbb{E}\left[\|\mathcal{Q}(x)\|_0\right]$$

assumption (holds for RandK, l2-quantization)

expected density

Complexity Bounds for PP-MARINA and MARINA

MARINA: $$\mathcal{O}\left(\frac{\Delta_0\left(1+\omega/\boxed{\sqrt{n}}\right)}{\varepsilon^2}\right)$$

PP-MARINA: $$\mathcal{O}\left(\frac{\Delta_0\left(1+(1+\omega)\boxed{\sqrt{n}/r}\right)}{\varepsilon^2}\right)$$

7. Experiments

The Problem

$$\min_{x\in\mathbb{R}^d}\left\{ f(x) = \frac{1}{N}\sum_{t=1}^{N}\left(1 - \frac{1}{1+\exp(-y_t a_t^\top x)}\right)^2 \right\}$$

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$y_t \in \{-1, 1\}$

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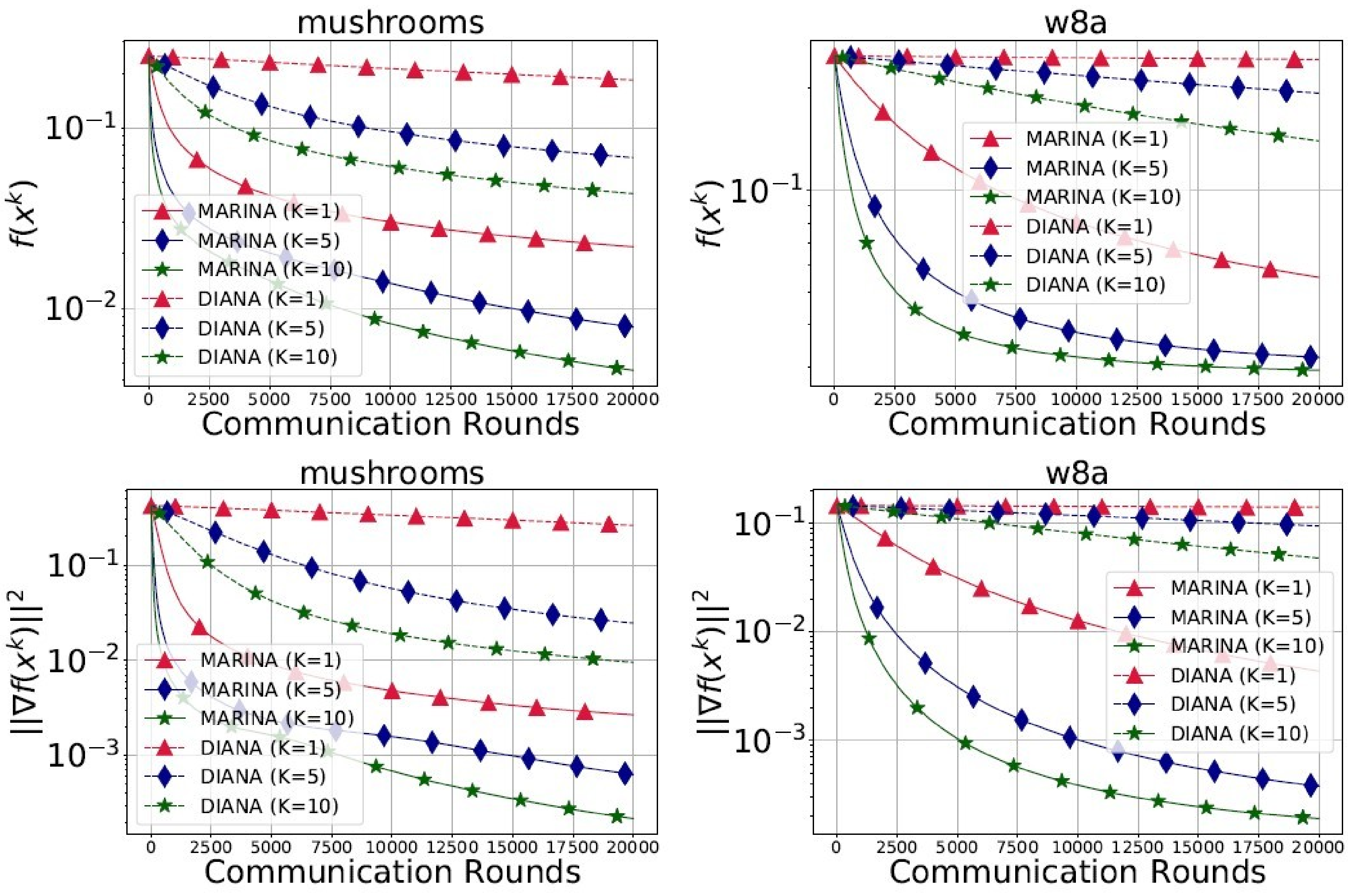
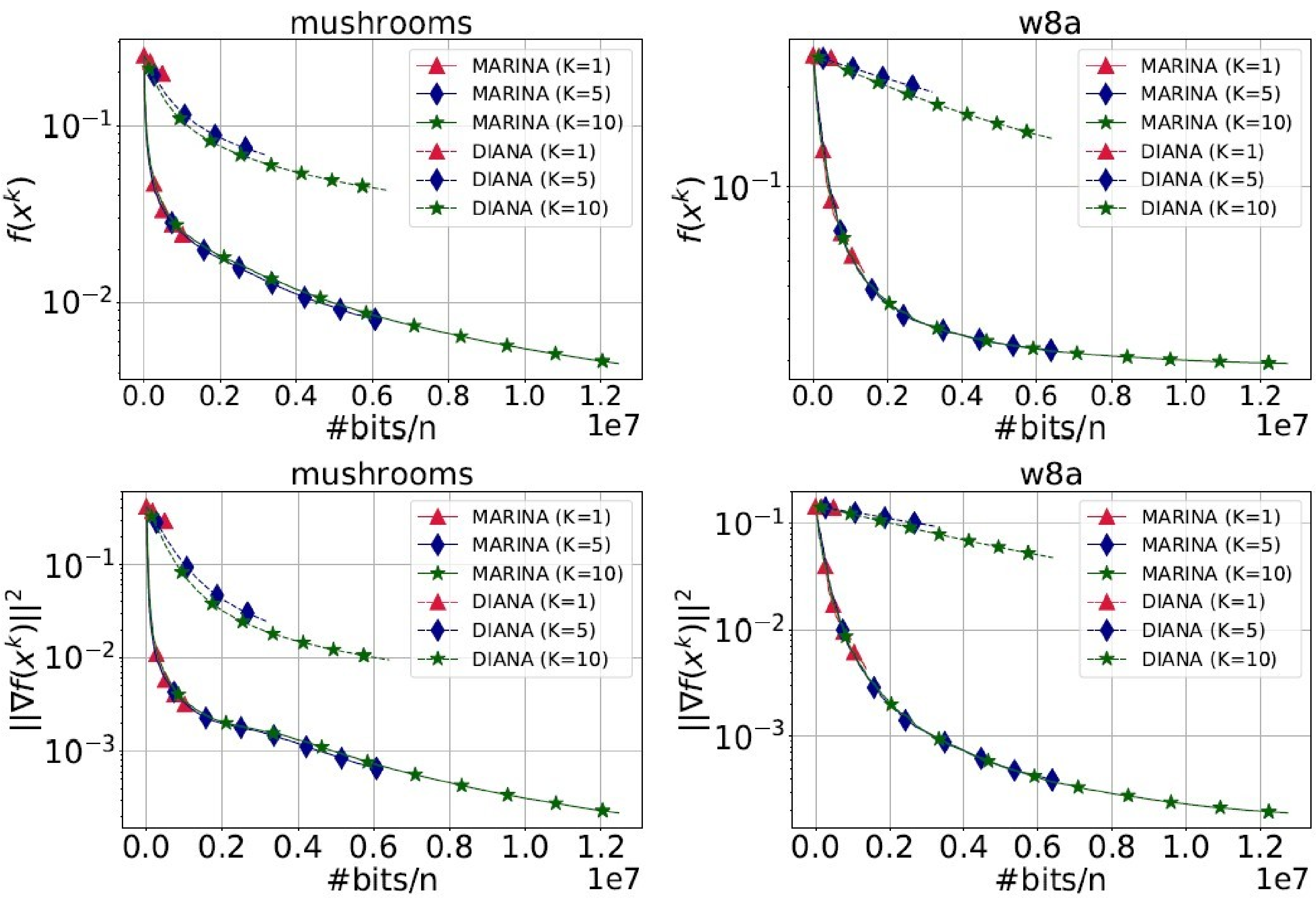
- The dataset was split into 5 equal parts among 5 clients
- Theoretical stepsizes
- For stochastic methods batchsize was 1% of local data

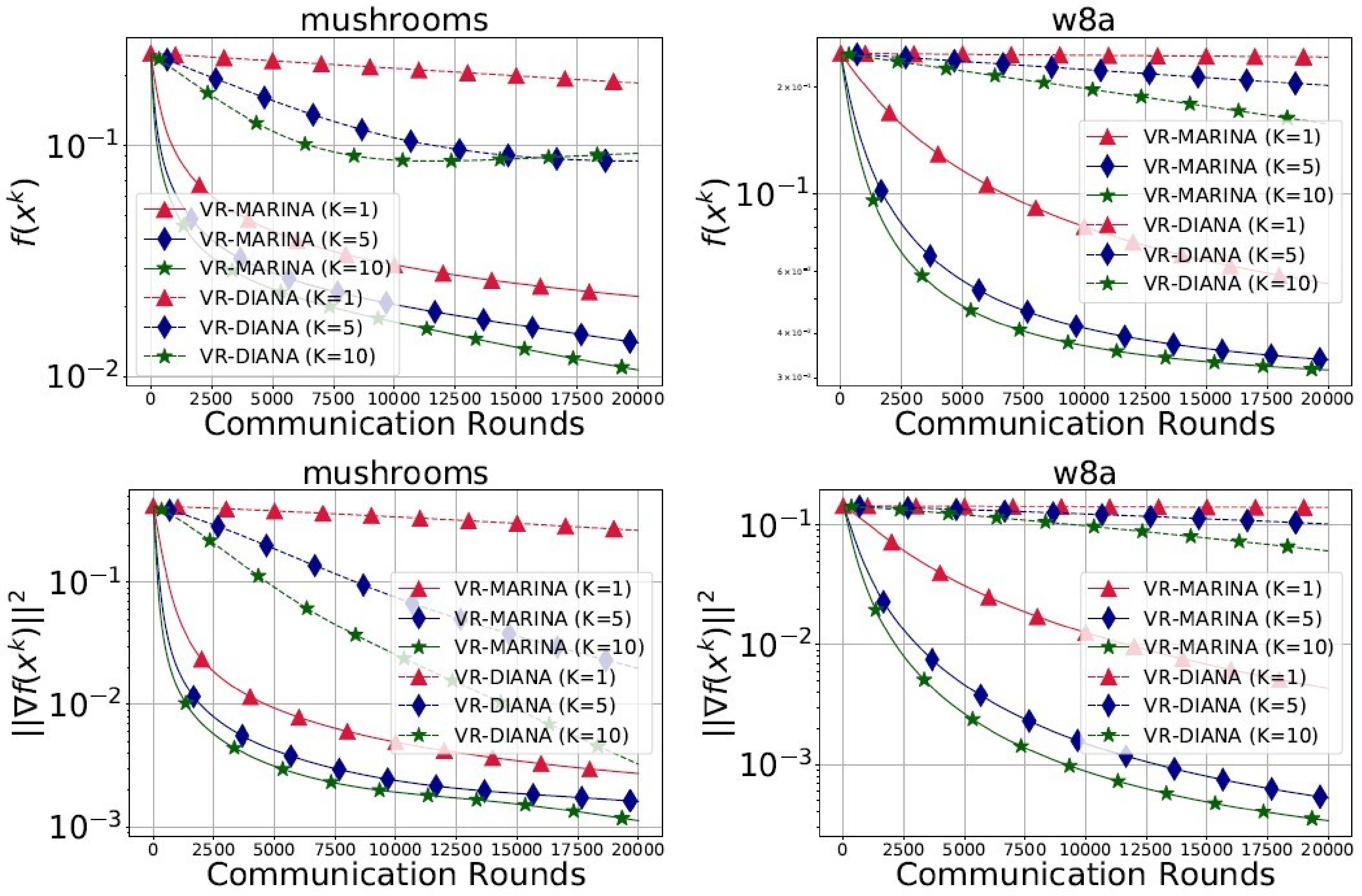
MARINA vs DIANA (RandK)



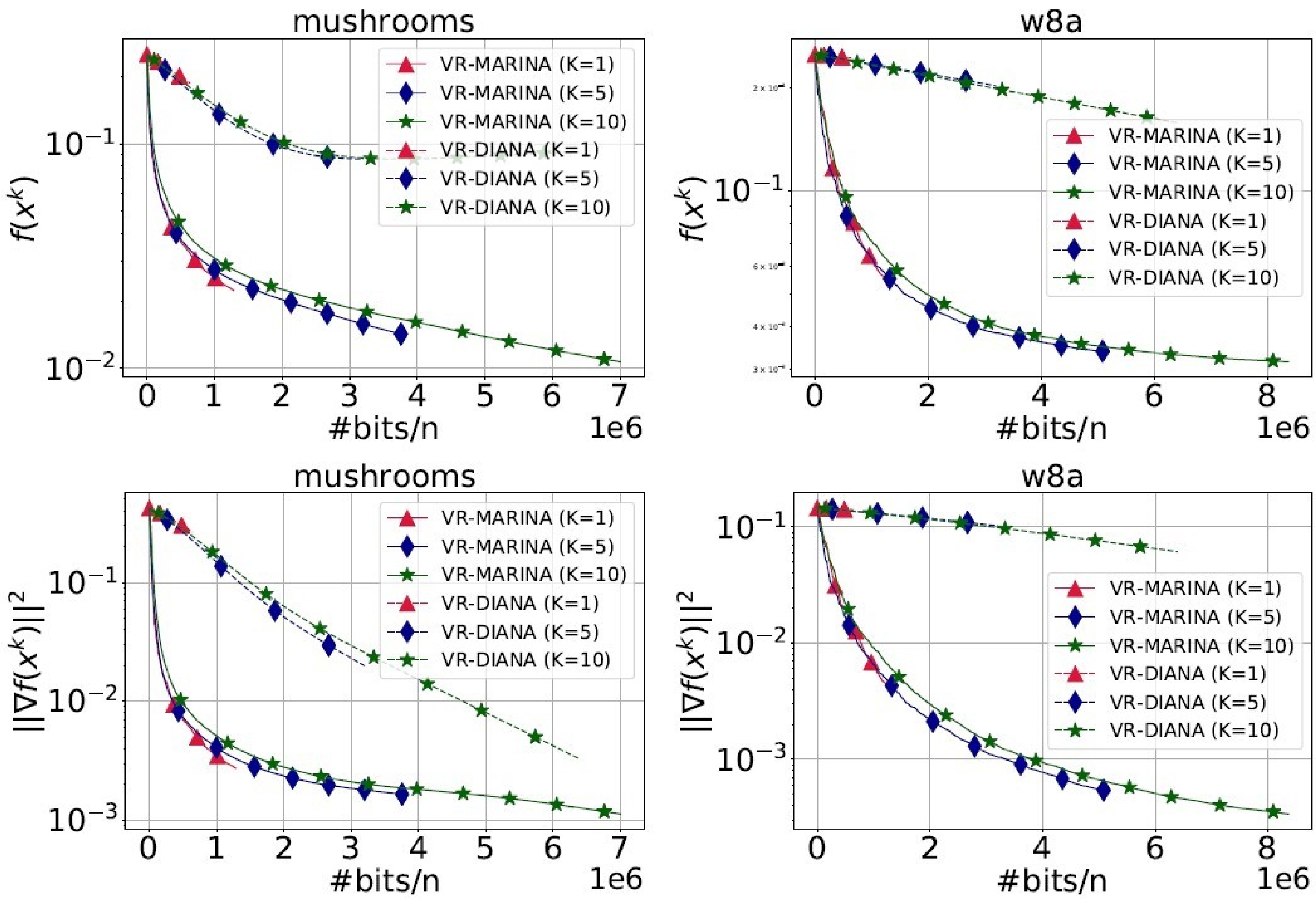
MARINA vs DIANA (RandK)



VR-MARINA vs VR-DIANA (RandK)



VR-MARINA vs VR-DIANA (RandK)



8. Extra Results

In the paper, we also have:

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- Rates under Polyak- Lojasiewicz Condition
- Explicit dependencies on smoothness constants, non-uniform sampling
- Simple proofs