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# *DETERMINATION OF LOCATION AND VOLUME OF LEAKS IN A PIPELINE NETWORK OF COMPLEX STRUCTURE FOR UNSTEADY FLUID FLOW*

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## *Statement of the problem*

We consider the problem of localization of leakage points in the pipelines of complex structure, differs from many previously cases examined for individual linear sections of main pipelines.

We consider the flow regimes in complex pipe networks for sufficiently general case of complex-shaped hydraulic network with loops containing segments and nodes. Sometimes it is difficult to decide which direction fluid will flow. The direction of flow is often obvious, but when it is not flow direction has to be assumed. We assume that, the flow directions in the segments are given a priori, and either the calculated value of flow rate is greater than zero in a segment which means that actual flow direction in this segment coincide with the given direction, or the calculated value of the flow rate less than zero then actual flow direction is opposite to the given one.

To simplify presentation of numerical schemes and to be specific, let us consider the pipe network, containing 8 segments as shown in figure 1.

## *Statement of the problem*

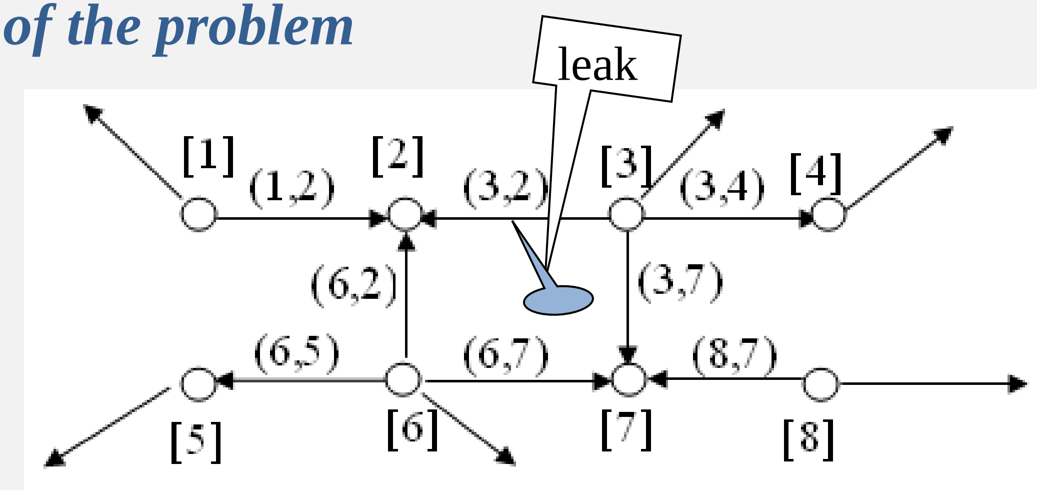



Fig.1 The scheme of pipe network with 8 nodes

Numbers in brackets identify the nodes (or junctions). The set of nodes we denote by $I$: $I=\{k_1,...,k_N\}$; where $k_i, i=\overline{1,N}$ are the nodes; $N=|I|$ is the numbers of nodes in the network. Two numbers in parentheses identify two-index numbers of segments. The flow in these segments goes from the first index to the second (for example, the flow in the segment (1,2) is obviously from the node 1 to node 2.

## *Statement of the problem*

Let $J$: $J=\{(k_i,k_j):k_i,k_j\in I\}$ denote the set of segments and $M=|J|$ denote it's quantity; $l_{k_ik_j}$, $d_{k_ik_j},k_i,k_j\in I$ is a length and diameter of the segment $(k_i,k_j)$ respectively.

Let $I_k^+$ denote the set of nodes connected with node $k$ by segments where flow goes into the node, let $I_k^-$ denote the set of nodes connected with node $k$ by segments where flow goes out of the node; $I_k=I_k^+\bigcup I_k^-$ denote the set of total nodes connected with node $k$ and $N_k=|I_k|, N_{k^+}=|I_k^+|, N_{k^-}=|I_k^-|$, $N_k=N_{k^-}+N_{k^+}$. The arrows in figure 1 formally indicate assumed (not actual) direction of the fluid flow.

## *Statement of the problem*

Note that, the arrows indicating direction of flow are selected so that each node may be only inflow or outflow. Such selection facilitate in certain extent the calculating process, but it is not principal (the reason for this will be clear further).

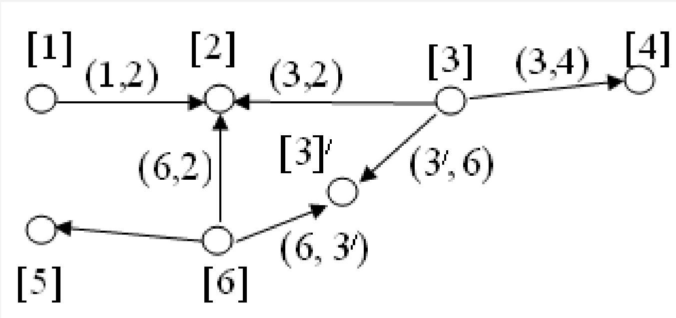



Fig. 2 The case of introduction of additional node

Such an appointment of directions can be done for any structure of the pipeline network, at the expense of artificial introduction of an additional node and partition of whole segment into two sections in the case of necessity.

For example, in the segment (6, 3) we introduced a new node [3'] as shown in figure 2, and the segment is divided into two sections: (6, 3') and (3',6).

## Statement of the problem

Beside of inflows and outflows in the segments of the network there can be external inflows (sources) and outflows (sinks) with the rate $\tilde{q}_i(t)$ at some nodes $i \in I$ of the network. Positive and negative values of $\tilde{q}_i(t)$ indicate the existence of external inflow or outflow at the node $i$. However, in general case, assuming that the case $\tilde{q}_i(t) \equiv 0$ for the sources is admissible one can consider all nodes of the network as the nodes with external inflows or outflows. Let $I^f \subset I$ denote the set of nodes $i \in I$, where $i$ is such that the set $I_i^+ \bigcup I_i^-$ consists of only one segment. It means that the node $i$ is a node of external inflow or outflow for the whole pipe network (for example $I^f = \{1,4,5,8\}$ in fig.1). Let $N_f = \left| I^f \right|$ denote the number of such nodes, it is obviously that $N_f \leq N$. Let $I^{\text{int}}$ denote the set of nodes not belonging to $I^f$, so $N_{\text{int}} = \left| I^{\text{int}} \right|$, i.e., $I^{\text{int}} = I / I^f$, $N_{\text{int}} = N - N_f$. In actual conditions, the pumping stations are placed, the measuring equipment is installed and the quantitative accounting is conducted at the nodes from the set $I^f$.

## *Statement of the problem*

We assume that at some instants of time $t \geq t_0$ at some points $\xi_{ks} \in (0, l)$, of any $(ks)$-th section of the pipeline network, fluid leakage with the flow rates $q_{ks}^{loss}(t)$ began. Using the generalized Dirac function δ(*x*), we can describe the motion of the liquid by the following linearized system of differential equations for unsteady flow of dripping liquid with constant density $\rho$ in a linear pipe $(k, s)$ of length $l_{ks}$ and diameter $d_{ks}$ of oil pipeline network can be written in the following form [15]:

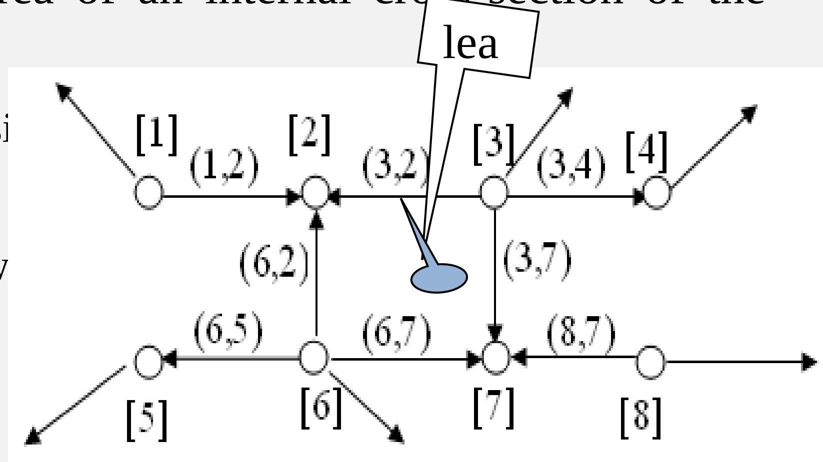
$$\begin{cases} -\dfrac{\partial P^{ks}(x,t)}{\partial x} = \dfrac{\rho}{S^{ks}} \dfrac{\partial Q^{ks}(x,t)}{\partial t} + 2a^{ks} \dfrac{\rho}{S^{ks}} Q^{ks}(x,t), x \in (0, l^{ks}), \\ -\dfrac{\partial P^{ks}(x,t)}{\partial t} = c^2 \dfrac{\rho}{S^{ks}} \dfrac{\partial Q^{ks}(x,t)}{\partial x} + c^2 \dfrac{\rho}{S^{ks}} q_{ks}^{loss}(t) \delta(x - \xi_{ks}), \ t \in (0, T], \end{cases} \quad s \in I_k^+, k \in I. \quad (1)$$

here $c$ is the sound velocity in oil; $S^{ks}$ is the area of an internal cross-section of the segment $(k, s)$; $a^{ks}$ is the coefficient of dissipation

(we may consider that the kinematic coefficient of viscosi

$\gamma$ is independent of pressure and the condition

$2a^{ks} = \dfrac{32\gamma}{(d^{ks})^2} = const$ is quite accurate for a laminar flow



## Statement of the problem

$Q^{k_i k_j}(x,t)$, $P^{k_i,k_j}(x,t)$ are the flow rate and pressure of flow, respectively, at the time instance $t$ in the point $x \in (0, l^{k_i,k_j})$ of the segment$(k_i, k_j)$ of the pipe network. $P^k(t), Q^k(t)$ are the pressure and flow rate at the node $k \in I$, respectively. The values $Q^{k_i k_j}(x,t)$ can be positive or negative. The positive or negative value $Q^{k_i k_j}(x,t)$ means that, an actual flow in the segment $(k_i, k_j)$ is directed from the node $k_i$ to the node $k_j$ or the flow direction is from $k_j$ to $k_i$, respectively. It is obvious that, each segment carries an inflow and outflow for some nodes, and the following conditions are satisfied:

$$Q^{k_i,k_j}(x,t) = -Q^{k_j k_i}(l^{k_i,k_j} - x, t),$$

$$P^{k_i,k_j}(x,t) = P^{k_j k_i}(l^{k_i,k_j} - x, t),\ x \in (0, l^{k_i k_j}),\ k_i \in I, k_j \in I^{+}_{k_i}, \qquad (2)$$

where $l^{k_j k_i} = l^{k_i k_j}$.

## *Statement of the problem*

The equation for each segment of the network appears in the system (1) only once. Indeed, the first index $k$ takes the values of all nodes from $I$, and the segments $(k,s)$ make up the set of all segments-inflows into the node $s$, $s \in I_k^+$. Taking into account that, each segment of the network is inflow for some node, consequently, there is an equation in (1) for each segment of the network and this equation takes place in (1) only once.

Instead of using (1),we can write the process of fluid flow in the network for the segments-outflows in the following form:

$$\begin{cases} -\dfrac{\partial P^{ks}(x,t)}{\partial x} = \dfrac{\rho}{S^{ks}} \dfrac{\partial Q^{ks}(x,t)}{\partial t} + 2a^{ks} \dfrac{\rho}{S^{ks}} Q^{ks}(x,t),\ x \in (0, l^{ks}), \\ -\dfrac{\partial P^{ks}(x,t)}{\partial t} = c^2 \dfrac{\rho}{S^{ks}} \dfrac{\partial Q^{ks}(x,t)}{\partial x} + c^2 \dfrac{\rho}{S^{ks}} q_{ks}^{loss}(t)\delta(x-\xi_{ks}),\ t \in (0,T], t \in (0,T], \end{cases} \quad s \in I_k^-, k \in I,$$

because each segment is an outflow for some node.

## *Statement of the problem*

The conditions of Kirchhoff's first law (total flow into the node must be equal to total flow out of the node) are satisfied at the nodes of the network at $t \in [0, T]$:

$$\sum_{s \in I_k^+} Q^{ks}(l^{ks}, t) - \sum_{s \in I_k^-} Q^{ks}(0, t) = \tilde{q}^k(t), \; k \in I . \quad (3)$$

Also, the following conditions of flow continuity for the nodes of the net (the equality of the values of pressures on all adjacent ends of the segments of the network) hold:

$$P^k(t) = P^{k_i k}(l^{k_i k}, t) = P^{k k_j}(0, t), \; k_i \in I_k^+, k_j \in I_k^-, k \in I , \quad (4)$$

where $\tilde{q}^k(t)$ is the external inflow $(\tilde{q}^k(t) > 0)$ or outflow $(\tilde{q}^k(t) < 0)$ for the node $k$, $P^k(t)$ is the value of the pressure in the node $k$.

We must note that they have significant specific features, consisting in the fact that the conditions (3) and (4) are non-separated (nonlocal) boundary conditions unlike classical cases of boundary conditions for partial differential equations.

## *Statement of the problem*

It is evident that (3) includes $N_f + N_{\text{int}} = N$ conditions. The number of independent conditions is $(N_k - 1)$ for every node $k$ from the set $I^{\text{int}}$ in (4) (i.e., one less than the total number of adjacent nodes), and two boundary conditions (4) are associated with every internal segment of the network.

The total number of conditions for all nodes from $I^f$ is $N_f$. So, the total number of conditions in (3) and (4) is $[N_f + N_{\text{int}}] + [(2M - N_f) - N_{\text{int}}] = 2M$.

As it was noted above the number of conditions in (3) is $N$, but in view of the condition of material balance ($\sum_{k \in I} \tilde{q}^k(t) = 0$) for the whole pipeline network, we conclude that the number of linear**ly** independent conditions is $N - 1$.

## *Statement of the problem*

So, it is necessary to add any one independent condition. As a rule the value of pressure at one of the nodes $s \in I^f$ is given for this purpose, in place of the flow rate $q^s(t)$:

$$P^s(t) = \tilde{P}^s(t). \quad (5)$$

In the case of unknown points of leakages and their rates $\xi_{ks}, q_{ks}^{loss}(t)$ we will assume that at the ends of the pipeline sections a constant and rather long observation on pressure is made, i.e., the values of $P_{mes}^n(t),\ n \in I_p^f$ or $Q_{mes}^m(t), m \in I_q^f$ are known. It is quite natural to suppose that the sought leak spots do not coincide with the points of observation of regimes.

In more general case, for every node from $I^f = I_q^f \cup I_p^f$, it is necessary to give the values of pressure ( $I_p^f \subset I^f$ denotes the set of such nodes) or the values of flow rate (the set $I_q^f \subset I^f$) and $I_p^f$ must not be an empty: $I_p^f \neq \varnothing$.

## *Statement of the problem*

So, we will add the following conditions to the condition (3):

$$\begin{cases} P^{n}(t) = P^{ns}(0,t) = P^{n}_{mes}(t), & s \in I^{+}_{n}, \ \text{если}\ I^{-}_{n} = \varnothing, \\ P^{n}(t) = P^{sn}(l^{sn},t) = P^{n}_{mes}(t), & s \in I^{-}_{n}, \ \text{если}\ I^{+}_{n} = \varnothing, \end{cases} \quad n \in I^{f}_{p}, \qquad (6)$$

$$\begin{cases} Q^{m}(t) = Q^{ms}(0,t) = Q^{m}_{mes}(t), & s \in I^{+}_{m}, \ \text{если}\ I^{-}_{m} = \varnothing, \\ Q^{m}(t) = Q^{sm}(l^{sm},t) = Q^{m}_{mes}(t), & s \in I^{-}_{m}, \ \text{если}\ I^{+}_{m} = \varnothing, \end{cases} \quad m \in I^{f}_{q}, \qquad (7)$$

When the spots of oil leakages from a pipeline and the rates of these leakages are known $\xi_{ks}, q_{ks}{}^{loss}(t),\ (ks) \in J$, it is sufficient to use one of the boundary-value conditions (6) or (7) to calculate the regime of liquid motion in the pipeline from (1) on the time interval $[t_0, T]$. One of them we will use in the functional, the form which will be given below.

## *Statement of the problem*

So, we assume that, the initial conditions for the process (1) are not precisely defined, at the initial time $t_0$, but some sets $Q = Q^{ks}$, $\mathrm{P} = \mathrm{P}^{ks}$, $(\mathrm{k}, \mathrm{s}) \in \mathrm{J}$ of admissible values of initial regimes are given, which are defined in this case by parametric set $D \subset R^{M+N}$ of admissible values of flow amounts on segments for steady flow:

$$\begin{aligned} &\hat{Q}^{ks}(x) = Q^{ks}(x, t_0; \gamma) = \gamma_q^{ks} = const, \\ &\hat{P}^{ks}(x) = P^{ks}(x, t_0; \gamma) = \gamma_p^{ks} - 2ax\gamma_q^{ks}, \\ &x \in (0, l^{ks}), \quad \gamma = (\gamma_p, \gamma_q) = (\gamma_p^{ks}, \gamma_q^{ks}) \in D \subset \\ &\subset R^{M+N}, (k, s) \in J, \; k \in I \end{aligned} \tag{6}$$

Here $\gamma_q^{ks}-$ are admissible values of flow amounts in $(k, s)-$th segment $(k, s) \in J$, $\gamma_p^{ks}-$ are admissible values of pressure in the nodes $k \in I$ at steady flow, the corresponding density functions of distributions, written in vector form as $\mu_D(\gamma)$ are given, which are known from practical considerations, $M$ and $N$ is the number of segments and nodes accordingly.

## *Statement of the problem*

The problem consists in the detection of the points of leakage $\xi = \{\xi_{ks}, (k,s) \in J\}$ and corresponding losses of raw material $q^{loss}(t) = \{q_{ks}^{loss}(t), (ks) \in J\}$ at $t \in [t_0, T]$ with the use of the given mathematical model and obtained information.

It is important to note that if process (1) is rather long, then, due to the presence of friction typical of any real physical system, the influence of the initial state of the pipeline on the regimes of oil motion in it becomes weaker with time. Therefore, when the process is observed for a long time, i.e., within a large time interval $[t_0, T]$, the influence of the initial regime of oil flow in a pipeline (at $t = t_0$) on the current state of the process decreases, and there exists such $\tau$ $(\tau < T)$ that at $t > \tau$ the regime of oil motion experiences only the influence of the boundary-value conditions on the time interval $[t_0, T]$, where the quantity $\tau$ is determined by the parameters of the process and the characteristics of the pipeline. Thus, we arrive at the problem without initial conditions [4-6].

## *Statement of the problem*

In order to solve the problem posed, we will consider the functional that determines the derivation of regimes of oil flow at the given points of the oil pipeline section from those predicted:

$$\Im(\xi, q^{loss}) = \int_{\mathrm{D}} \left[ \Phi(\xi, q^{loss}; \gamma) + \Re(\xi, q) \right] \mu_{\mathrm{D}}(\gamma) d\gamma \to \min, \tag{9}$$

$$\Phi(\xi, q^{loss}; \gamma) = \sum_{m \in \tilde{I}_q^f} \int_{\tau}^{T} [Q^m(t; \xi, q(t), \gamma) - Q_{mes}^m(t)]^2 dt, \tag{10}$$

$$\Re(\xi, q) = \varepsilon_1 \left\| q(t) \text{-} \hat{q} \right\|_{L_2^Z[t_0, T]}^2 + \varepsilon_2 \left\| \xi \text{ - } \hat{\xi} \right\|_{R^Z}^2, \tag{10*}$$

where $Q^m(t; \xi, q(t), \gamma), m \in \tilde{I}_q^f$ – is the solution of the problem (1)–(5),(7),(8),(10) or (1)–(6),(8),(9) at the given values of $(\xi, q^{loss}(t))$, $[\tau, T]$ is the time interval of monitoring the process whose regimes already do not depend on the initial conditions; $\tilde{\xi}, \tilde{q} \in R^m, \varepsilon_1, \varepsilon_2$ – are the regularization parameters. Since the initial conditions at time $t_0$ do not influence the process in the interval $[\tau, T]$, exact knowledge of the initial value of $t_0$ is not of primary importance.

## *Statement of the problem*

**Remark:** If the number of the section on which leakage has taken place is not known beforehand, then it is necessary to solve the aforementioned problem for all the sections. It is clear that the case under which the minimal value of the functional has been obtained corresponds to the solution to the given problem.

Proceeding from the meaning of the problem considered, technological conditions, and technical requirements, we will assume that are restrictions on the identified functions and parameters:

$$0 < \xi_{ks} \le l^{ks}, \quad \underline{q} \le q^{loss}(t) \le \bar{q}, \quad t \in [t_0, T], \tag{11}$$

where $\underline{q}, \bar{q}$ are the given quantities.

As is seen, as to the determination of the points and rates of leakages the posed problem is the problem of parametric optimal control of an object described by a hyperbolic system. For its solution we use numerical methods (projections of the conjugated gradient) based on iteration procedures of first order optimization. To carry out this procedure, it is necessary to obtain formulas for the gradient of functional. If as a result of the solution of posed problem we obtain that $\left|q^{loss}(t)\right| \le \varepsilon, \ t \in [\tau, T]$, this will mean that in this section of the pipeline network there is no leakage of raw material.

## *Numerical method of the solution to the problem*

Let the functions $\varphi^{ks}(x,t)$, $\psi^{ks}(x,t), s \in I_k^+, k \in I$ are the solutions to the next adjoint boundary value problem:

$$
\left.
\begin{aligned}
&-\frac{\partial \varphi^{ks}(x,t)}{\partial x} = \frac{\partial \psi^{ks}(x,t)}{\partial t}, \\
&-\frac{\partial \varphi^{ks}(x,t)}{\partial t} = c^2 \frac{\partial \psi^{ks}(x,t)}{\partial x} - 2a^{ks}\varphi^{ks}(x,t),
\end{aligned}
\right. \quad t \in (0,T],\ x \in (0, l^{ks}), s \in I_k^+, k \in I. \tag{12}
$$

$$\varphi^{ks}(x,T) = 0,\ \psi^{ks}(x,T) = 0,\ \ x \in [0, l^{ks}],\ \ s \in I_k^+, k \in I,$$

$$\psi(l^m, t) = -2\frac{S^{ks}}{\rho}[Q^m(t;\xi,q^{loss}) - Q^m_{mes}(t)],\ s \in I_m^-,\ \ \text{если}\ I_m^+ = \varnothing, m \in I_q^f,$$

$$\psi(0, t) = -2\frac{S^{ks}}{\rho}[Q^m(t;\xi,q^{loss}) - Q^m_{mes}(t)],\ s \in I_m^+,\ \ \text{если}\ I_m^- = \varnothing,\ m \in I_q^f,$$

$$\sum_{s \in I_k^+} \varphi^{ks}(l^{ks}, t) - \sum_{s \in I_k^-} \varphi^{ks}(0,t) = 0,\ \ k \in I,$$

$$\psi^{k_i k}(l^{k_i k}, t) = \psi^{k k_j}(0,t),\ k_i \in I_k^+, k_j \in I_k^-, k \in I,$$

## The formulas for the functional gradient

$$grad_{q^{\bar{k}\bar{s}}} \Im(\xi, q) = \int_{D} \left\{ grad_{q^{\bar{k}\bar{s}}} \Phi(\xi, q; \gamma) + 2\varepsilon(q^{\bar{k}\bar{s}}(t) - \tilde{q}^{\bar{k}\bar{s}}(t)) \right\} \mu_D(\gamma) d\gamma, \quad (13)$$

$$grad_{\xi^{\bar{k}\bar{s}}} \Im(\xi, q) = \int_{D} \left\{ grad_{\xi^{\bar{k}\bar{s}}} \Phi(\xi, q; \gamma) + 2\varepsilon(\xi^{\bar{k}\bar{s}} - \hat{\xi}^{\bar{k}\bar{s}}) \right\} \mu_D(\gamma) d\gamma, \quad (14)$$

$$grad_{q^{\bar{k}\bar{s}}} \Im(\xi, q) = \int_{D} \left\{ c^2 \frac{\rho}{S^{\bar{k}\bar{s}}} \psi^{\bar{k}\bar{s}}(\xi^{\bar{k}\bar{s}}, t; \gamma) + 2\varepsilon(q^{\bar{k}\bar{s}}(t) - \tilde{q}^{\bar{k}\bar{s}}(t)) \right\} \mu_D(\gamma) d\gamma, \; t \in [t_0, T], \quad (15)$$

$$grad_{\xi^{\bar{k}\bar{s}}} \Im(\xi, q(t)) = \int_{D} \left\{ c^2 \frac{\rho}{S^{\bar{k}\bar{s}}} \int_{t_0}^{T} q^{\bar{k}\bar{s}}(t) (\psi^{\bar{k}\bar{s}}(x,t))'_x \big|_{x=\xi^{\bar{k}\bar{s}}} dt + 2\varepsilon_2(\xi^{\bar{k}\bar{s}} - \hat{\xi}^{\bar{k}\bar{s}}) \right\} \mu_D(\gamma) d\gamma \quad (16)$$

## *The results of numerical experiments*

**Problem 1.** We will consider the following specially constructed test problem for oil pipeline network consisting of 5 nodes, as shown in figure 3. Here

$$N = 6, M = 5, I^{f}\{1,3,4,6\}, N_{f} = 4, N_{\text{int}} = 2.$$

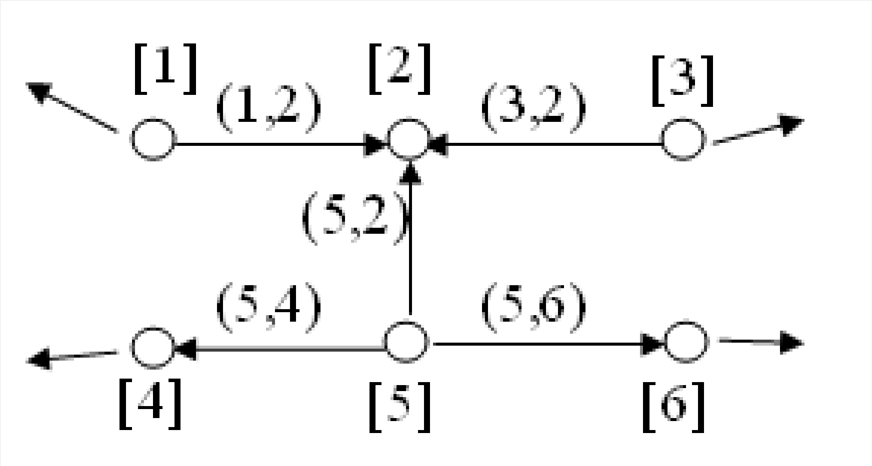



Fig.3. The scheme of oil pipeline network with 5 nodes

There are no external inflows and outflows inside the network.

## *The results of numerical experiments*

We assume that in the coarse of 30 min we observe the process (mode of operation of pumping plants at the ends of the sections) of oil transportation with the kinematic viscosity $\nu = 1.5 \cdot 10^{-4} \ (m^2 / s)$ and density $\rho = 920 (kg / m^3)(2a = 0.017$ for case being considered; the sound velocity in oil is $1200 ( m / s))$ in the sections of pipeline of diameter 530 (mm), of the lengths of the segments:

$$l^{(1,2)} = 100 \ (\kappa м), l^{(5,2)} = 30 \ (\kappa м), l^{(3,2)} = 70 \ (\kappa м), l^{(5,4)} = 100 \ (\kappa м), l^{(5,6)} = 60 \ (\kappa м).$$

Let there was regime in the pipes at initial time instance $t = 0$ with the following values of pressure and flow rate in the pipes:

$$\hat{P}^{1,2}(x) = 2300000\text{-}5.8955x \text{ (Pa)}, \ \hat{P}^{5,2}(x) = 1745669\text{-}1.17393x \text{ (Pa)},$$
$$\hat{P}^{3,2}(x) = 1827844\text{-}1.677043x \text{ (Pa)}, \ \hat{P}^{5,4}(x) = 1827844\text{-}2.35786x \text{ (Pa)},$$
$$\hat{P}^{5,6}(x) = 1827844\text{-}0.94415x \text{ (Pa)}. \quad (17)$$

$$\hat{Q}^{1,2}(x) = 300 \ (m^3 / hour), \hat{Q}^{5,2}(x) = 200 \ (m^3 / hour), \hat{Q}^{3,2}(x) = 100 \ (m^3 / hour),$$
$$\hat{Q}^{5,4}(x) = 120 \ (m^3 / hour), \ \hat{Q}^{5,6}(x) = 80 \ (m^3 / hour), \quad (18)$$

## *The results of numerical experiments*

Let the oil flow rate at the ends of this pipeline section be defined by the functions:

$$\tilde{P}_0^1(t) = 2000000 + 300000 e^{-0.0003t} \text{ (Pa)}, \quad \tilde{P}_0^3(t) = 1900000 - 72156 e^{-0.0004t} \text{ (Pa)}, \quad (19)$$

$$\tilde{P}_l^4(t) = 1800000 - 66571 e^{-0.0007t} \text{ ( Pa)}, \quad \tilde{P}_l^6(t) = 1600000 + 86372 e^{-0.0002t} \text{ ( Pa)}.(20)$$

**Case 1.** On the assumption that the point of leakage is located at the point $\xi = 30\ (km)$ of the first section of pipeline network and the rate of leakage is determined by the function $q^{loss}(t) = 50 - 10e^{-0.0003t}\ (m^3/h)$, we solved the boundary-value problem (1)-(6) numerically and determined the numerical values of pressure at the ends of the section $P^n(t), n \in I_p^f$. Thereafter, with the aid of the probe of uniformly distributed random numbers these values were changed within _2% (to simulate the error of measurements) and used as the observed regimes of the process. The point and rate of leakage $\xi$, $q^{loss}(t)$ "forgotten" in this case.

## *The results of numerical experiments*

It is required to determine $\xi$, $q^{loss}(t)$ with the aid of the above-suggested method of solving problem (1)–(5),(7),(8),(10) . For this purpose, we used the method of the projection of conjugate gradients. The numerical solution of the boundary-value problem (1)–(5),(7) was made using the scheme of the sweep method introduced in [7], on the grids with the pitches $h_x = 10 м$ and $h_t = 100 (сек)$.

**Table1. The results of experiments for the case 1(1-st section)**

| $\xi_0$ (km) | $q_0^{loss}(t)$ $(m^3/hour)$ | $\xi_*$ (km) | $\Im_0$ | $\Im_*$ | Number of iter. |
|---|---|---|---|---|---|
| 60,00 | $90-10e^{-0.0003t}$ | 30,00 | 76,10 | $5,73\cdot 10^{-7}$ | 6 |
| 20,00 | $20-10e^{-0.0003t}$ | 29,99 | 16,70 | $1,26\cdot 10^{-7}$ | 5 |
| 90,00 | $30-10e^{-0.0003t}$ | 30,00 | 11,66 | $3,19\cdot 10^{-6}$ | 16 |
| 10,00 | $66+20e^{-0.0003t}$ | 29,99 | 42,48 | $1,85\cdot 10^{-6}$ | 14 |
| 45,68 | $66+20e^{-0.0003t}$ | 29,99 | 57,63 | $7,43\cdot 10^{-7}$ | 8 |

## *The results of numerical experiments*

**Table 2 The results of experiments for the case 1 (section (1, 2))**

| $\eta$ % | | $(\xi^{(1,2)}, q^{(1,2)}(t))_1^0$ | $(\xi^{(1,2)}, q^{(1,2)}(t))_2^0$ | $(\xi^{(1,2)}, q^{(1,2)}(t))_3^0$ | $(\xi^{(1,2)}, q^{(1,2)}(t))_4^0$ | $(\xi^{(1,2)}, q^{(1,2)}(t))_5^0$ |
|---|---|---|---|---|---|---|
| 0 | $\Im_0$ | 76,104 | 16,704 | 11,664 | 42,48 | 57,636 |
| | $\Im_*$ | $5{,}73 \cdot 10^{-7}$ | $1{,}26 \cdot 10^{-7}$ | $3{,}19 \cdot 10^{-6}$ | $1{,}85 \cdot 10^{-6}$ | $7{,}43 \cdot 10^{-7}$ |
| | $\tilde{\xi}^{(1,2)}$ | 30,003 | 29,998 | 30,008 | 29,994 | 29,998 |
| | $\delta\xi^{(1,2)}$ | 0,00009 | 0,00006 | 0,0003 | 0,0002 | 0,00006 |
| | $\delta q^{(1,2)}$ | 0,0003 | 0, 0006 | 0,0003 | 0,0002 | 0,0003 |
| 0,5 | $\Im_0$ | 76,352 | 16,837 | 11,683 | 42,148 | 57,732 |
| | $\Im_*$ | 0,023 | 0,014 | 0,024 | 0,017 | 0,020 |
| | $\tilde{\xi}^{(1,2)}$ | 29,841 | 30,332 | 30,068 | 29,654 | 29,796 |
| | $\delta\xi^{(1,2)}$ | 0,005 | 0,011 | 0,002 | 0,011 | 0,006 |
| | $\delta q^{(1,2)}$ | 0,043 | 0,030 | 0,049 | 0,041 | 0,042 |
| 1 | $\Im_0$ | 77,119 | 16,924 | 11,832 | 43,744 | 57,413 |
| | $\Im_*$ | 0,067 | 0,071 | 0,073 | 0,062 | 0,065 |
| | $\tilde{\xi}^{(1,2)}$ | 28,527 | 29,392 | 29,923 | 30,597 | 29,839 |
| | $\delta\xi^{(1,2)}$ | 0.049 | 0.020 | 0.002 | 0.020 | 0.005 |
| | $\delta q^{(1,2)}$ | 0.109 | 0.092 | 0.107 | 0.094 | 0.093 |

## *The results of numerical experiments*

Table 1 presents the obtained results of the minimization of functional (10) for different initial values of the identified parameters $(\xi,\ q^{loss}(t))^0$, as well as the required number of iterations (one-dimensional minimizations) of the method of projection of conjugate gradients.

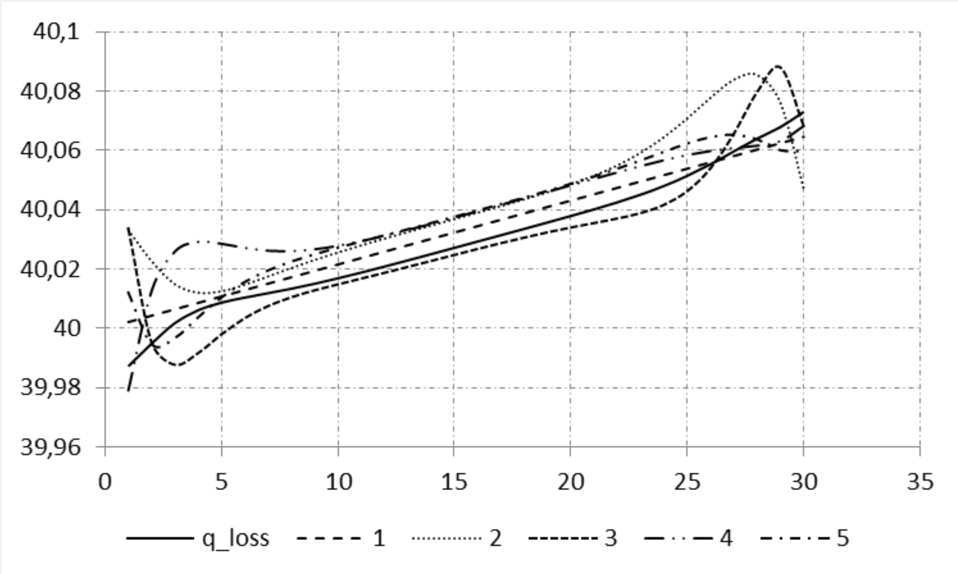



**Fig.1** The exact and experimental time dependences of raw material leakage.

## *The results of numerical experiments*

Then we will assume that the point of leakage is located on the fifth section. The results of numerical experiments are given on the table 3.

**Table 3 The results of experiments for the case 1 (5-th section)**

| $\xi_0$ $(km)$ | $q_0^{loss}(t)$ $(m^3/hour)$ | $\xi_*$ $(km)$ | $\Im_0$ | $\Im_*$ | Number of iter. |
|---|---|---|---|---|---|
| 45,68 | $66+20e^{-0.0003t}$ | 30,00 | 813,93 | $3,19\cdot10^{-6}$ | 57 |
| 20,00 | $30+10e^{-0.0003t}$ | 29,95 | 1,19 | $5,57\cdot10^{-5}$ | 35 |
| 35,00 | $10+10e^{-0.0003t}$ | 30,30 | 117,08 | $2,19\cdot10^{-4}$ | 22 |

**Case 2.** Now on the assumption that the point of leakage is located at the point $\xi=15\ (km)$ of the forth section of the pipeline network, with the rate of leakage is determined by the function $q^{loss}(t)=10+10e^{-0.0003t}\ (m^3/h)$. The results of numerical experiments are given on the table 3.

## *The results of numerical experiments*

**Table 4 The results of experiments for the case 2 (4-th section)**

| $\xi_0$ (*km*) | $q_0^{loss}(t)\ (m^3/hour)$ | $\xi_*$ (*km*) | $\Im_0$ | $\Im_*$ | Number of iter. |
|---|---|---|---|---|---|
| 25,00 | $5+10e^{-0.0003t}$ | 15,22 | 2,556 | $3{,}55\cdot 10^{-5}$ | 165 |
| 10,00 | $30+10e^{-0.0003t}$ | 14,78 | 44,615 | $3{,}55\cdot 10^{-5}$ | 204 |
| 22,56 | $15+10e^{-0.0003t}$ | 15,22 | 3,467 | $3{,}59\cdot 10^{-5}$ | 197 |

**Problem 2.** On the assumption that there are two points of leakage which are located at $\xi_{(1,2)}=50\ (km)$ and $\xi_{(5,4)}=60\ (km)$ of the first and forth sections of the pipeline network, with the rates of leakages $q_{(1,2)}^{loss}(t)=30-10e^{-0.0003t}\ (m^3/h)$ and $q_{(5,4)}^{loss}(t)=40-10e^{-0.0003t}\ (m^3/h)$. The results of numerical experiments are given on the table 4.

# *The results of numerical experiments*

**Table 5. The results of experiments for the problem 2**

| $(\xi^0_{(1,2)};\xi^0_{(5,4)})$ *km* | $(q^{loss}_{(1,2)}(t);q^{loss}_{(5,4)}(t))^0$ $(m^3/h)$ | $(\xi^*_{(1,2)};\xi^*_{(5,4)})$ *km* | $\Im_0$ | $\Im_*$ | **Number of iter.** |
|---|---|---|---|---|---|
| (40,00; 48,00) | $(80-10e^{-0.0003t}$ ; $70-10e^{-0.0003t}$ ) | (50,67; 60,37) | 80,16 | $0,1\cdot 10^{-4}$ | 35 |
| (80,00; 40,00) | $(40+10e^{-0.0003t}$ ; $50+10e^{-0.0003t}$ ) | (49,79; 59,29) | 40,36 | $0,11\cdot 10^{-3}$ | 7 |
| (60,00; 70,00) | $(60-10e^{-0.0004t}$ $90-10e^{-0.0002t}$ ) | (49,45; 59,58) | 98,86 | $3,48\cdot 10^{-5}$ | 65 |
| (35,00; 45,00) | (80; 80) | (49,97; 59,79) | 148,48 | $2,54\cdot 10^{-5}$ | 58 |
| (25,00; 35,00) | (80; 10) | (50,50; 60,40) | 107,29 | $3,59\cdot 10^{-5}$ | 64 |

## *The results of numerical experiments*

**Problem 3.**

We will assume that there is a point of leakage located at <u>unknown</u> section of the pipeline network. But we will assume that there is a point of leakage $\xi = 30\ (km)$ located at the third section of the pipeline network with the rate $q^{loss}(t) = 30 + 10e^{-0.0003t}\ (m^3/h)$, to formulate the test observed values of the rates of leakages on the ends of network.

These computations are carried out for all sections, beginning from the first section, on the assumption that the point of leakage is located at same section. The functional gets its minimal value at the section with number 3, as expected. The results of numerical experiments are given on the table 5.

# *The results of numerical experiments*

**Table 6. The results of experiments for the problem 3**

| $\xi_0$ $(km)$ | $q_0^{loss}(t)$ $(m^3/h)$ | Number of sections | $\xi_*$ $(km)$ | $\Im_0$ | $\Im_*$ | Number of iter. |
|---|---|---|---|---|---|---|
| 25000 | $15+10e^{-0.0003t}$ | 1 | 59,90 | 43,27 | 39,61 | 6 |
| | | 2 | 22,79 | 139,53 | 60,38 | 58 |
| | | 3 | 29,97 | 10,18 | $1{,}22\cdot 10^{-5}$ | 12 |
| | | 4 | 22,43 | 54,18 | 37,95 | 79 |
| | | 5 | 29,96 | 72,39 | 20,62 | 185 |
| 5000 | $5+10e^{-0.0003t}$ | 1 | 59,86 | 46,83 | 39,61 | 5 |
| | | 2 | 23,05 | 84,13 | 60,40 | 5 |
| | | 3 | 29,93 | 32,14 | $2{,}44\cdot 10^{-5}$ | 12 |
| | | 4 | 18,98 | 38,62 | 37,72 | 150 |
| | | 5 | 29,47 | 21,78 | 20,62 | 72 |

## *The results of numerical experiments*

The graphics for the rate of the leakages shown in the figure 2, these graphics are same for both experiments.

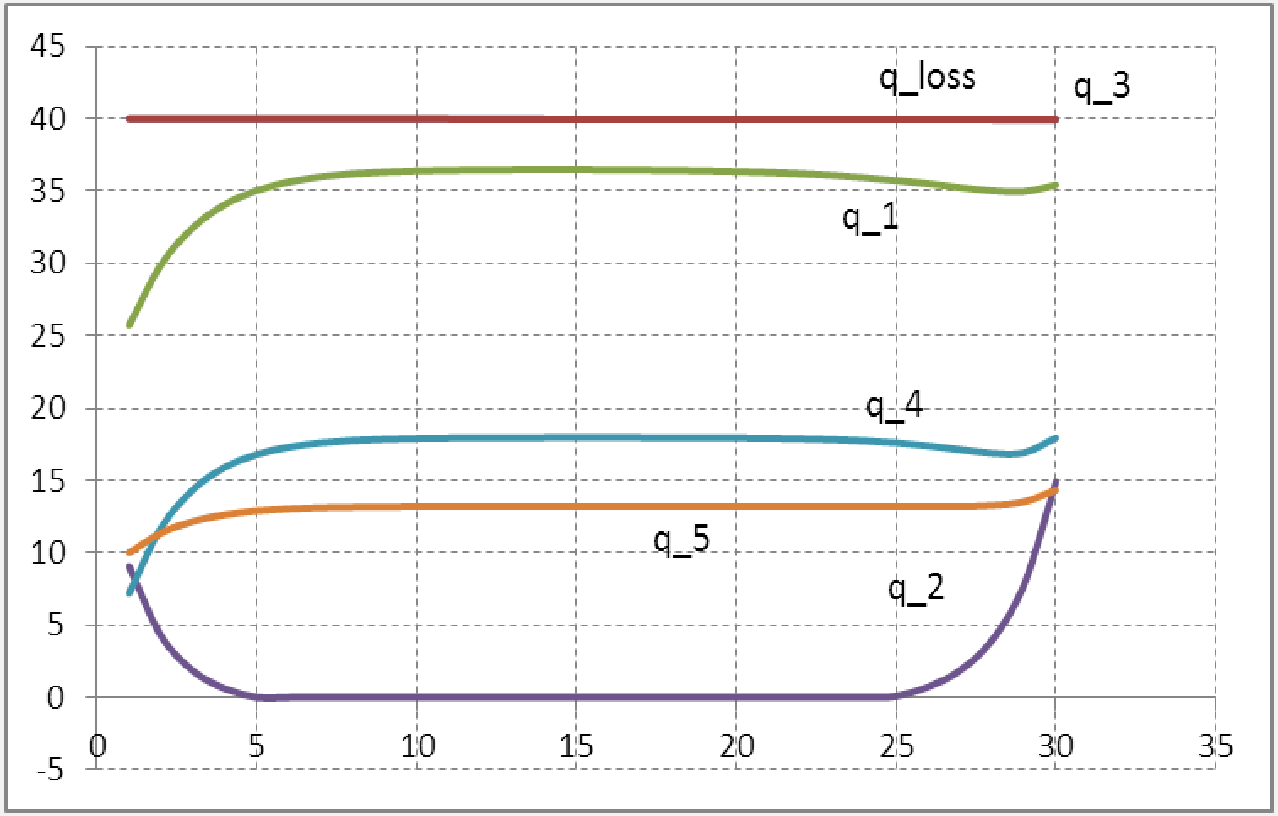



Fig. 2. The graphic of th rate of leakages

## *The results of numerical experiments*

$$300 \le \hat{Q}_0^{(1,2)n}(x) \le 320; \quad 1.7\cdot 10^6 \le \hat{P}_0^{(1,2)n}(x) \le 2.7\cdot 10^6,$$
$$200 \le \hat{Q}_0^{(5,2)n}(x) \le 220; \quad 1.6\cdot 10^6 \le \hat{P}_0^{(5,2)n}(x) \le 2.1\cdot 10^6,$$
$$100 \le \hat{Q}_0^{(3,2)n}(x) \le 120; \quad 1.6\cdot 10^6 \le \hat{P}_0^{(3,2)n}(x) \le 2.2\cdot 10^6, \qquad (21)$$
$$120 \le \hat{Q}_0^{(5,4)n}(x) \le 140; \quad 1.6\cdot 10^6 \le \hat{P}_0^{(5,4)n}(x) \le 2.2\cdot 10^6,$$
$$80 \le \hat{Q}_0^{(5,6)n}(x) \le 100; \quad 1.7\cdot 10^6 \le \hat{P}_0^{(5,6)n}(x) \le 2.2\cdot 10^6,$$

$$h_N = 20/N$$

$$D_N = \{(\hat{Q}_0^n(x), \hat{P}_0^n(x)):$$
$$\hat{Q}_0^{(1,2)n}(x) = 300 + nh_N, \hat{P}_0^{(1,2)n}(x) = 2.3\cdot 10^6 - 5.8955x + 2\cdot 10^4 nh_N,$$
$$\hat{Q}_0^{(5,2)n}(x) = 200 + nh_N, \hat{P}_0^{(5,2)n}(x) = 1.7\cdot 10^6 - 1.174x + 2\cdot 10^4 nh_N,$$
$$\hat{Q}_0^{(3,2)n}(x) = 100 + nh_N, \hat{P}_0^{(3,2)n}(x) = 1.8\cdot 10^6 - 1.67x + 2\cdot 10^4 nh_N, \qquad (22)$$
$$\hat{Q}_0^{(5,4)n}(x) = 120 + nh_N, \hat{P}_0^{(5,4)n}(x) = 1.8\cdot 10^6 - 2.358x + 2\cdot 10^4 nh_N,$$
$$\hat{Q}_0^{(5,6)n}(x) = 80 + nh_N, \hat{P}_0^{(5,6)n}(x) = 1.8\cdot 10^6 - 0.944x + 2\cdot 10^4 nh_N, x \in [0,l], n = \overline{1,N}\}$$

# *The results of numerical experiments*

**Table 7 The results of experiments at $N = 3$**

| ${\xi^{(1,2)}}^0$ | ${q^{(1,2)}}^0$ | $\tilde{\xi}^{(1,2)}$ | $\Im_0$ | $\Im_*$ | $\delta\xi^{(1,2)}$ | $\delta q^{(1,2)}$ |
|---|---|---|---|---|---|---|
| 60 | $90-10e^{-0.0003t}$ | 30,145 | 155,248 | $8,2\cdot10^{-4}$ | 0,004 | 0,0005 |
| 20 | $20-10e^{-0.0003t}$ | 29,996 | 22,323 | $2,7\cdot10^{-4}$ | 0,0001 | 0,0004 |
| 90 | $30-10e^{-0.0003t}$ | 30,190 | 22,323 | $1,0\cdot10^{-3}$ | 0,006 | 0,0001 |
| 10 | $66+20e^{-0.0003t}$ | 30,022 | 84,303 | $2,2\cdot10^{-4}$ | 0,0007 | 0,0005 |
| 45,685 | $66+20e^{-0.0003t}$ | 30,029 | 118,809 | $7,4\cdot10^{-5}$ | 0,0009 | 0,0005 |

# *The results of numerical experiments*

**Table 8 The results of experiments at** $N = 5$

| $\xi^{(1,2)^0}$ | $q^{(1,2)^0}$ | $\tilde{\xi}^{(1,2)}$ | $\Im_0$ | $\Im_*$ | $\delta\xi^{(1,2)}$ | $\delta q^{(1,2)}$ |
|---|---|---|---|---|---|---|
| 60 | $90 - 10e^{-0.0003t}$ | 29,963 | 155,390 | $1{,}6 \cdot 10^{-4}$ | 0.001 | 0,0006 |
| 20 | $20 - 10e^{-0.0003t}$ | 30,093 | 33,608 | $3{,}9 \cdot 10^{-4}$ | 0.003 | 0,0003 |
| 90 | $30 - 10e^{-0.0003t}$ | 30,100 | 22,443 | $4{,}3 \cdot 10^{-4}$ | 0.003 | 0,0005 |
| 10 | $66 + 20e^{-0.0003t}$ | 29,985 | 84,086 | $2{,}8 \cdot 10^{-4}$ | 0.0005 | 0,0004 |
| 45,685 | $66 + 20e^{-0.0003t}$ | 29,995 | 118,854 | $6{,}4 \cdot 10^{-5}$ | 0.0001 | 0,0002 |

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***Thank you for attention!***