On perturbations of one-parameter semigroups determined by covariant operator valued measures on the half-axis

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A one-parameter family of contractions $T_t : X \to X,\ t \geq 0$, acting on a Banach space X is said to be a semigroup if $T_{t+s} = T_t T_s,\ t, s \geq 0$, and $T_0 = I$ (the identical transformation). If orbits of the semigroup $T = (T_t)$ are continuous in some topology then there exists a linear operator $\mathcal{L}$ with the domain $\mathcal{D}(\mathcal{L})$ dense in X in the same topology such that $T_t = exp(t\mathcal{L}),\ t \geq 0$. The operator $\mathcal{L}$ is said to be a generator of the semigroup T [1]. For the case $X = B(H)$ (the algebra of all bounded operators in a Hilbert space H) a perturbation of $\breve{T}$ of the semigroup T can be defined as a solution to the integral equation [2]

$$\breve{T}_t - \int\limits_0^t \mathcal{M}(ds)\breve{T}_{t-s} = T_t,\ t \geq 0,$$

where $\mathcal{M}$ is a measure on the half-axis taking values in the set of all completely positive maps on the algebra $B(H)$. To define a semigroup the measure $\mathcal{M}$ should be covariant with respect to the action of T in the sense

$$T_r \circ \mathcal{M}([t, s)) = \mathcal{M}([t + r, s + r)),\ r \geq 0,\ s \geq t \geq 0.$$

Let us go back to an arbitrary Banach space X. If we consider a perturbation of the generator $\mathcal{L}$ by a bounded operator Δ in X, then the operator $\breve{\mathcal{L}} = \mathcal{L} + \Delta$ having the same domain $\mathcal{D}(\breve{\mathcal{L}}) = \mathcal{D}(\mathcal{L})$ is a generator of the semigroup that is a solution to the integral equation determined by the covariant measure

$$\mathcal{M}([t, s)) = \int\limits_t^s T_r \Delta dr,\ s, t \geq 0.$$

More complicated cases that lead to a change of the domain of a generator are defined by non-trivial cohomologies of T. We consider two examples in which a crucial role is played by the semigroup $S = (S_t)$ consisting of right shifts in the Hilbert space $H = L^2(\mathbb{R}_+)$. In one example, perturbations of the semigroup S are introduced in [3-4]. The second example determines the construction of perturbation for the semigroup $T = (T_t)$ acting in $X = B(F(H))$ by the formula [5]

$$T_t(x) = \hat{S}_t x \hat{S}_t^*,\ x \in B(F(H)),$$

where $\hat{S}_t$ acts in the antisymmetric Fock space $F(H) = \{\mathbb{C}\Omega\} \oplus H \oplus H^{\otimes_a^2} \oplus \dots \oplus H^{\otimes_a^n} \oplus \dots$ over one-particle Hilbert space $H = L^2(\mathbb{R}_+)$ by the formula

$$\hat{S}_t\Omega = \Omega,\ \hat{S}_t(f_1 \Lambda f_2 \Lambda \dots \Lambda f_n) = S_t f_1 \Lambda S_t f_2 \Lambda \dots \Lambda S_t f_n,\ t \geq 0,\ f_j \in H.$$

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