Functional Analysis Seminar

Local and 2-local derivations and automorphisms of Octonian algebras

Shavkat Ayupov, Karimbergen Kudaybergenov

V.I.Romanovskiy Institute of Mathematics, Uzbekistan Academy of Sciences

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Outline

What is a derivation?

In mathematics, a derivation is a mapping on an algebra A which captures fundamental features of the classical derivative. Let $A = C^\infty$ be the set of all infinitely differentiable functions on $(-\infty, \infty)$. The derivative operator $\frac{d}{dt} : C^\infty \to C^\infty$ $\left(\frac{d}{dt} : f \mapsto f'\right)$ is a fundamental prototype of a derivation.

Definition 1.1

Let A be an algebra. A derivation $D : A \to A$ is a linear mapping satisfying the Leibniz rule, i.e.,

$$D(xy) = D(x)y + xD(y),\ x, y \in A.$$

Local derivations

A linear mapping $\Delta : A \to A$ is said to be a local derivation, if for every $x \in A$ there exists a derivation D_x on A (depending on x) such that $\Delta(x) = D_x(x)$.

This notion was introduced and investigated independently by R.V. Kadison [a] and D.R. Larson and A.R. Sourour[b]. The above papers gave rise to a series of works devoted to the description of mappings which are close to automorphisms and derivations of C^*-algebras and operator algebras. R.V. Kadison set out a program of study for local maps, suggesting that local derivations could prove useful in building derivations with particular properties.

[a] R.V. Kadison, Local derivations, *J. Algebra*, **130** (1990) 494-509.

[b] D. R. Larson, A. R. Sourour, Local derivations and local automorphisms of $B(X)$, *Proc. Sympos. Pure Math.* **51** (1990) 187-194.

Local derivations

R.V.Kadison proved that each continuous local derivation of a von Neumann algebra M into a dual Banach M-bimodule is a derivation. This theorem gave way to studies on derivations on C^*-algebras, culminating with a result due to B.E. Johnson, which asserts that every local derivation of a C^*-algebra A into a Banach A-bimodule is automatically continuous, and hence is a derivation.[a] For details we refer to the papers[b][c].

[a] B.E. Johnson, Local derivations on C^*-algebras are derivations, *Transactions of the American Mathematical Society.* **353** (2001), 313–325.

[b] Sh. A. Ayupov, K. K. Kudaybergenov, 2-local derivation on von Neumann algebras. *Positivity*, **19** (2015), 445-455.

[c] Sh. A. Ayupov, K. K. Kudaybergenov, A. M. Peralta, A survey on local and 2-local derivations on C^*-algebras and von Neumann algebras, in Topics in Functional Analysis and Algebra, *Contemporary Mathematics AMS*, **672** (2016) 73-126.

2-Local derivations

In 1997, P. Šemrl[a] introduced the concepts of 2-local derivations. Recall that a mapping $\Delta : A \to A$ (not necessary linear) is said to be a 2-local derivation, if for every pair $x, y \in A$ there exists a derivation $D_{x,y} : A \to A$ (depending on x, y) such that $\Delta(x) = D_{x,y}(x), \Delta(y) = D_{x,y}(y)$.

P. Šemrl have described 2-local derivations on the algebra $B(H)$ of all bounded linear operators on the infinite-dimensional separable Hilbert space H, by proving that every 2-local derivation on $B(H)$ is a derivation. A detailed discussion of 2-local derivations on operator algebras can be found in the survey [b].

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[a] P. Šemrl, Local automorphisms and derivations on $B(H)$, *Proc. Amer. Math. Soc.*, **125** (1997) 2677-2680.

[b] Sh. A. Ayupov, K. K. Kudaybergenov, A. M. Peralta, A survey on local and 2-local derivations on C^*-algebras and von Neumann algebras, in Topics in Functional Analysis and Algebra, *Contemporary Mathematics AMS*, **672** (2016) 73-126.

Outline

Local automorphisms

Let A be an algebra (not necessary associative). Recall that a linear bijection $\Phi : A \to A$ is said to be an automorphism, if $\Phi(xy) = \Phi(x)\Phi(y)$ for all $x, y \in A$. A linear mapping Δ is said to be a local automorphism, if for every $x \in A$ there exists an automorphism Φ_x on A (depending on x) such that $\Delta(x) = \Phi_x(x)$.

The latter notion was introduced D.R. Larson and A.R. Sourour[a]. They proved that if $A = B(X)$, the algebra of all bounded linear operators on a Banach space X, then every invertible local automorphism of A is an automorphism. Thus automorphisms on $B(X)$ are completely determined by their local actions.

[a] D. R. Larson, A. R. Sourour, Local derivations and local automorphisms of $B(X)$, *Proc. Sympos. Pure Math.* **51** (1990) 187-194.

2-Local automorphisms

In 1997, P. Semrl also introduced the concept 2-local automorphisms.

Recall that a mapping $\Delta : A \to A$ (not necessary linear) is said to be a 2-local automorphism, if for every pair $x, y \in A$ there exists an automorphism $\Phi_{x,y} : A \to A$ (depending on x, y) such that $\Delta(x) = \Phi_{x,y}(x)$, $\Delta(y) = \Phi_{x,y}(y)$.[a]

[a] P. Šemrl, Local automorphisms and derivations on $B(H)$, *Proc. Amer. Math. Soc.*, **125** (1997) 2677-2680.

Outline

A Gleason-Kahane-Żelazko theorem

The Gleason-Kahane-Żelazko theorem[a][b] a fundamental contribution in the theory of Banach algebras, asserts that every unital linear functional F on a complex unital Banach algebra A such that, $F(a)$ belongs to the spectrum, $\sigma(a)$, of a for every $a \in A$, is multiplicative. In the above terminology, this is equivalent to say that every unital linear local homomorphism from a unital complex Banach algebra A into C is multiplicative.

[a] A.M. Gleason, A characterization of maximal ideals, *J. Analyse Math.* **19**, 171-172 (1967)

[b] J.P. Kahane, W. Żelazko, A characterization of maximal ideals in commutative Banach algebras, *Studia Math.* **29**, 339-343 (1968)

A Kowalski-Słodkowski theorem

After the Gleason-Kahane-Żelazko theorem was established, Kowalski and Słodkowski[a] showed that at the cost of requiring the local behavior at two points, the condition of linearity can be dropped, that is, suppose A is a complex Banach algebra (not necessarily commutative nor unital), then every (not necessarily linear) mapping $T : A \to C$ satisfying $T(0) = 0$ and $T(x-y) \in \sigma(x-y)$, for every $x, y \in A$, is multiplicative and linear.

[a] S.Kowalski, Z. Słodkowski, A characterization of multiplicative linear functionals in Banach algebras, *Studia Math.* **67**, 215-223 (1980)

A Kowalski-Słodkowski theorem

According to the above notation, the result established by Kowalski and Słodkowski proves that every (not necessarily linear) 2-local homomorphism T from a (not necessarily commutative nor unital) complex Banach algebra A into the complex field C is linear and multiplicative. Consequently, every (not necessarily linear) 2-local homomorphism T from A into a commutative C^*-algebra is linear and multiplicative.

Local mappings of non-associative algebras

In recent years non-associative analogues of classical constructions become of interest in connection with their applications in many branches of mathematics and physics. The notions of local and 2-local derivations (automorphisms) have also become popular for some nonassociative algebras such as Lie and Leibniz algebras. The main problems concerning these notions are to find conditions under which every local (or 2-local) automorphism or derivation automatically becomes an automorphism (respectively, a derivation), and also to present examples of algebras with local or 2-local automorphisms (respectively, derivations) that are not automorphisms (respectively, derivations).

Discussions with professor E.Zelmanov

The investigation of local maps on non-associative algebras came out from discussions with professor E.Zelmanov (University of California, San Diego) during USA–Uzbekistan Conference held at the California State University, Fullerton, on May, 2014. Later he suggested to consider the case of octonians during the First China – Central Asia Joint Meeting in Mathematics September 15–20, 2019, Sichuan University, Chengdu, China. The authors gratefully acknowledges Efim Zelmanov for those and some recent discussions.

Local derivations

Let us present a list of finite or infinite dimnensional algebras for which all local derivations are derivation:

- von Neumann algebras, in particular, the algebra $M_n(C)$ of all square matrices of order n over the field of complex numbers[a];
- the complex polynomial algebra $C[x]$[b];
- finite dimensional semisimple Lie algebras over a algebraically closed field of the characteristic zero[c].

[a] R.V. Kadison, Local derivations, *J. Algebra*, **130** (1990) 494-509.
[b] R.V. Kadison, Local derivations, *J. Algebra*, **130** (1990) 494-509.
[c] Sh. A. Ayupov, K. K. Kudaybergenov, Local derivations on finite dimensional Lie algebras, *Linear Algebra Appl.*, **493** (2016) 381-398.

Local derivations

- finite dimensional simple Leibniz algebras over a algebraically closed field of the characteristic zero[a];
- infinite dimensional Witt algebras over a algebraically field of the characteristic zero[b];
- Witt algebras over a field of the prime characteristic zero[c].

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[a] Sh.A. Ayupov, K.K. Kudaybergenov, B.A. Omirov, Local and 2-local derivations and automorphisms on simple Leibniz algebras, *Bull. Malays. Math. Sci. Soc.* **43** (2020) 2199-2234.

[b] Y. Chen, K. Zhao, Y. Zhao, Local derivations on Witt algebras, *Linear and multilinear algebra*, https://doi.org/10.1080/03081087.2020.1754750.

[c] Y. F. Yao, Local derivations on the Witt algebra in prime characteristic, *Linear and multilinear algebra*, https://doi.org/10.1080/03081087.2020.1819189.

On the other hand, algebras close to nilpotent algebras, as a rule, admit pure local derivations, that is, local derivations that are not derivations. Below a short list of some classes of algebras which admits pure local derivations:

- the algebra $C(x)$ of rational functions[a];
- finite dimensional filiform Lie algebras[b].

[a] R.V. Kadison, Local derivations, *J. Algebra*, **130** (1990) 494-509.

[b] Sh. A. Ayupov, K. K. Kudaybergenov, Local derivations on finite dimensional Lie algebras, *Linear Algebra Appl.*, **493** (2016) 381-398.

Local derivations which are not derivations

- finite dimensional filiform Leibniz algebras[a];
- solvable Leibniz algebras with abelian nilradicals, which have one dimensionial complementary space[b];
- an algebra of niltriangular $n \times n$-matrices[c].

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[a] Sh.A. Ayupov, K.K. Kudaybergenov, B.A. Omirov, Local and 2-local derivations and automorphisms on simple Leibniz algebras, *Bull. Malays. Math. Sci. Soc.* **43** (2020) 2199-2234.

[b] Sh.A. Ayupov, A. Khudoyberdiyev, B. Yusupov, Local and 2-local derivations of solvable Leibniz algebras. *Internat. J. Algebra Comput.* **30** (2020), no. 6, 1185-1197.

[c] A.P. Elisova, I.N. Zotov, V.M. Levchuk, G.S. Suleimanova, Local automorphisms and local derivations of nilpotent matrix algebras, *The Bulletin of Irkutsk State University. Series Mathematics*, **4**:1 (2011), 9-19.

2-Local derivations

Now let us present a list of finite or infinite dimnensional algebras for which all 2-local derivations are derivation:

- von Neumann algebras, in particular, the algebra $M_n(C)$ of all square matrices of order n over the field of complex numbers[a][b][c];
- finite dimensional semisimple Lie algebras over a algebraically closed field of the characteristic zero[d].

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[a] P. Šemrl, Local automorphisms and derivations on $B(H)$, *Proc. Amer. Math. Soc.*, **125** (1997) 2677-2680.

[b] S.O. Kim, J.S. Kim, Local automorphisms and derivations on M_n, Proc. Amer. Math. Soc. 132 (2004) 1389–1392.

[c] Sh.A.Ayupov, K.K. Kudaybergenov, 2-local derivations on von Neumann algebras. Positivity **19** (2015), no. 3, 445–455.

[d] Sh. A. Ayupov, K. K. Kudaybergenov, I.Rakhimov, 2-Local derivations on finite dimensional Lie algebras, *Linear Algebra Appl.*, **474** (2015) 1-11.

2-Local derivations

- finite dimensional simple Leibniz algebras over a algebraically closed field of the characteristic zero[a];
- infinite dimensional Witt algebras over a algebraically field of the characteristic zero[bc].

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[a] Sh.A. Ayupov, K.K. Kudaybergenov, B.A. Omirov, Local and 2-local derivations and automorphisms on simple Leibniz algebras, *Bull. Malays. Math. Sci. Soc.* **43** (2020) 2199-2234.

[b] Sh.A. Ayupov, K.K. Kudaybergenov, B.Yusupov, 2-Local derivations on generalized Witt algebras, *Linear and multilinear algebra*, doi/full/10.1080/03081087.2019.1708846

[c] Y. Zhao, Y. Chen and K. Zhao, 2-Local derivations on Witt algebras, *Journal of Algebra and Its Applications*, doi.org/10.1142/S0219498821500687.

2-Local derivations which are not derivations

Below a short list of some classes of algebras which admits pure 2-local derivations:

- an algebra of niltriangular 2×2-matrices;[a]
- finite dimensional nilpotent Lie and Leibniz algebras[b];
- p-filiform Leibniz algebras and some classes of solvable Leibniz algebras.[cd]

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[a] J.H. Zhang, H.X. Li, 2-Local derivations on digraph algebras, Acta Math. Sinica (Chin. Ser.) 49 (2006) 1401–1406.

[b] Sh.A. Ayupov, K.K. Kudaybergenov, B.A. Omirov, Local and 2-local derivations and automorphisms on simple Leibniz algebras, *Bull. Malays. Math. Sci. Soc.* **43** (2020) 2199-2234.

[c] Sh.A.Ayupov, K.K Kudaybergenov, B.B Yusupov, J. Math. Sci. 245 (2020), no. 3, 359–367.

[d] Sh.A.Ayupov, A.Khudoyberdiyev, B.B. Yusupov, Internat. J. Algebra Comput. 30 (2020), no. 6, 1185–1197.

Local automorphisms

In[a] it was proved that every local automorphism on the special linear Lie algebra sl_n is an automorphism or anti-automorphism. Further Constantini[b] extended the above result for an arbitrary simple Lie algebra g. These results can be reformulate as follows

$$LAut(g) = Aut(g) \ltimes \{\mathrm{id}_g, -\mathrm{id}_g\},$$

where id_g is the identical automorphism of g. In particular, $Aut(g)$ is a normal subgroup of $LAut(g)$.

[a] Ayupov Sh.A., Kudaybergenov K.K., Local automorphisms on finite-dimensional Lie and Leibniz algebras, *Algebra, complex analysis and plupotential theory*, 31–44, Springer Proc. Math. Stat., 264, Springer, 2018.

[b] M. Constantini, Local automorphisms of finite-dimensional simple Lie algebras, *Linear Algebra Appl.*, **562 (1)** (2019), 123-134.

Local automorphisms

In general, for a Lie algebra g such that $Aut(g)$ is a normal subgroup of $LAut(g)$, the quotient group $LAut(g)/Aut(g)$ measures how big is the set of local autmorphisms of the algebra g which are not automorphisms. From the above observation the following problems arise.

Let A be an arbitrary algebra.

- Is $Aut(A)$ a normal subgroup of $LAut(A)$? (e.g., this is true for simple Lie and Leibniz algebras);
- Describe the quotient group $LAut(A)/Aut(A)$; (e.g., it is Z_2 for simple Li algebras);
- Describe algebras for which the quotient group $LAut(A)/Aut(A)$ is trivial (e.g., this is true for non-Lie Leibniz algebras).

Outline

Let F be a field with the characteristic zero. Recall that an algebra A is said to be alternative, if

$$x(xy) = (xx)y, \ \ (xy)y = x(yy), \ \forall\, x, y \in A.$$

Let O_F be an octonion algebra over F, i.e., O_F is a 8-dimensional unital alternative algebra over F with a nondegenerate multiplicative quadratic form N, that is,

$N : O_F \to F$ is a mapping such that

i) $N(\lambda x) = \lambda^2 N(x)$ for all $\lambda \in F, x \in O_F$;

ii) the mapping $\langle \cdot, \cdot \rangle : O_F \times O_F \to F$ defined by

$$\langle x, y \rangle = \frac{1}{2}\left(N(x+y) - N(x) - N(y)\right)$$

is bilinear;

iii) $N(xy) = N(x)N(y)$ for all $x, y \in O_F$;

iv) if $\langle x, y \rangle = 0$ for all $y \in O_F$, then $x = 0$.

Octonion algebras

The octonion algebra O_F is said to be of type I if it has an orthonormal basis $\{1, e_1, \ldots, e_7\}$ such that $e_i^2 = -1$ for all i, and it is of type II otherwise.
We shall consider the octonion algebra O_F of type I with a unit $e_0 = 1$. The multiplication table in the algebra O_F is fully determined by the following conditions[a]:

$$e_i e_j = -e_j e_i, \quad i \neq j,$$

$$e_i e_{i+1} = e_{i+3 (mod\, 7)},$$

$$e_i e_j = e_k \Longrightarrow e_{\sigma(i)} e_{\sigma(j)} = e_{\sigma(k)},$$

for any cyclic permutation σ and $1 \leq i, j, k \leq 7$.

[a] Kuzmin E.N., Shestakov I.P., Non-associative structures. Algebra, VI, 197–280, Encyclopaedia Math. Sci., 57, Springer, Berlin, 1995.

Fano plane

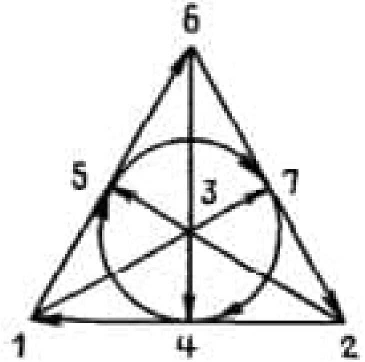

Fig. 1

Outline

- The space of all derivations $Der\left(O_F\right)$ of O_F equipped with the Lie bracket is isomorphic to the exceptional Lie algebra $g_2(F)$.[a][b]
- simple Lie algebras

$$sl_{n+1},\ so_{2n+1},\ sp_{2n},\ so_{2n},$$
$$g_2,\ f_4,\ e_6,\ e_7,\ e_8.$$

[a] N. Jacobson, Composition algebras and their automorphisms, *Rend. Circ. Mat. Palermo* **7** (1958) 55–80.

[b] J. Humphreys, Introduction to Lie algebras and representation theory, Graduate Text in Math. vol 255, 1972.

Outline

Some derivations of octonion algebras

We do not need in our proofs a general form of derivations on octonion algebras, but let us present only four derivations which we shall use.

For every pair $i \neq j \in \{1, \ldots, 7\}$ denote by $\Delta_{i,j}$ the linear mapping on O_F which acts on basis elements as:

$$\Delta_{ij}(e_i) = e_j,\ \Delta_{ij}(e_j) = -e_i,\ \Delta_{ij}(e_k) = 0, k \neq i, j. \tag{1}$$

Note that the following linear mappings are derivations:

$$\Delta_{12} + \Delta_{75},\ \Delta_{35} + \Delta_{76},\ \Delta_{36} + \Delta_{57},\ \Delta_{37} + \Delta_{65}. \tag{2}$$

Local derivations of octonion algebras

Theorem 3.1

Let O_R be a real octonion algebra. Then a linear mapping Δ on O_R is a local derivation if and only if its matrix is skew-symmetric with zeros in the first row and the first column. In particular, the space $LocDer(O_R)$ of all local derivations on O_R is closed under the Lie brackets. Moreover, $LocDer(O_R)$, when equipped with the Lie brackets, is isomorphic to the Lie algebra $so_7(R)$ of all real skew-symmetric 7×7-matrices.

Remark 3.2

As we mentioned above, octonion algebra O_R is a simple non associative algebra and the Lie algebra $Der\,(O_R)$, of all its derivations, is isomorphic to the 14-dimensional Lie algebra $g_2(R)$. The dimension of the Lie algebra $LocDer\,(O_R)$ is equal to 21. Therefore, as far as we know, the real octonion algebra O_R is the first example of simple non associative algebra, which admits pure local derivations, i.e. local derivations which are not derivations.

2-Local derivations of octonion algebras

At the same time, in the case of 2-local derivations on octonion algebras over an algebraically closed field, the picture is completely different.

Theorem 3.3

Let O_F be an octonian algebra over an algebraically closed field F of characteristic zero. Then any 2-local derivation on O_F is a derivation.

Remark 3.4

It should be noted that a description of local derivations on octonian algebras over an algebraically closed field, as well as a description of 2-local derivations on the real octonion algebra O_R remain as open problems.

Outline

Theorem 3.5

Let O_F be an octonion algebra over a field F of the characteristic zero and let Δ be a linear bijection on O_F. Then Δ is a local automorphism if and only if it leaves the unit fixed and its matrix is orthogonal. In particular, the group $LocAut\,(O_F)$ of all local automorphisms on O_F is isomorphic to the group $O(7, F)$ of all orthogonal 7×7-matrices over F.

Remark 3.6

It should be noted that the group of all automorphisms $Aut\,(\mathbb{O}_F)$ *of* $\mathbb{O}_F$ *is a Lie group of type* G_2 *which is isomorphic to the subgroup of the special orthogonal group* $SO(7, F)$ *of all orthogonal* 7×7*-matrices with determinant* 1. *This means that* $Aut\,(\mathbb{O}_F)$ *is a proper (but not normal) subgroup of* $LocAut\,(\mathbb{O}_F)$. *Therefore the octonian algebras over a field* F *of the characteristic zero always admit local automorphisms which are not automorphisms.*

Local derivations and automorphisms of octonion algebras

Remark 3.7

An octonion algebra $\mathbb{O}_F$ equipped with the multiplication

$$x \circ y = \frac{1}{2}(xy + yx)$$

becomes a Jordan algebra $\left(\mathbb{O}_F^+, \circ\right)$. *Using the description of the Lie algebra of derivations (group of automorphsims) of the algebra $\mathbb{O}_F^+$ and the above results we obtain the following isomorphisms (E.Zelmanov):*

$$LocDer(\mathbb{O}_{\mathbb{R}}) \cong Der(\mathbb{O}_{\mathbb{R}}^+) \cong so_7(\mathbb{R});$$
$$LocAut(\mathbb{O}_F) \cong Aut(\mathbb{O}_F^+) \cong O(7, F),$$

where $charF = 0$.

At the same time, in the case of 2-local automorphisms on octonion algebras, the picture is completely different.

Theorem 3.8

Let O_F be an octonian algebra over an algebraically closed field F of characteristic zero. Then any 2-local automorphism on O_F is an automorphism.

What for the real octonian algebra we have

Theorem 3.9

Let O_R be the real octonion algebra and let Δ be a bijection on O_R. Then Δ is a 2-local automorphism if and only if it is a linear mapping which leaves the unit fixed and its matrix is orthogonal. This means that a bijection on O_R is a 2- local automorphism if and only if it is a local automorphism. In particular, the groups of all 2-local automorphisms and local automorphisms on O_R are isomorphic to the group $O(7, F)$ of all orthogonal 7×7-matrices over F.

Recall that an algebra $(A, \cdot)$ over a field F is called a Malcev algebra[a] if its multiplication is anticommutative and satisfies the following identity:

$$J(x, y, xz) = J(x, y, z)x,$$

where $J(x, y, z) = (xy)z + (yz)x + (zx)y$ is the Jacobian of the elements x, y, z.
This class of algebras was first introduced by A. I. Malcev under the name of "Moufang-Lie algebras".
Let O_F be an octonian algebra over a field F and define a bracket $[\cdot, \cdot]$ by $[x, y] = \frac{1}{2}(xy - yx)$, $x, y \in O_F$. Set $M_7(F) = \{x \in O_F : t(x) = 0\}$, where t is the trace on O_F. Then $M_7(F)$ with the bracket $[\cdot, \cdot]$, is a simple Malcev algebra.

[a] A.I. Malcev, Analytic loops, *Mat. Sb.* **36 (78)** (1955) 569–575.

It is known[a] that every automorphism of an octonion algebra O_F defines an automorphism on $M_7(F)$, and conversely. More precisely, if Φ is an automorphism on O_F, then the restriction $\Phi|_{M_7(F)}$ is an automorphism on $M_7(F)$. Conversely, if Ψ is an automorphism on $M_7(F)$, then its extension onto O_F defined as

$$\Phi(\lambda e_0 + x) = \lambda e_0 + \Psi(x), \lambda \in F, x \in M_7(F),$$

is an automorphism on O_F. Therefore a similar correspondence between local (2-local) automorphisms of the octonion algebra O_F and the Malcev algebra $M_7(F)$ are also true.
Note that similar correspondence also true for (local, 2-local) derivations.

[a] A. Elduque, H. Ch. Myung, Mutations of alternative algebras. Kluwer Academic Publishers Group, Dordrecht, 1994.

After my talk on LieJor Seminar on 15.04.2021 in Sao-Paolo, we have discussions with Professor Alberto Elduque (University of Zaragoza, Spain) of the above problems concerning local and 2-local derivations (see Remark 3.4). We have obtained more general results which contain complete descriptions of such maps for octonions over arbitrary fields. In particular, we have the following picture which gives the solutions of the above problems.

Conclusion

- $$LocDer(O_F) \cong LocDer(M_7(F)) \cong so_7(F) \supset g_2(F) \cong Der(O_F) \cong Der(M_7(F));$$
  for all F with $charF = 0$;
- $$2LocDer(O_F) \cong 2LocDer(M_7(F)) \cong g_2(F) \cong Der(O_F) \cong Der(M_7(F)),$$
  where F – algebraically closed;
- $$2LocDer(O_R) = LocDer(O_R) \cong LocDer(M_7(R)) \cong so_7(R).$$

Conclusion

- $$LocAut(O_F) \cong LocAut(M_7(F) \cong O(7,F)) \geq G_2(F) \cong Aut(O_F) \cong Aut(M_7(F)$$

  for all F with $charF = 0$;

- $$2LocAut(O_F) \cong 2LocAut(M_7(F)) \cong G_2(F) \cong Aut(O_F) \cong Aut(M_7(F),$$

  where F – algebraically closed;

- $$2LocAut(O_R) \cong 2LocAut(M_7(R)) \cong LocAut(O_R) \cong O(7,R) \geq G_2(R) \cong Aut(O_R) \cong Aut(M_7(F).$$