The Holomorphic Extension of Functions with the Boundary Morera Properties in Domains with Piecewise-Smooth Boundary

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Preface

This report contains some results related to the holomorphic extension of functions given on the boundary of a bounded domain $D \subset \mathbb{C}^n$, $n > 1$, to this domain. Here we will discuss boundary multidimensional variants of the Morera theorem. It consists in the vanishing of integrals of a given function over the intersection of the boundary of the domain with complex lines (complex planes).

Preface

A domain D in $\mathbb{C}^n$ has a boundary ∂D of class $\mathcal{C}^k$ (we write $\partial D \in \mathcal{C}^k$), if

$$D = \{z \in \mathbb{C}^n : \rho(z) < 0\},$$

where ρ is the real-valued function of class $\mathcal{C}^k$ in some neighborhood of the closure of D, and the differential $d\rho \neq 0$ on ∂D.

If $k = 1$, then we say that D is a domain with a smooth boundary. We will call the function ρ a defining function for the domain D. The orientation of the boundary ∂D is induced by the orientation of D.

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Preface

A domain D with a piecewise-smooth boundary ∂D will be understood as a smooth polyhedron, that is, a domain of the form

$$D = \{z \in \mathbb{C}^n : \ \rho_j(z) < 0, \ j = 1, \dots, m\},$$

where the real-valued functions ρ_j are class $\mathcal{C}^1$ in some neighborhood of the closure $\overline{D}$, and for every set of distinct indices $j_1, \dots, j_s$ we have $d\rho_{j_1} \wedge \dots \wedge d\rho_{j_s} \neq 0$ on the set $\{z \in \mathbb{C}^n : \rho_{j_1}(z) = \dots = \rho_{j_s}(z) = 0\}$. It is well known that the Stokes's formula holds for such domains D and surfaces ∂D.

Functions with the Morera property

First, I will tell you some well-known facts about functions that satisfy the boundary Morera property.

Functions with the Morera property

Definition 1

We say that a continuous function f on ∂D ($f \in \mathcal{C}(\partial D)$) satisfies the *Morera property (condition)* along a complex plane l of dimension k, $1 \leqslant k \leqslant n-1$, if

$$\int\limits_{\partial D \cap l} f(\zeta)\beta(\zeta) = 0 \qquad (1)$$

for any differential form β of type $(k, k-1)$ with constant coefficients.

Functions with the Morera property

It is assumed that the plane l transversally intersects the boundary of the domain D.

If l is a complex line intersecting ∂D transversally, then the Morera property along l consists of the equality

$$\int_{\partial D \cap l} f(z+bt)dt = \int_{\partial D \cap l} f(z_1+b_1t, \dots, z_n+b_nt)dt = 0$$

for the given parameterization $\zeta = z + bt$ $(z, b \in \mathbb{C}^n,\ t \in \mathbb{C})$ of the complex line l.

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Functions with the Morera property

Clearly, the boundary values of functions $F \in \mathcal{A}(D)$ (i.e., functions holomorphic in D and continuous in the closure of the domain $\overline{D}$) satisfy this property. Moreover, the same is true for continuous CR-functions f on ∂D.

Functions with the Morera property

Recall that

Definition 2

A function $f \in \mathcal{C}(\partial D)$ is called a *CR*-function on ∂D if

$$\int_{\partial D} f(\zeta)\,\bar{\partial}\alpha(\zeta) = 0 \qquad (2)$$

for all exterior differential forms α of type $(n, n-2)$ with coefficients of class $\mathcal{C}^\infty$ in the $\overline{D}$.

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Functions with the Morera property

The Hartogs-Bochner theorem, which is now classical, tells us that any continuous function f on ∂D is a CR-function if and only if it is holomorphically extended into D up to a certain function $F \in \mathcal{A}(D)$ (the boundary of D is connected).

Functions with the Morera property

Globevnik and Stout considered the following inverse problem: let a function $f \in \mathcal{C}(\partial D)$ satisfy the Morera property (1) along any complex k-plane l intersecting ∂D transversally. Is it true that f is a CR-function on ∂D?

Functions with the Morera property

Obviously, the greater the dimension k of the complex plane, the weaker the Morera property along complex k-planes. Therefore, if the Morera property holds along all complex lines, so it does along all complex hyperplanes. The following theorem is the first sufficiently general assertion on the solution of this problem.

Functions with the Morera property

Theorem 3 (Globevnik, Stout)

Let $1 \leqslant k \leqslant n - 1$, and let a function $f \in \mathcal{C}(\partial D)$ satisfy the Morera property (1) along any complex k-plane l intersecting ∂D transversally, then f is a CR-function on ∂D (and, therefore, it is holomorphically continued to D by the Hartogs-Bochner theorem).

Functions with the Morera property

Next, we will consider one-dimensional complex lines l of the form

$$l = \{\zeta \in \mathbb{C}^n : \ \zeta_j = z_j + b_j t, \ j = 1, \dots, n, \ t \in \mathbb{C}\} \qquad (3)$$

passing through a point $z \in \mathbb{C}^n$ in the direction of a vector $b \in \mathbb{CP}^{n-1}$ (the direction of b is determined with an accuracy of up to multiplication by a complex number $\lambda \neq 0$).

By Sard's theorem, for almost all $z \in \mathbb{C}^n$ and almost all $b \in \mathbb{CP}^{n-1}$, the intersection $l \cap \partial D$ is a finite set of piecewise-smooth curves (except for the degenerate case where $\partial D \cap l = \varnothing$).

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Functions with the Morera property

We now formulate the assertion belonging to J. Globevnik and E. L. Stout (a particular case of Theorem 3).

Functions with the Morera property

Theorem 4 (Globevnik, Stout)

Let a function $f \in \mathcal{C}(\partial D)$, and for almost all $z \in \mathbb{C}^n$ and almost all $b \in \mathbb{CP}^{n-1}$, let

$$\int\limits_{\partial D \cap l} f(z + bt)\, dt = \int\limits_{\partial D \cap l} f(z_1 + b_1 t, \dots, z_n + b_n t)\, dt = 0. \qquad (4)$$

Then the function f is holomorphically extended into D up to a function $F \in \mathcal{C}(\overline{D})$. (If $\partial D \cap l = \varnothing$, then the integral in (4) is assumed to be equal to zero.)

We note that without the connectivity condition of the boundary of the domain, Theorem 4 is obviously false.

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We note that without the connectivity condition of the boundary of the domain, Theorem 4 is obviously false.

Functions with the Morera property

A generalization of this theorem is the following result.

Theorem 5

Let k be a fixed nonnegative integer and let a function $f \in \mathcal{C}(\partial D)$. If, for almost all $z \in \mathbb{C}^n$ and almost all $b \in \mathbb{CP}^{n-1}$, the condition

$$\int\limits_{\partial D \cap l} f(z_1 + b_1 t, \dots, z_n + b_n t) t^k \, dt = 0$$

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Functions with the Morera property

Definition 6

The condition

$$\int_{\partial D \cap l} f(z_1 + b_1 t, \ldots, z_n + b_n t) t^k \, dt = 0 \tag{5}$$

will be called the generalized Morera property.

Functions with the Morera property

The problem of finding *sufficient* sets of complex lines $\mathfrak{L} = \{l\}$ for which the condition (4) for $l \in \mathfrak{L}$ implies a holomorphic extension of the function f to D was posed by Globevnik, Stout. For example, is a set $\mathfrak{L}_V$ of lines l intersecting a certain open set $V \subset D$ such a sufficient set?

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Functions with the Morera property

Theorem 7

For a fixed k and a function $f \in \mathcal{C}(\partial D)$, let the condition (5) hold for almost all lines l (of the form (3)) intersecting an open set $V \subset D$ (or an open set $V \subset \mathbb{C}^n \setminus \overline{D}$), then the function f is holomorphically extended into D.

Corollary 8

Let A be an algebraic hypersurface in $\mathbb{C}^n$. If the condition (5) for a function f holds for almost all complex lines l intersecting A, then the function f is holomorphically extended into D.

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Functions with the Morera property

So let us consider a set of complex lines intersecting the germ of a real-analytic manifold of real dimension $(2n-2)$ to be a sufficient set.

Functions with the Morera property

Let $D \subset \mathbb{C}^n$ $(n > 1)$ be a bounded domain with a connected smooth boundary. Consider complex lines l of the form (3). Let Γ be the germ of a real-analytic manifold of real dimension $(2n-2)$. Let a domain $D \subset \mathbb{C}^n$ satisfy a conditions

$$\sum_{j=1}^{n} \frac{\partial \rho}{\partial \zeta_j} b_j \neq 0, \tag{6}$$

for the points z, lying in the neighbourhood of a manifold Γ such that $\partial D \cap \Gamma = \varnothing$.

Functions with the Morera property

Let $D \subset \mathbb{C}^n$ $(n > 1)$ be a bounded domain with a connected smooth boundary. Consider complex lines l of the form (3). Let $\varGamma$ be the germ of a real-analytic manifold of real dimension $(2n - 2)$. Let a domain $D \subset \mathbb{C}^n$ satisfy a conditions

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for the points z, lying in the neighbourhood of a manifold $\varGamma$ such that $\partial D \cap \varGamma = \varnothing$.

Functions with the Morera property

Theorem 9

Let a domain $D \subset \mathbb{C}^n$ satisfy a conditions (6) *and a function $f \in \mathcal{C}(\partial D)$ satisfy the generalized Morera property* (5), *i.e.,*

$$\int\limits_{\partial D \cap l} f(z_1 + b_1 t, \ldots, z_n + b_n t) t^k dt = 0 \tag{7}$$

for all $z \in \Gamma$, $b \in \mathbb{CP}^{n-1}$ and for a fixed integer non-negative number k. Then the function f has the holomorphically extension into the domain D.

Functions with the Morera property

For $k = 0$ the condition (7) takes us to the boundary Morera property

$$\int\limits_{\partial D \cap l} f(z_1 + b_1 t, \dots, z_n + b_n t)\, dt = 0. \tag{8}$$

Corollary 10

Let a domain D satisfy the conditions of Theorem 9, and a function $f \in \mathcal{C}(\partial D)$ satisfy the condition (8) for all $z \in \Gamma$ and $b \in \mathbb{CP}^{n-1}$, then f is holomorphically extended into the domain D.

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Functions with the Morera property

We also obtained an analog of the theorem 5 for domains with a piecewise-smooth boundary.

Theorem 11

Let D be a bounded domain in $\mathbb{C}^n$ $(n > 1)$ with a connected piecewise-smooth boundary and let k be a fixed nonnegative integer and let a function $f \in \mathcal{C}(\partial D)$. If, for almost all $z \in \mathbb{C}^n$ and almost all $b \in \mathbb{CP}^{n-1}$, the condition (5)

$$\int_{\partial D \cap l} f(z_1 + b_1 t, \ldots, z_n + b_n t) t^k \, dt = 0,$$

holds, then f is holomorphically extended into D.

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Let D be a bounded domain in $\mathbb{C}^n$ $(n > 1)$ *with a connected piecewise-smooth boundary and let k be a fixed nonnegative integer and let a function* $f \in \mathcal{C}(\partial D)$*. If, for almost all* $z \in \mathbb{C}^n$ *and almost all* $b \in \mathbb{CP}^{n-1}$*, the condition* (5)

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holds, then f is holomorphically extended into D.

Functions with the Morera property

We also obtained an analog of the theorem 7 for domains with a piecewise-smooth boundary.

Theorem 12

Let D be a bounded domain in $\mathbb{C}^n$ $(n > 1)$ with a connected piecewise-smooth boundary and let for a fixed k and a function $f \in \mathcal{C}(\partial D)$ the condition (5) hold for almost all lines l of the form (3) intersecting an open set $V \subset D$ (or an open set $V \subset \mathbb{C}^n \setminus \overline{D}$), then the function f is holomorphically extended into D.

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Functions with the one-dimensional holomorphic extension property

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Functions with the one-dimensional holomorphic extension property

Definition 13

The function $f \in \mathcal{C}(\partial D)$ has the one-dimensional holomorphic extension property along complex lines l of the form (3) if for any line l such that $\partial D \cap l \neq \varnothing$, there exists a function F having the following properties:

1. $F \in \mathcal{C}(\overline{D} \cap l)$,
2. $F = f$ on the set $\partial D \cap l$,
3. the function F is holomorphic at interior (with respect to the topology of l) points of the set $\overline{D} \cap l$.

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Functions with the Morera property

It is shown that for a class of continuous functions given on the boundary of a ball a family of complex lines passing through finite points in the ball will be a sufficient family.

Functions with the one-dimensional holomorphic extension property

Let $B = \{z \in \mathbb{C}^n : |z| < 1\}$ be a unit ball in $\mathbb{C}^n$ centered at the origin and let $S = \partial B$ be the boundary of the ball.

We denote by $\mathcal{A}$ a set of points $a_k \in B \subset \mathbb{C}^n$, $k = 1, \ldots, n+1$, lying outside the complex hyperplane in $\mathbb{C}^n$.

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Functions with the one-dimensional holomorphic extension property

Theorem 14

Let a function $f(\zeta) \in \mathcal{C}(S)$ has the one-dimensional holomorphic extension property along the family $\mathfrak{L}_{\mathcal{A}}$, then $f(\zeta)$ extends holomorphically into B.

Functions with the one-dimensional holomorphic extension property

In what follows we will need the definition of a domain with the Nevanlinna property. Let $G \subset \mathbb{C}$ be a simply connected domain and $t = k(\tau)$ be a conformal mapping of the unit circle $\Delta = \{\tau : |\tau| < 1\}$ on G.

Functions with the one-dimensional holomorphic extension property

Domain G is *a domain with the Nevanlinna property*, if there are bounded holomorphic functions u and v in G such that almost everywhere on $S = \partial\Delta$, the equality

$$\bar{k}(\tau) = \frac{u(k(\tau))}{v(k(\tau))}$$

holds in terms of the angular boundary values. Essentially this means

$$\bar{t} = \frac{u(t)}{v(t)} \quad \text{on} \quad \partial G.$$

Functions with the one-dimensional holomorphic extension property

Give a characterization of a domains with the Nevanlinna property. Domain G is a domain with the Nevanlinna property if and only if $k(\tau)$ admits a holomorphic pseudocontinuation through S in $\overline{\mathbb{C}} \setminus \overline{\Delta}$, i.e., there are bounded holomorphic functions u_1 and v_1 such that the function $\tilde{k}(\tau) = \dfrac{u_1(\tau)}{v_1(\tau)}$ coincides almost everywhere with the function $k(\tau)$ on S.

Functions with the one-dimensional holomorphic extension property

In our further consideration we will need the domain G to possess *the strengthened Nevanlinna property*, that is the function $u_1(\tau) \neq 0$ in $\mathbb{C} \setminus \Delta$ and $\tilde{k}$ has at infinity zero of no more than first order. If $G = \Delta$ then $\overline{\tau} = \dfrac{1}{\tau}$ on $\partial\Delta$. Therefor meromorphic function $\dfrac{1}{\tau}$ has a zero of the first order at ∞.

For example, such domain will include domains for which $k(\tau)$ is a rational function with no poles on $\overline{\Delta}$ and no zeros in $\mathbb{C} \setminus \Delta$.

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Functions with the one-dimensional holomorphic extension property

We denote by $\mathcal{A}$ the set of points $a_k \in D \subset \mathbb{C}^n$, $k = 1, \ldots, n+1$, which do not lie on the complex hyperplane in $\mathbb{C}^n$.

Theorem 15

Let D be a bounded strictly convex circular domain with twice smooth boundary in $\mathbb{C}^n$ and possess the strengthened Nevanlinna property in the points from the set $\mathcal{A}$ and the function $f(\zeta) \in \mathcal{C}(\partial D)$ have the one-dimensional holomorphic extension property along the family $\mathfrak{L}_{\mathcal{A}}$, then the function $f(\zeta)$ extends holomorphically into D.

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