ADVENTURES BEYOND POSSIBLE WORLDS

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I will survey a recent line of research [16] on generalizations of possible world semantics, building on the "possibility semantics" of Humberstone [17] and on general algebraic semantics for nonclassical logic. So far these investigations have focused on the following areas of logic:

Boolean algebra. The standard Stone duality for Boolean algebras is non-constructive, relying on the ultrafilter principle. Inspired by work on possibility semantics [11], a choice-free analogue of Stone duality for Boolean algebras has been developed in [6].

Modal logic. Humberstone [17] originally gave possibility semantics for propositional modal logic. His framework has been generalized and systematically investigated in [10, 11, 2, 19, 20]. Possibility semantics for first-order modal logic has also been developed in [1, 9, 16].

One motivation for going beyond possible world semantics is the existence of modal logics that cannot be characterized by any class of possible world frames (Kripke incompleteness). Modal incompleteness results for even more general semantics are covered in [14].

Modal logic with propositional quantifiers. Kripke incompleteness arises in an especially natural way with more expressive languages, such as the language of modal logic with propositional quantifiers. Algebraic semantics for this language are investigated in [13, 12, 7, 8].

Intuitionistic logic. Classical possibility semantics has been generalized to "nuclear semantics" for intuitionistic logic in [3, 4, 18].

Inquisitive logic. Inquisitive logic aims to expand the purview of logic beyond the logic of statements to include the logic of questions. In [5], the choice-free Stone duality of [6] is used to give semantics for inquisitive logic on a classical base. In [15], the nuclear perspective of [4] is used to give semantics for inquisitive logic on an intuitionistic base.

In addition to an overview of this work, I will discuss several open problems and directions for future research.

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